Uncertainty determination of ERPs from LLR by parameter variation during data analysis

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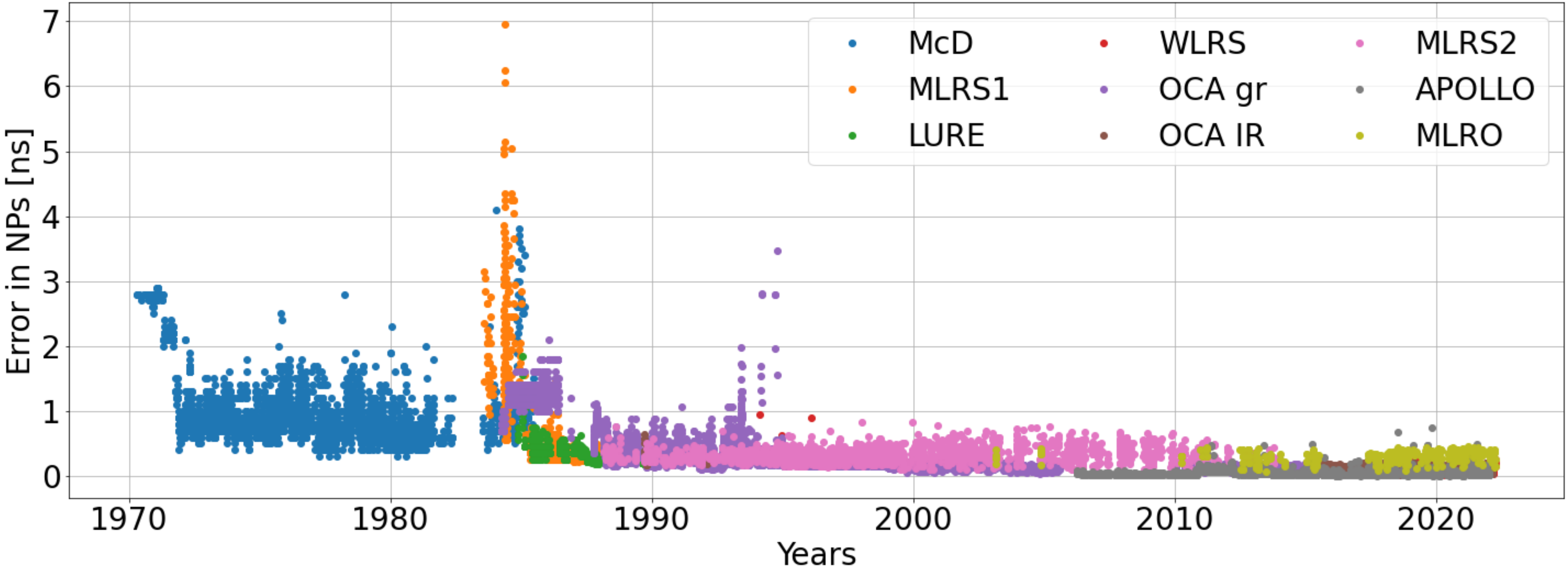
LLR Normal Points: Standard Deviation



30172 NPs (April 1970 - April 2022)

Mean Error (2022) = 0.10 ns ≈ 3.02 cm

Uncertainty estimation: Gauß-Markov Model

Observation vector $\mathbf{l}$,
parameter vector $\mathbf{x}$

$$\hat{\sigma}_0^2 = \frac{\mathbf{v}^T\mathbf{P}\mathbf{v}}{n-u},\ \mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \hat{\sigma}_0^2 \cdot (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}$$

$$\mathbf{x_0} = \hat{\mathbf{x}}$$

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big|_{\mathbf{l_0},\mathbf{x_0}},\ \mathbf{\Delta l} = \mathbf{l} - \mathbf{l_0}$$

pass

fail

iterate

$$\widehat{\mathbf{\Delta x}} = (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\mathbf{\Delta l}$$
$$\widehat{\mathbf{\Delta l}} = \mathbf{A}\,\widehat{\mathbf{\Delta x}}$$
$$\mathbf{v} = \widehat{\mathbf{\Delta l}} - \mathbf{\Delta l}$$
$$\hat{\mathbf{x}} = \mathbf{x_0} + \widehat{\mathbf{\Delta x}}$$
$$\hat{\mathbf{l}} = \mathbf{l_0} + \widehat{\mathbf{\Delta l}}$$

Check validation:

$$\hat{\mathbf{l}} - f(\hat{\mathbf{x}}) = 0$$

and define stop criterion

$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}$ = variance covariance matrix
Diagonal elements of $\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}$ = variance
Uncertainty (σ, on further slides) = square root of variance

[Alkhatib, 2021]
[Niemeier, 2008]

Uncertainty estimation: Gauß-Markov Model

For any estimated parameter 'x', $\boxed{x} \pm \boxed{\Delta x}$

estimated value

uncertainty

For a standard calculation*, Δx = 3σ

Personal communication, Prof. Peter Bender**, 1991:

- Correlations between NPs unaccounted
- Different uncertainty obtained for different groups of adjusted parameters
- Systematic errors

[Hofmann, 2018]
Franz Hofmann, Liliane Biskupek, Jürgen Müller *Contributions to reference systems from Lunar Laser Ranging using the IfE analysis model,* 2018. Journal of Geodesy, 92:975-987. doi:10.1007/s00190-018-1109-3

[Singh et al., 2021]
Vishwa Vijay Singh, Liliane Biskupek, Jürgen Müller, Mingyue Zhang. *Impact of non-tidal station loading in LLR,* 2021. Advances in Space Research, 67(12), 3925–3941. doi:10.1016/j.asr.2021.03.018.

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**https://www.iau.org/administration/membership/individual/2805/

Earth Rotation Parameters (ERP) from LLR

- Pre-analysis of NPs to identify best subsets
- A-priori ERP: IERS 14C04 series
  fixed for those nights that were not considered
- See Biskupek (2015) for partial derivatives calculation of ERPs
- For the adjusted non-ERP parameters, all LLR NPs used (see Singh et al. (2021) for a full list)

- Determination of either ΔUT1, x_p and y_p, only x_p, only y_p
- Recent results (study conducted with 28093 NPs):
  - Estimated ERPs: Singh and Biskupek (2022)
  - Article: Singh et al. (2022)

[Singh and Biskupek, 2022]
Dataset: Earth Rotation Parameters from LLR with NPs for timespan 1970 - 2021, Research data repository of the Leibniz University Hannover. doi:10.25835/3h1r07a7.
[Singh et al., 2022]
Vishwa Vijay Singh, Liliane Biskupek, Jürgen Müller, Mingyue Zhang. *Earth rotation parameter estimation from LLR, 2022. Advances in Space Research*, 70(8), 2383-2398. doi:10.1016/j.asr.2022.07.038.
[Biskupek et al., 2022]
Liliane Biskupek, Vishwa Vijay Singh Jürgen Müller. *Estimation of Earth Rotation Parameter UT1 from Lunar Laser Ranging Observations,* 2022. In: *International Association of Geodesy Symposia*, Springer, Berlin, Heidelberg. doi:10.1007/1345_2022_178.

ERP from LLR: Subset Definition

Subset	Explanation	Number of nights	Time span
all_10	Nights selected with NPs from all LLR observatories, with at least 10 NPs per night, from at least one observatory	971	30.09.1983 - 13.03.2022
all2_10	Nights selected with NPs from all LLR observatories, with at least 10 NPs per night, from <u>at least two observatories</u>	370	09.04.1984 - 10.02.2022
all_15	Nights selected with NPs from all LLR observatories, with at least 15 NPs per night, from at least one observatory	491	09.04.1984 - 13.03.2022
all2_15	Nights selected with NPs from all LLR observatories, with at least 15 NPs per night, from <u>at least two observatories</u>	212	09.04.1984 - 10.02.2022

370 best nights

491 best nights

212 best nights

Uncertainty: ERPs

Cases:

1: A-priori values of velocity of LLR observatories from standard solution

2: A-priori values of velocity of LLR observatories from (and fixed to) ITRF2020*

Sub-cases:

.1: Non-ERP + ERP adjusted

.2: Only ERP adjusted

$x_{1.1} \pm \Delta x_{1.1}$

$x_{1.2} \pm \Delta x_{1.2}$

$x_{2.1} \pm \boldsymbol{\Delta x_{2.1}}$

$x_{2.2} \pm \Delta x_{2.2}$

For each case $\rightarrow 3\sigma$

range = max - min

$3\sigma \approx$ range?

*APOLLO velocity fixed to GPS station 'P027'

Uncertainty: xp

Units: mas

Subset	Time Span	3σ (1.1)	3σ (1.2)	3σ (2.1)	3σ (2.2)
all_10	<2000.0	18.12	17.98	18.50	18.35
	>2000.0	1.72	1.71	1.75	1.74
all2_10	<2000.0	13.78	13.70	14.02	13.95
	>2000.0	1.90	1.89	1.93	1.92
all_15	<2000.0	15.36	15.14	15.67	15.44
	>2000.0	1.40	1.39	**1.42**	1.41
all2_15	<2000.0	9.41	9.34	9.57	9.51
	>2000.0	1.30	1.29	**1.32**	1.31

Units: mas

Range
1.46
0.37
1.43
0.27
1.11
0.22
0.69
0.22

xp and yp adjusted separately

Slightly better results when adjusting **without** LLR parameters

3σ > Range

Values in table = WRMS (weighted based on number of NPs per night)

xp and yp estimated separately

Uncertainty: yp

Units: mas

Subset	Time Span	3σ (1.1)	3σ (1.2)	3σ (2.1)	3σ (2.2)
all_10	<2000.0	11.78	11.60	12.03	11.85
	>2000.0	2.15	2.14	2.19	2.19
all2_10	<2000.0	9.99	9.93	10.18	10.12
	>2000.0	1.94	1.92	1.97	1.96
all_15	<2000.0	11.29	11.14	11.53	11.37
	>2000.0	1.74	1.73	**1.77**	1.77
all2_15	<2000.0	9.53	9.47	9.71	9.65
	>2000.0	1.50	1.49	**1.53**	1.52

Units: mas

Range
2.58
0.45
0.87
0.29
1.40
0.23
0.69
0.21

xp and yp adjusted separately

Slightly better results when adjusting **without** LLR parameters

3σ > Range

Values in table = WRMS (weighted based on number of NPs per night)

xp and yp estimated separately

Uncertainty: ΔUT1

Units: μs

Subset	Time Span	3σ (1.1)	3σ (1.2)	3σ (2.1)	3σ (2.2)
all_10	<2000.0	128.34	137.91	131.05	143.56
	>2000.0	22.13	23.88	22.58	24.86
all2_10	<2000.0	96.04	98.24	97.80	100.30
	>2000.0	21.16	21.70	21.55	22.16
all_15	<2000.0	115.52	118.36	117.90	120.63
	>2000.0	18.17	18.69	**18.53**	19.04
all2_15	<2000.0	85.93	87.75	87.47	89.54
	>2000.0	16.13	16.44	**16.42**	16.77

Units: μs

Range
188.41
18.34
125.30
18.56
74.79
11.96
88.16
12.25

worst case section of nights, 3σ < Range

Slightly better results when adjusting with LLR parameters

3σ ≈ Range (overall)

best case section of nights, 3σ > Range

Values in table = WRMS (weighted based on number of NPs per night)

Uncertainty: Other parameters

Cases (indicating fixed and adjusted parameters; ERP not estimated in any case):

1. Standard solution (see Singh et al. (2021) for a full list of adjusted parameters)

   The following cases indicate the modifications made to the standard solution:

2. Reflector coordinates fixed
3. Station coordinates fixed
4. Station velocities fixed
5. Station biases fixed
6. Station coordinates + velocities + biases fixed
7. Station coordinates + velocities fixed
8. All dynamical parameters fixed

Results: 3σ > Range

Conclusions and Further Studies

- Adjusting different subsets of parameters
  - Change between adjusted $\Delta UT \approx$ adjustment errors
  - Uncertainty of ΔUT from LLR should be given as three times obtained standard deviation (3σ) from LSA
  - 2σ uncertainty for ΔUT in future?
- 1σ uncertainty for all standard parameters (not shown) and for terrestrial pole coordinates sufficient

- Best results (all2_15, after 2000.0):
  xp and yp estimated separately
  ΔUT (3σ) = 16.42 µs, xp (1σ) = 0.44 mas, yp (1σ) = 0.51 mas
  ~ 7.55 mm, ~ 1.32 cm, ~ 1.53 cm

- For relativistic parameters, similar tests will be performed

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