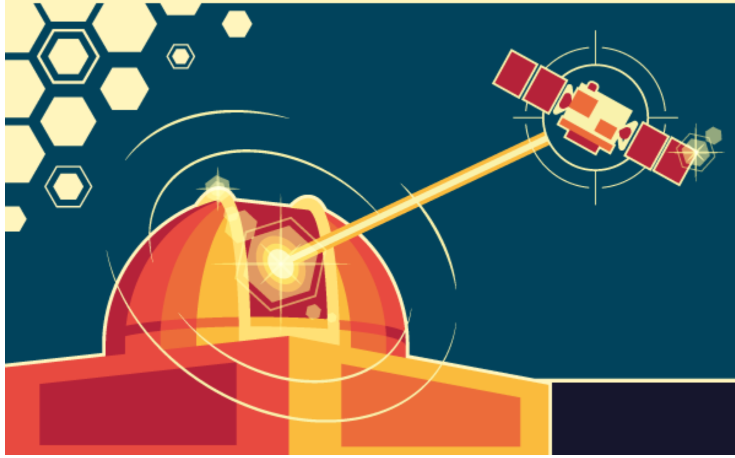

22ND INTERNATIONAL WORKSHOP ON LASER RANGING

7-11 November 2022
Yebes, Spain

RECONNECTING THE ILRS COMMUNITY

Fundamental Physics results in testing Gravitation with Laser-Ranged satellites: the LARASE and SaToR-G experiments

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LARASE
LASERRANGEDSATELLITESEXPERIMENT

SaToR-G

Guadalajara, 8th november 2022

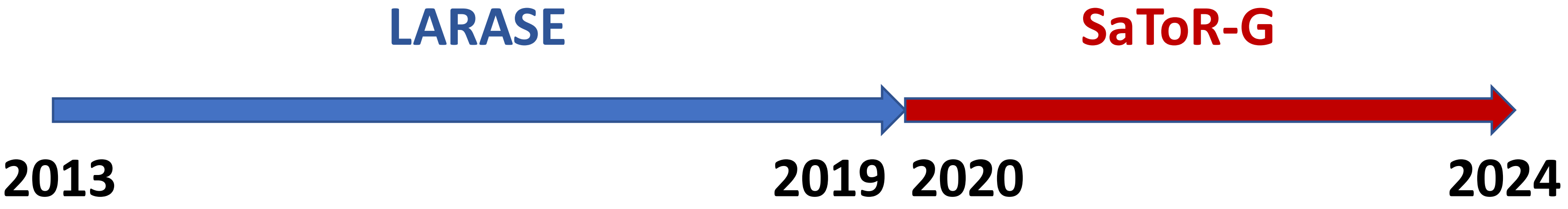
INTRODUCTION

The **LARASE** (2013-2019) and **SaToR-G** (started on 2020) are two experiments, funded by the Italian National Institute for Nuclear Physics (**INFN-CSN2**), devoted to measurements of the gravitational interaction in the **Weak-Field** and **Slow-Motion** limit of **General Relativity** by means of laser tracking to geodetic passive satellites orbiting around the Earth.

A main point of these activity is the modeling of both gravitational and non-gravitational perturbations.

Today we will present:

- Measurement of Lense-Thirring effect,
- Preliminary measurement to constrain Yukawa-like interactions.



LARASE

SaToR-G

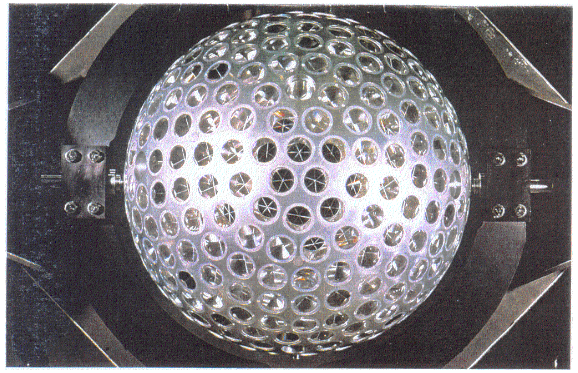
2013 **2019** **2020** **2024**

EXPERIMENTAL FRAMEWORK

The predictions of **GR** on the orbits of **geodetic satellites**, which play the role of **test masses**, are compared with measured ones.

Parameter	Unit	Symbol	LAGEOS	LAGEOS II	LARES
Semi-major axis	km	a	12 270.00	12 162.08	7 820.31
Eccentricity	-	e	0.0044	0.0138	0.0012
Inclination	deg.	i	109.84	52.66	69.49
Radius	cm	R	30.0	30.0	18.2
Mass	kg	M	406.9	405.4	383.8
Area/Mass	m^2/kg	A/M	6.94×10^{-4}	6.97×10^{-4}	2.69×10^{-4}


LAGEOS (NASA, 1976)


LAGEOS II (ASI/NASA, 1992)

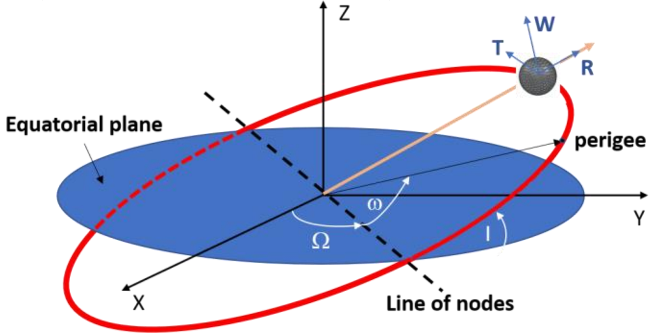

LARES (ASI, 2012)

THE LENSE-THIRRING PRECESSION

The so-called **Lense-Thirring** effect consists of a precession of the orbit of a satellite around a primary produced by its rotation, i.e. by its **angular momentum *J*** (**mass currents**).

This precession produces a secular effect in two orbital elements:

- the right ascension of the ascending node (RAAN), Ω
- the argument of pericenter, ω



$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \mu \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{sec} = -\mu \frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$

$$\mu = \begin{cases} 1 \text{ in General Relativity} \\ 0 \text{ in Newtonian physics} \end{cases}$$

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}_{LT}$	+30.67	+31.50	+118.48
$\dot{\omega}_{LT}$	+31.23	-57.31	-334.68

$J_\oplus$	is the source of the effect
G, c	are two fundamental constants of nature
a, e, i	mean Keplerian elements

SYSTEMATIC ERRORS

Among the main perturbations to consider we have:

- **The gravitational perturbations**
  - Earth’s gravitational field
  - Tides
    - Ocean
    - Solid
  - General relativity

Main source of error

- **The non-gravitational perturbations**
  - Thermal thrust
  - …

SYSTEMATIC ERRORS

- The **Lense-Thirring** precession is very small compared to the classical precession of the orbit due to the deviation from the spherical symmetry for the distribution of the Earth's mass, or even compared to the same relativistic **Schwarzschild** precession produced by the mass of the primary (≈ 3350 mas/yr for **LAGEOS**)

$$\boxed{\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \mu \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}}$$

$$\boxed{\dot{\Omega}^{Class} \cong -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{\left(1-e^2\right)^2} \left\{ J_2 + J_4 \left[\frac{5}{8} \left(\frac{R_\oplus}{a}\right)^2 \left(7\sin^2 I - 4\right) \frac{\left(1+\frac{3}{2}e^2\right)}{\left(1-e^2\right)^2} \right] + \ldots \right\}}$$

Lense-Thirring

Classic

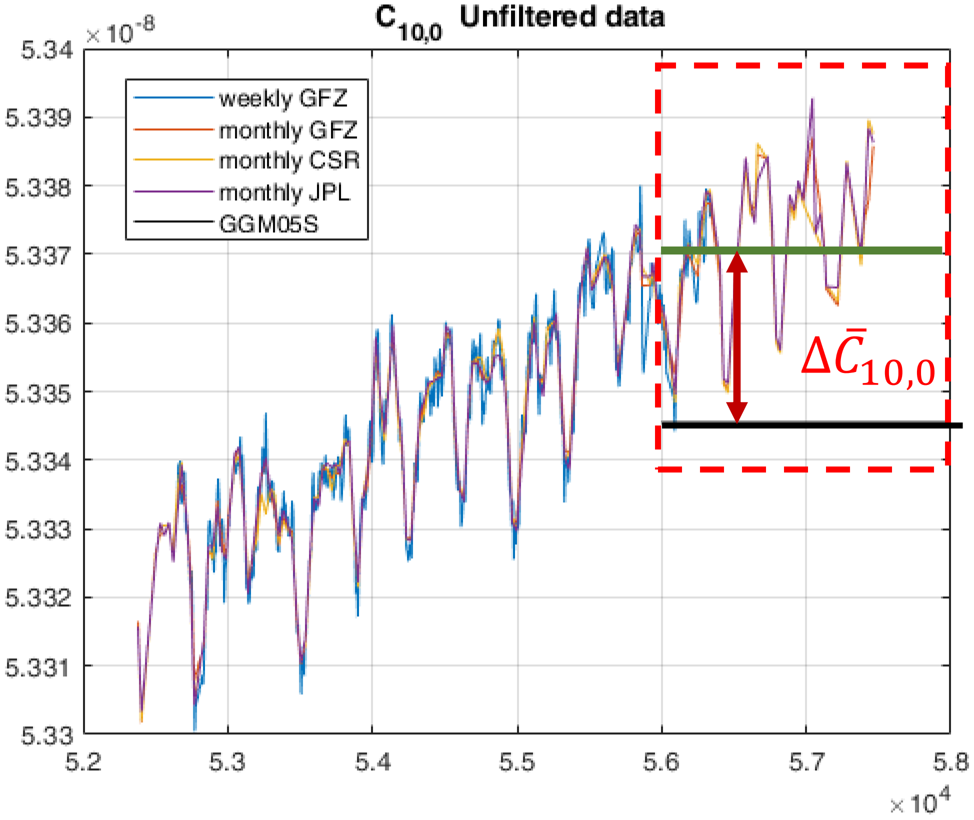
Therefore, the **correct modelling of the even zonal harmonics** (ℓ = even, m = 0) represents the main challenge in this kind of measurements, since they have the same signature of the relativistic effect but much larger amplitudes. **These harmonics are the main sources of systematic errors**

THE MODELING OF THE EVEN ZONAL HARMONICS OF THE GRAVITATIONAL FIELD

- We considered several static models for the background gravitational field of the Earth

- To **reduce the impact** of the harmonics, we modeled the first **10** even zonal harmonics exploiting their significant time dependency as well evidenced by their **Temporal Solutions** (**TS**) provided by the **GRACE** (NASA/DLR) mission

$\bar{C}_{10,0}$



PRECISE ORBIT DETERMINATION

The data reduction of the satellites orbit has been done with **GEODYN II** (NASA/GSFC) on a time span of about 6.5 years (2359 days) from **MJD 56023**, that is from April 6th 2012, and we computed the residuals on the orbit elements of **LAGEOS**, **LAGESOS II** and **LARES**:

- Background gravity model: GGM05S + other fields from GRACE
- Arc length of 7 days
- No empirical accelerations
- Thermal thrust effects (Yarkovsky Schach and Rubincam) not modelled
- General relativity modelled with the exception of the Lense-Thirring effect

1. EIGEN-GRACE02S (2004)
2. GGM05S (2014)
3. ITU_GRACE16 (2016)
4. Tonji-Grace02s (2017)

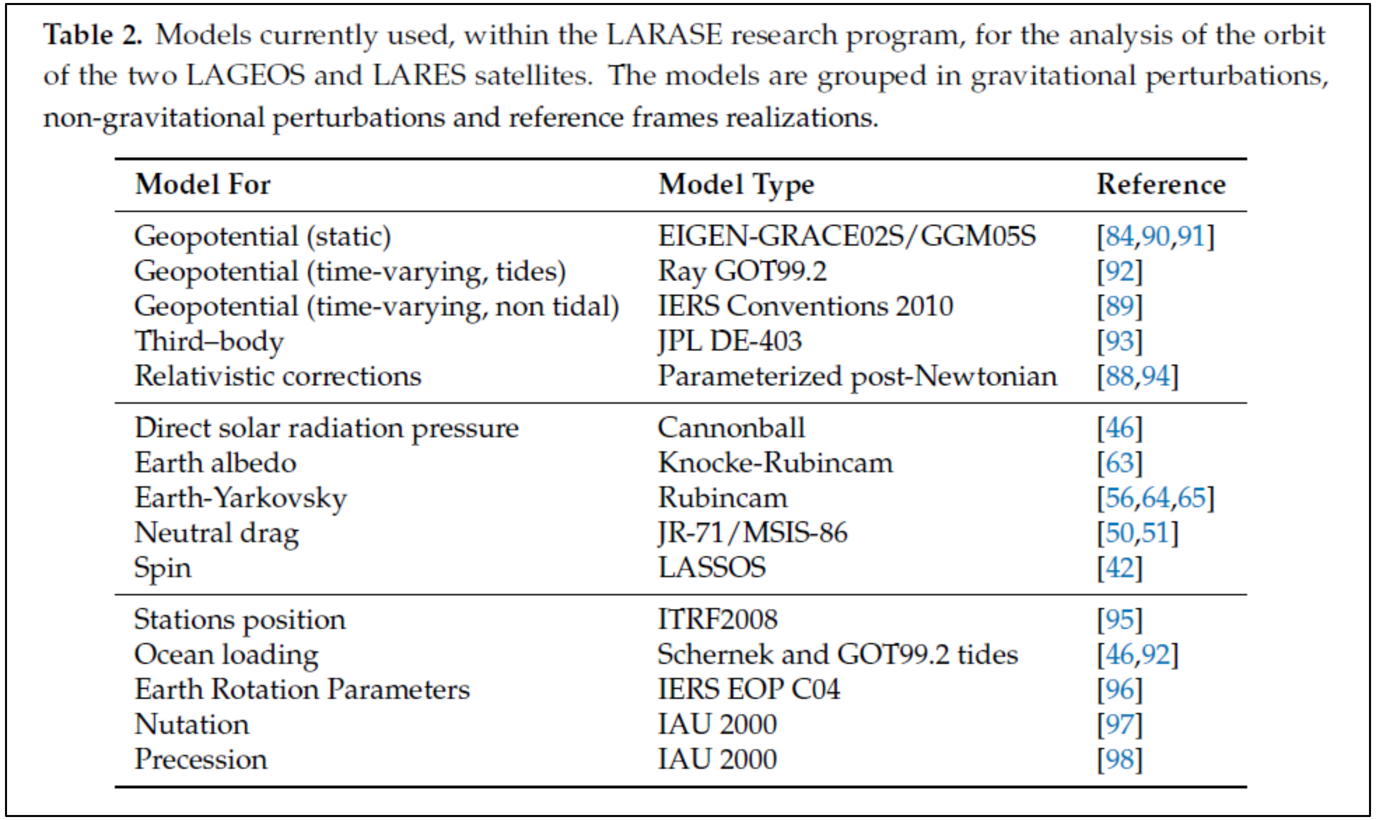


Table 2. Models currently used, within the LARASE research program, for the analysis of the orbit of the two LAGEOS and LARES satellites. The models are grouped in gravitational perturbations, non-gravitational perturbations and reference frames realizations.

Model For	Model Type	Reference
Geopotential (static)	EIGEN-GRACE02S/GGM05S	[84,90,91]
Geopotential (time-varying, tides)	Ray GOT99.2	[92]
Geopotential (time-varying, non tidal)	IERS Conventions 2010	[89]
Third–body	JPL DE-403	[93]
Relativistic corrections	Parameterized post-Newtonian	[88,94]
Direct solar radiation pressure	Cannonball	[46]
Earth albedo	Knocke-Rubincam	[63]
Earth-Yarkovsky	Rubincam	[56,64,65]
Neutral drag	JR-71/MSIS-86	[50,51]
Spin	LASSOS	[42]
Stations position	ITRF2008	[95]
Ocean loading	Schernek and GOT99.2 tides	[46,92]
Earth Rotation Parameters	IERS EOP C04	[96]
Nutation	IAU 2000	[97]
Precession	IAU 2000	[98]

ANALYSIS METHOD

- By solving a linear system of three equations in three unknowns, we can solve for the relativistic precession while reducing the impact in the measurement of the non perfect knowledge of the Earth’s gravitational field:

$$\boxed{\begin{aligned} \dot{\Omega}_2^{L1}\delta J_2 + \dot{\Omega}_4^{L1}\delta J_4 + \dot{\Omega}_{LT}^{L1}\mu + \cdots &= \delta\dot{\Omega}_{res}^{L1} \\ \dot{\Omega}_2^{L2}\delta J_2 + \dot{\Omega}_4^{L2}\delta J_4 + \dot{\Omega}_{LT}^{L2}\mu + \cdots &= \delta\dot{\Omega}_{res}^{L2} \\ \dot{\Omega}_2^{LR}\delta J_2 + \dot{\Omega}_4^{LR}\delta J_4 + \dot{\Omega}_{LT}^{LR}\mu + \cdots &= \delta\dot{\Omega}_{res}^{LR} \end{aligned}}$$

$$(\mu, \delta J_2, \delta J_4)$$

$$\dot{\Omega}_{GR}^{comb} = 50.17\ mas/yr$$

$$\boxed{\dot{\Omega}^{comb} = \delta\dot{\Omega}_{res}^{L1} + k_1\delta\dot{\Omega}_{res}^{L2} + k_2\delta\dot{\Omega}_{res}^{LR}}$$

$$\mu = \frac{\dot{\Omega}^{comb}}{\dot{\Omega}_{GR}^{comb}} = \begin{cases} 1 \text{ in General Relativity} \\ 0 \text{ in Newtonian physics} \end{cases}$$

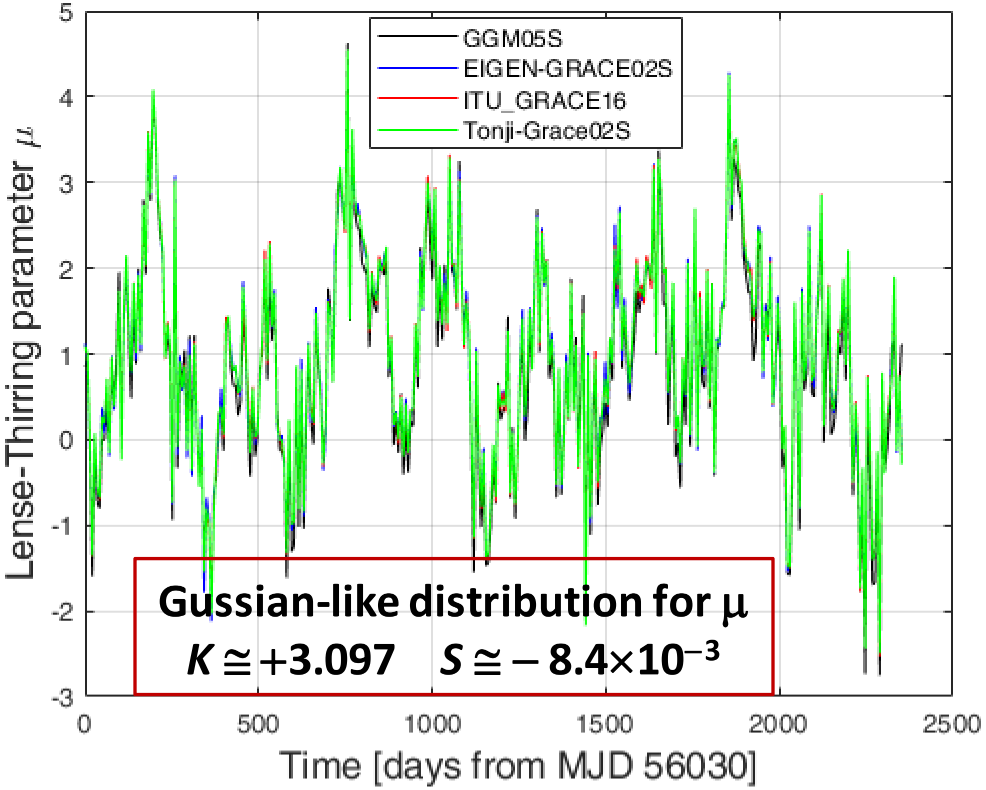
$k_1 \cong 0.345$

$k_2 \cong 0.073$

LENSE-THIRRING MESUREMENT

- Different models for the gravitational field were considered: GGM05S, EIGEN-GRACE02S, ITU_GRACE16, Tonj-Grace02S

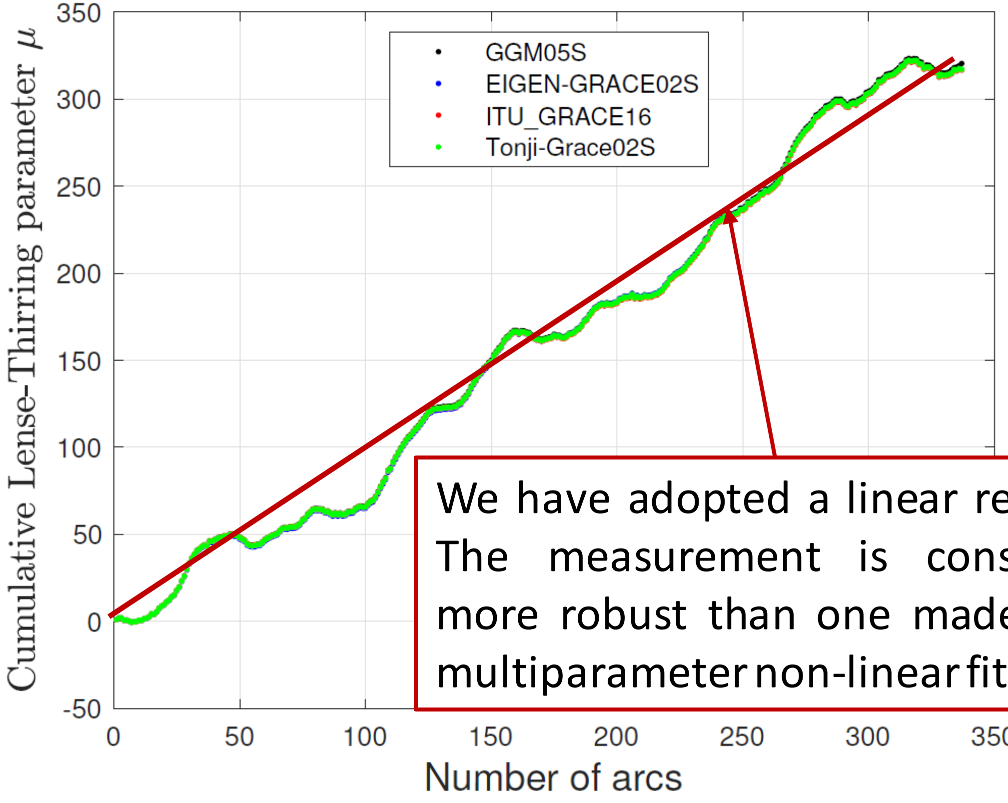
Results for μ from the linear system



Gussian-like distribution for μ

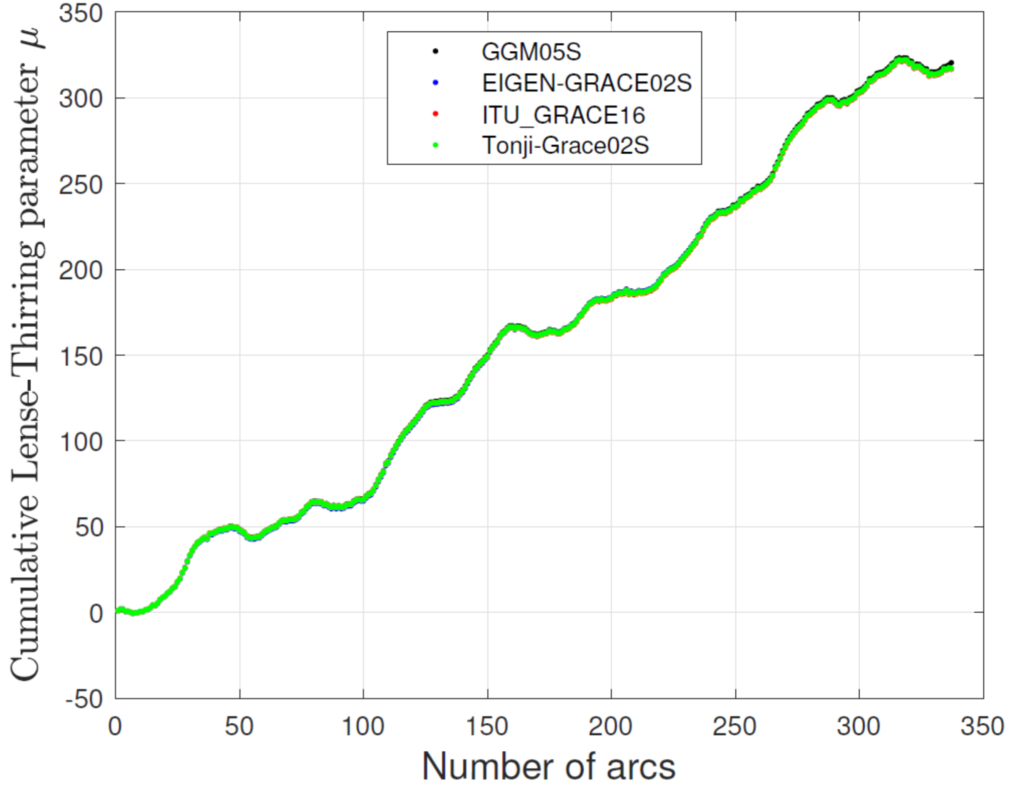
$K \cong +3.097 \quad S \cong -8.4\times10^{-3}$

Cumulative sum for μ



We have adopted a linear regression. The measurement is consequently more robust than one made using a multiparameter non-linear fit.

LENSE-THIRRING MEASUREMENT



$$\mu_{meas} - 1 = 1.5 \times 10^{-3} \pm 7.4 \times 10^{-3} \pm 16 \times 10^{-3}$$

Model	$\mu \pm \delta\mu$	$\mu - 1$
GGM05S	1.0053 ± 0.0074	$+0.0053$
EIGEN-GRACE02S	1.0002 ± 0.0074	$+0.0002$
ITU_GRACE16	0.9996 ± 0.0074	-0.0004
Tonji-Grace02s	1.0008 ± 0.0074	$+0.0008$

Perturbations	$\delta\mu_{sys}$ [%]
Gravitational field	1.0
Tides	0.6
Periodic effect	1.0
De Sitter effect	0.3
RSS	1.6
SAV	2.9

Errors @ 95% CL

This is indeed a very **precise** and **accurate** measurement

D. Lucchesi et al: *An improved measurement of the Lense-Thirring precession on the orbits of laser-ranged satellites with an accuracy approaching the 1% level*, arXiv:1910.01941, oct 2019

D. Lucchesi et al.: *1% Measurement of the Gravitomagnetic Field of the Earth with Laser-Tracked Satellites*, Universe **2020**, 6, 139

SaToR-G EXPERIMENT

- The analysis of the satellites' orbits lets to measure some gravitational effects and put constrains on different gravitation theories. The main possible measurements are:
  - Constrains on long range Yukawa-like interations (5th force, …)
  - **PPN** parameters and their combinations: β, γ, α_1, α_2
  - Relativistic precessions and not linear gravitational interation
  - **EEP** and its strong formulation (**SEP** and Nordtvedt effect: $\eta_n = 4\beta - \gamma - 3$)

In General Relativity

$\boldsymbol{\beta} = 1$
$\boldsymbol{\gamma} = 1$
$\boldsymbol{\alpha_1} = \boldsymbol{\alpha_2} = 0$
$\boldsymbol{\eta_n} = 0$

- The final goal is to perform precise and accurate measurements, i.e. to valuate the systematic errors affecting the measurements to get meaningful constrains on the different theories.

YUKAWA-LIKE INTERATION

- A Yukawa-like potential produces a radial acceleration $\Re = -\frac{G_\infty M_\oplus}{a^2}\left(\frac{a}{r}\right)^2 \alpha\left(1+\frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}$ that gives secular effect only on two orbital parameters. The effects are function of the mean orbital parameters a,e, of the true anomaly f and of the mean motion n.

Argument of pericenter

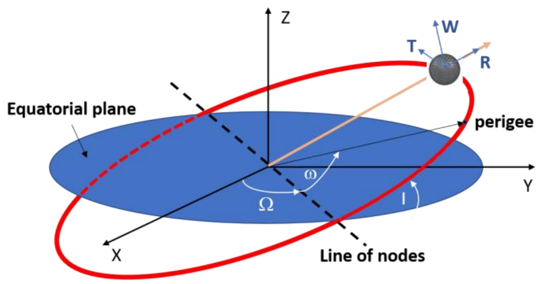
$$\dot{\omega}(\alpha,\lambda) = -\frac{\sqrt{1-e^2}}{n\,a\,e}\Re\cos f$$

Mean anomaly

$$\dot{M}(\alpha,\lambda) = n + \frac{1}{na}\Re\left(\frac{\cos u(f,e)}{e(1-e^2)} - \sqrt{1-e^2}\sin f\,\sin u(f,e) + 2\,\frac{(1-e^2)}{(1+e\cos f)}\right)$$

- The effect of this interaction must be compared with the precession predicted by General Relativity

RELATIVISTIC PRECESSIONS



Argument of pericenter

$$\dot{\omega}_{Schw} = \frac{3\,(GM_{\oplus})^{3/2}}{c^2\,a^{5/2}(1-e^2)}$$

$$\dot{\omega}_{LT} = -\frac{6G}{c^2a^3}\frac{J_{\oplus}}{(1-e^2)^{3/2}}\cos i$$

$$\dot{\omega}_{J_2}^{indir} = -\frac{3n^3J_2R_{\oplus}^2}{8c^2}\frac{5(1+e^2)-(17+25e^2)cos^2 i}{(1-e^2)^3}$$

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\dot{\omega}_{Schw}$	+3278.78	+3352.58	+10110.15
$\dot{\omega}_{LT}$	+31.23	-57.33	-124.53
$\dot{\omega}_{J2}$	-3.62	+2.69	-26.03
Total	**+3306.38**	**+3352.58**	**+9959.59**

$$\dot{\omega}_{J_2}^{dir} = \frac{3n^3J_2R_{\oplus}^2}{8c^2(1-e^2)^{7/2}}\left[30+19\sqrt{1-e^2}+6e^2\left(-5+6\sqrt{1-e^2}\right)+\left(9\left(6+5\sqrt{1-e^2}\right)+6e^2\left(-9+10\sqrt{1-e^2}\right)\right)\cos i\right]$$

Mean anomaly

$$\dot{M}_{Schw} = -\sqrt{1-e^2}\,\dot{\omega}_{Schw}$$

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\dot{M}_{Schw}$	-3278.75	-3352.26	-10110.14
$\dot{M}_{J_2 rel}$	-0.92	$+0.15$	-6.71
Total	**-3279.67**	**-3352.11**	**-10116.85**

MEASUREMENT OF PERIGEE AND MEAN ANOMALY

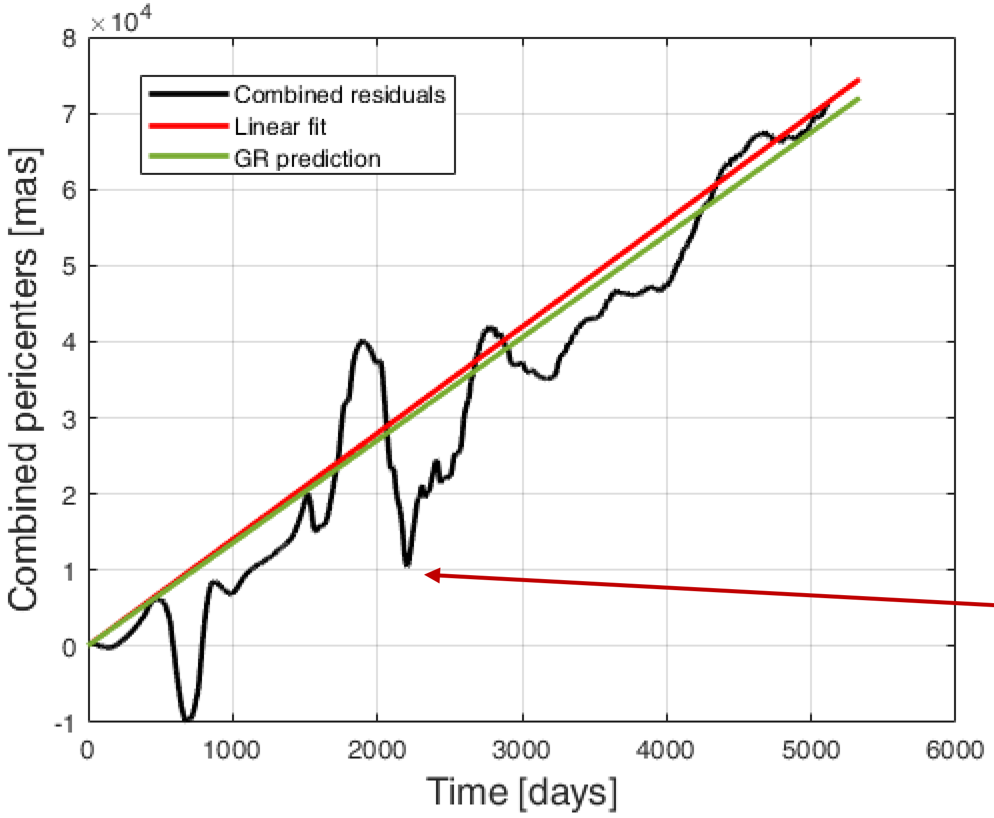
We are therefore interested in new analyzes of the long-term and secular effects on the orbits of the two **LAGEOS** and (possibly) of **LARES** to further constrain a possible **long-range force** described by a **Yukawa-like** potential

- Improve the results of a previous measurement (2014) obtained with LAGEOS II argument of pericenter including LAGEOS
- Perform the analysis over the entire life of LAGEOS II, about 28 years
- Prefer linear fits to non-linear ones
- Compare the results with the predictions of **GR** and of other **ATG**

D. Lucchesi, R. Peron, *LAGEOS II pericenter general relativistic precession (1993-2005): Error budget and constraints in gravitational physics*. Phys. Rev. D 89, 082002, doi:10.1103/PhysRevD.89.082002, 2014

PRELIMINARY ANALYSIS: ARGUMENTS OF PERICENTER LAGEOS AND LAGEOS II

- The analysis was done on a time span of 13.7 years to reduce systematic errors introduced by gravitational field
- The use of two observables allows to cancel the errors due to J_2



$$\dot{\omega}^{L1}_{res} + k\dot{\omega}^{L2}_{res} \qquad k \cong +0.489594$$

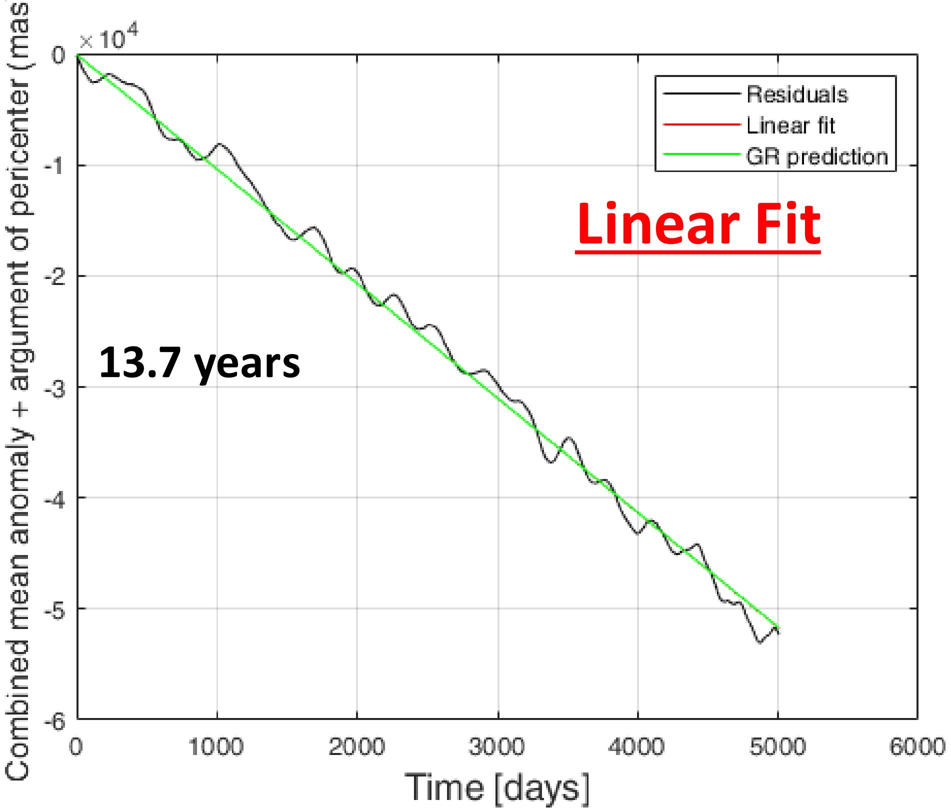
$$\dot{\omega}_{tot} = \boldsymbol{\varepsilon}\dot{\omega}_{GR} + \dot{\omega}_{GP} + \dot{\omega}_{NGP} + \cdots$$

$$\boldsymbol{\varepsilon - 1 \cong 3 \times 10^{-2} \pm 1 \times 10^{-2} \pm \ldots}$$

But the unmodelled thermal thrust effects are too large on LAGEOS

PRELIMINARY ANALYSIS: LAGEOS II ARGUMENT OF PERICENTER AND MEAN ANOMALY

- The analysis was done on a time span of 13.7 years to reduce systematic errors introduced by gravitational field
- The use of two observables allows to cancel the errors due to J_2



Linear Fit

13.7 years

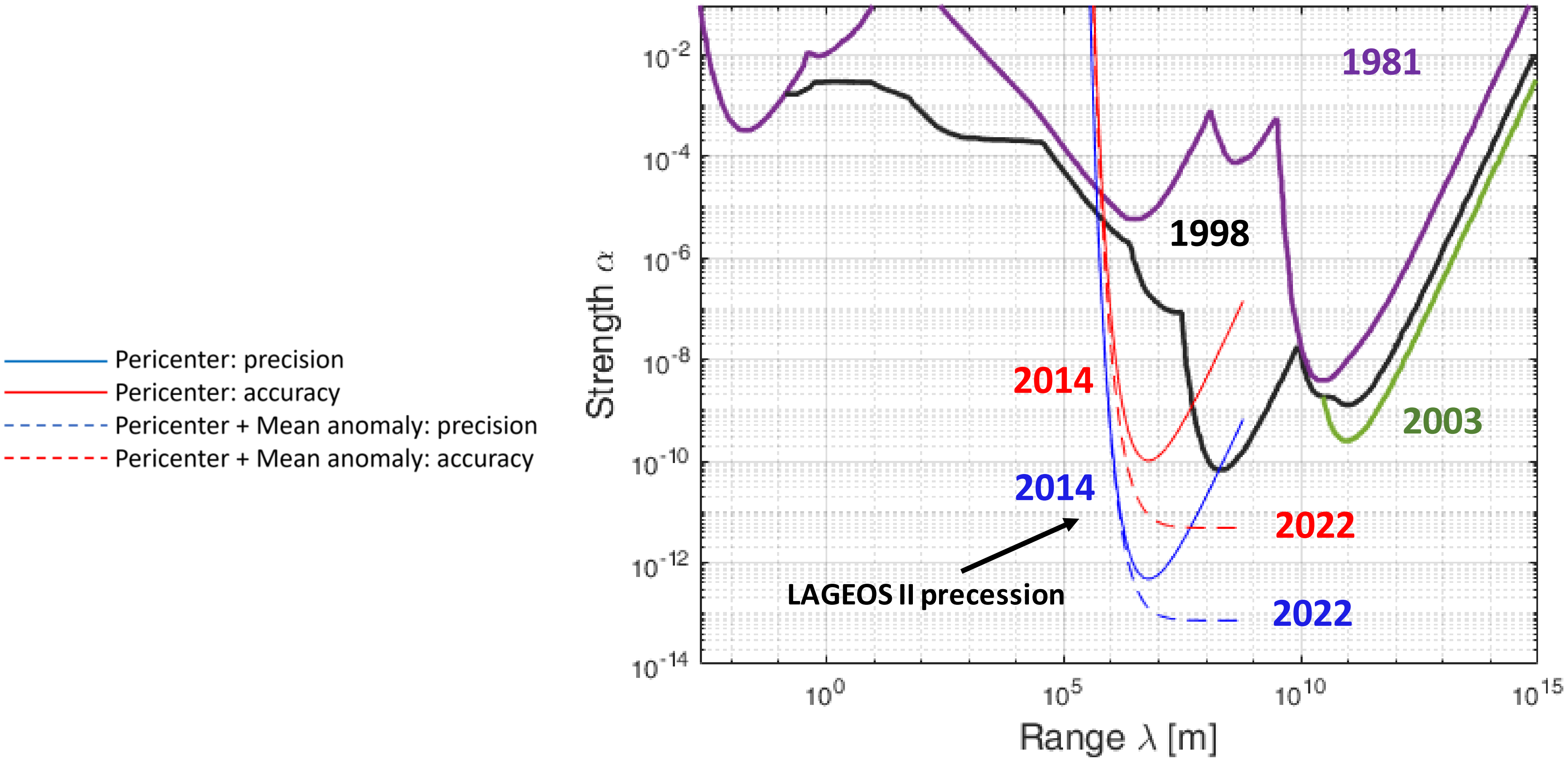
$$\dot{M}^{L2}_{res} + k\dot{\omega}^{L2}_{res} \qquad k \cong -0.123500$$

$$\boxed{\varepsilon - 1 \cong (+0.35 \pm 2.42) \times 10^{-3} \pm 0.8 \cdot 10^{-2}}$$

A previous measurement in 2014 was made using a no linear fit:

$$\varepsilon - 1 = (-0.12 \pm 2.10) \cdot 10^{-3} \pm 2.5 \cdot 10^{-2}$$

COMPARISATION WITH PREVIOUS RESULTS



CONCLUSIONS

- The LARASE (2013-2019) and SaToR-G (2020-) experiments, funded by the Italian National Institute for Nuclear Physics (INFN-CSN2), were devoted to measurements of the gravitational interaction in the Weak-Field and Slow-Motion limit of General Relativity by means of laser tracking to geodetic passive satellites orbiting around the Earth.

- One of the main objectives of the LARASE was a robust and reliable measurement of the Einstein-Thirring-Lense precession. This objective was achieved after a careful evaluation of the main sources of systematic error, in particular those due to the even zonal harmonic coefficients (ℓ = even and $m = 0$) using GRACE's time solutions.

- The goal of SaToR-G is to place constraints on different theories of gravitation beyond General Relativity.

- A preliminary measurement to constrain Yukawa-like interactions (deviation from $1/r^2$ dependance) was presented.

- A further important activity carried out by LARASE and SaToR-G is to improve the dynamic model of the three satellites, in particular that of the main non-conservative forces acting on their surface.

MANY THANKS FOR YOUR KIND ATTENTION