Processing of SLR observations with an optimal Wiener filter – an alternative way to calculate normal points

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Applying iterative editing criteria in the calculation of normal points is a standard procedure at many ILRS stations. However recent quality analysis has shown systematic dependencies when plotting rms versa residual of the resulting normal points. A thorough investigation of SOS-W single photon data, showing clear satellite signature effects, revealed the iterative editing procedure as the underlying error source, since it's convergence properties are dominated by the skew nature of the data distribution. In order to eliminate the systematic errors inherent in the iterative editing procedure, a normal point algorithm based on an optimal Wiener or deconvolution filter has been set up. By deconvolving the satellite dependent signature effect from the measured signal, the proposed algorithm permits to reach rms values comparable to those obtained from calibration measurements to a flat target. In the present paper the algorithm is used to reprocess data from some geodetic satellites gathered in 2017 and the resulting bias series are compared to those obtained from the standard iterative procedure.

1. Introduction

The motivation of this study was triggered by Toshimichi Otsubo's systematic bias analysis presented at the Riga Workshop in 2017 [1]. Figure 1 shows a sample plot of the Satellite Observing System Wettzell (SOS-W), where a clear systematic effect is visible when plotting normal point residuals versa normal point rms. For LAGEOS data gathered with the SOS-W this systematic effect matches a linear trend of 1mm residual per 1mm rms. Analyzing our on site normal point residual data gathered from LAGEOS1 and LAGEOS2 in 2017, the same trend can be found and leads to the suspicion that this behavior can be attributed to the convergence properties of the iterative data clipping procedure incorporated in the normal point algorithm. To overcome this error source, a new normal point procedure based on an optimal Wiener[2] or deconvolution filter has been designed and tested with the advantage of getting rid of the iterative clipping procedure and apply spectral techniques instead to filter the raw measurements when forming the normal point.

2. A new normal point algorithm

In order to apply a spectral filtering technique like a Wiener filter to SLR data, the residuals have to be calculated with great care to avoid any remaining trend. Therefore the raw data, after applying a coarse 5 sigma filter, is subject to a short arc fit adjusting long track and

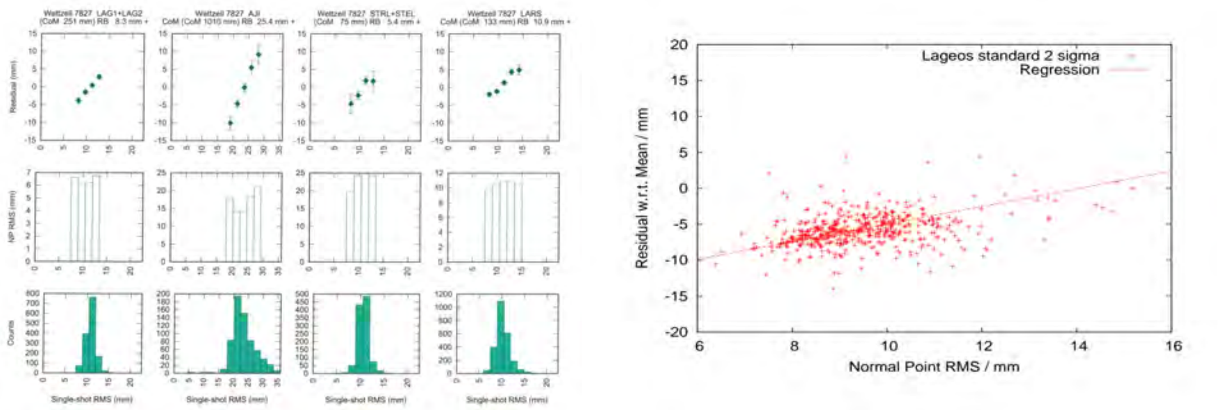

Figure 1: Toshimichi Otsubo's global bias analysis for the SOS-W on the left side. For LAGEOS a trend function of 1mm residual per 1mm residual rms is clearly visibly. The same trend can be seen in the on site generated SOS-W normal points on the right side.

radial errors as well as their first and second derivatives, yielding an orbit referring to the mean of the data distribution. We calculate histograms from the obtained residuals for all data in one satellite specific normal point interval. Figure 2 shows an example on simulated data using an analytical LAGEOS transfer function. In order to apply the Wiener filter, the normal point histograms, the transfer function and a histogram of the calibration pulse have to be fourier transformed, ending up in a measured signal spectrum C, the satellite transfer function R and the calibration pulse spectrum U. By convolution of U and R we obtain a modeled signal spectrum, in the following denoted by S. The high frequency components of C are further used to model the noise and extrapolate it to the low frequency components. This is performed by a least squares adjustment of a polynomial starting at a certain cutoff frequency. The resulting spectral shapes are shown in figure 3. From N and S we further calculate filter coefficients phi=$S^2/(N^2+S^2)$ and the spectrum of the filtered signal Us is obtained by Us=C*phi/R. Transforming back to the spatial scale, a filtered signal is obtained as illustrated by the yellow line in figure 2, which resembles the input calibration pulse very well. In this way the satellite signature is removed from the measurements and we perform statistical calculations on the filtered signal in the same way as we do with the calibration data.

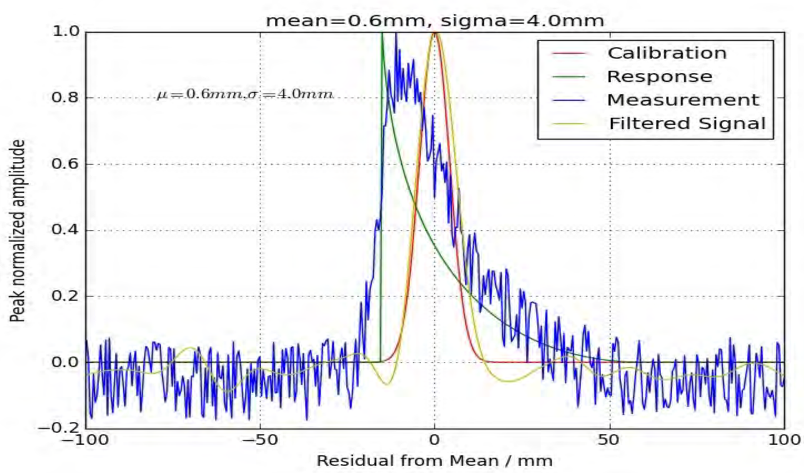

Figure 2: Example of a Wiener filter applied to simulated residual data (blue) using an analytical transfer function (green). The filtered signal resembles the input calibration pulse very well, since the satellite signature is removed by the Wiener filter.

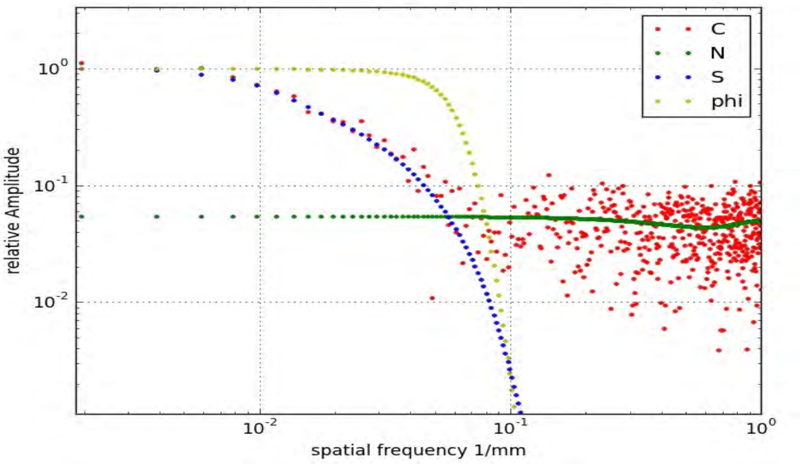

Figure 3: Spectral shapes of all signals involved in the Wiener filter algorithm. C is the measured signal, N models the noise from high frequency components, S is the signal model and phi the resulting filter coefficients.

3. Bias Analysis

For the bias analysis two independent normal-point-datasets of SOS-W-observations have been processed independently for the satellites LAGEOS1, LAGEOS2, AJISAI, ETALON1 and ETALON2. One dataset comprises the standard 2 sigma edited normal-point-data submitted to the Eurolas Data Center (EDC) from 2017, in the following referred to as S2S. The other dataset comprises normal-points reprocessed with the Wiener filter approach as described in the previous section, in the following referred to as WF. The applied Center Of Mass (COM) corrections for each satellite array were obtained through a wavelength specific transfer function supplied by Rodriguez (ETALON and LAGEOS) and Otsubo (AJISAI), evaluated for the S2S and WF approaches. This leads to different COM values since the WF approach strictly refers to the mean of the transfer function, whereas the S2S approach refers to a data editing dependent COM value. The resulting COM values are compiled in table 1. It should be noted that in case of the LAGEOS array in contrast to Otsubo and Appleby [3], where a value for the decay of the transfer function n=1.1 is recommended, a value of n=1.0 is used in this study. This affects the resulting COM correction at the millimeter level.

Table 1: COM-values obtained for the various reflector arrays and normal point algorithms for a wavelength of 850nm.

Reflector Array	S2S COM@850nm / mm	WF COM@850nm / mm
LAGEOS (n=1.0)	245	241
AJISAI (n=1.2)	993	962
ETALON (n=1.3)	579	554.5

The analysis was performed using the DGFI-TUM Orbit and Geodetic parameter estimation Software (DOGS). For LAFEOS-1/-2, AJISAI and ETALON-1/-2, DGFI-TUM computed independent 7-day arcs estimating range biases for the SOS-W station. Station coordinates as well as Earth orientation parameters and gravity field model coefficients were kep fixed to their corresponding background models. In general the normal-point-data-yield as well as normal-point-data-quality can be considered equal for both the S2S- and the WF- normal point algorithms, as figures 4 to 6 suggest.

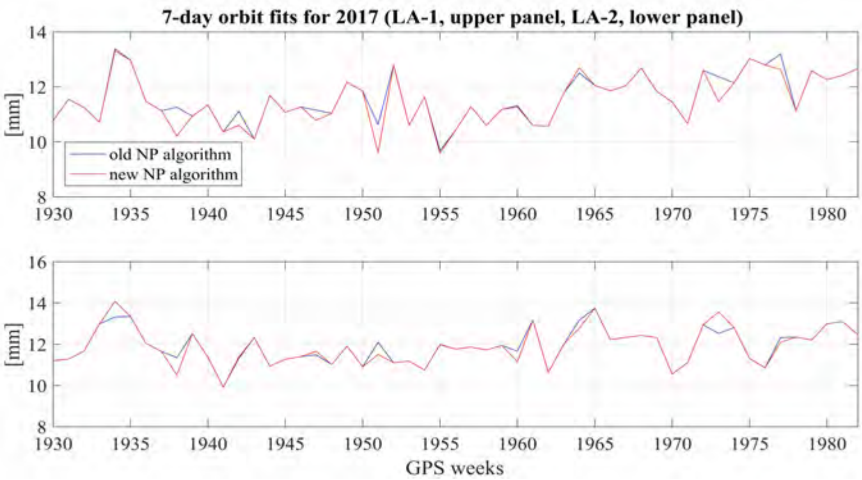

Figure 4: Orbit fit quality for LAGEOS1 and LAGEOS2 for the old normal point algorithm with iterative 2 sigma editing (blue) and the Wiener filter based normal point algorithm (red).

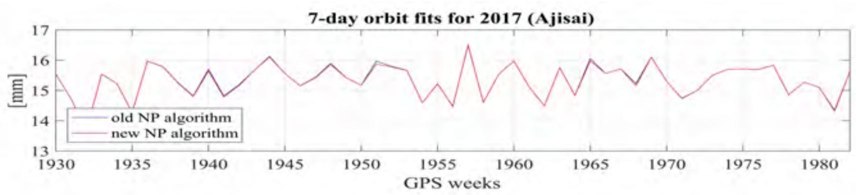

Figure 5: Orbit fit quality obtained for AJISAI for the old normal point algorithm with iterative 2 sigma editing(blue) and the Wiener filter based normal point algorithm (red).

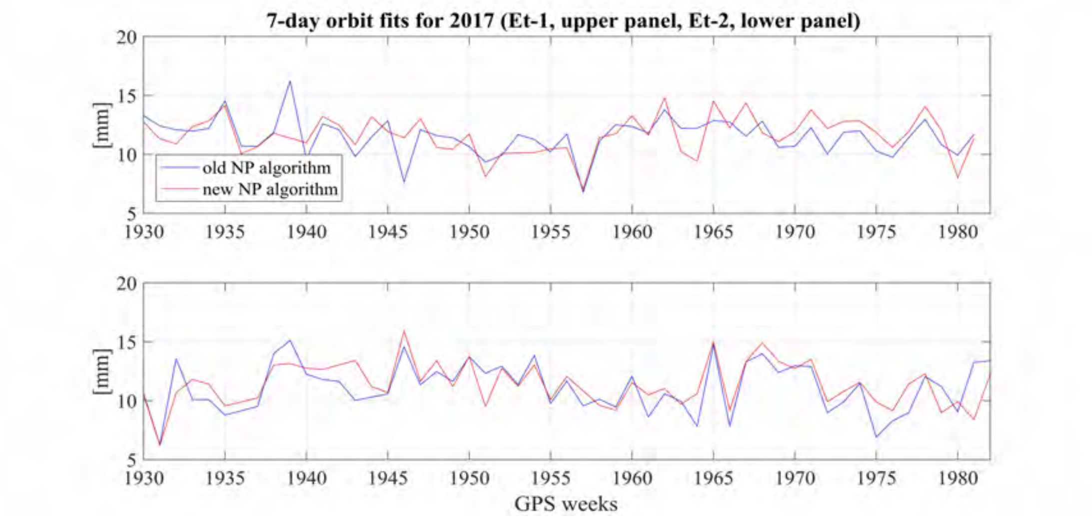

Figure 6: Orbit fit quality obtained for ETALON1 and ETALON2 for the old normal point algorithm with iterative 2 sigma editing(blue) and the Wiener filter based normal point algorithm (red).

Figures 7 to 9 show the results of the bias estimation obtained from 7 day global orbit fits for the satellite reflector arrays under consideration. There is a discrepancy of about 3.5mm between the S2S and the WF normal point algorithm visible in the data of both LAGEOS reflector arrays, which can be attributed to the fact, that we used a different exponent than recommended (n=1.0 instead of n=1.1) for the modeling of the decay of the transfer function. It is also clear to see that both arrays, even though constructed identically, show different biases by about 1.7mm for both normal point algorithms.
The results for AJIASI show the largest discrepancy of about 15mm when comparing the bias obtained for both normal point algorithms. From the authors point of view this is caused by the limited accuracy with which the transfer function can be modeled, which is backed also by the fact that this is the largest array under investigation.
When comparing the bias results for ETALON1 and ETALON2 we find sub millimeter agreement for the absolute bias and the WF algorithm performs superior with respect to the S2S approach, yielding also a sub millimeter systematic error. In contrast to the bias estimates performed pass-wise for LAGEOS-1/-2 (fig. 7) and AJISAI (fig. 8), we only estimated weekly range biases for both ETALON satellites (fig. 9) due to the much smaller amount of normal points.

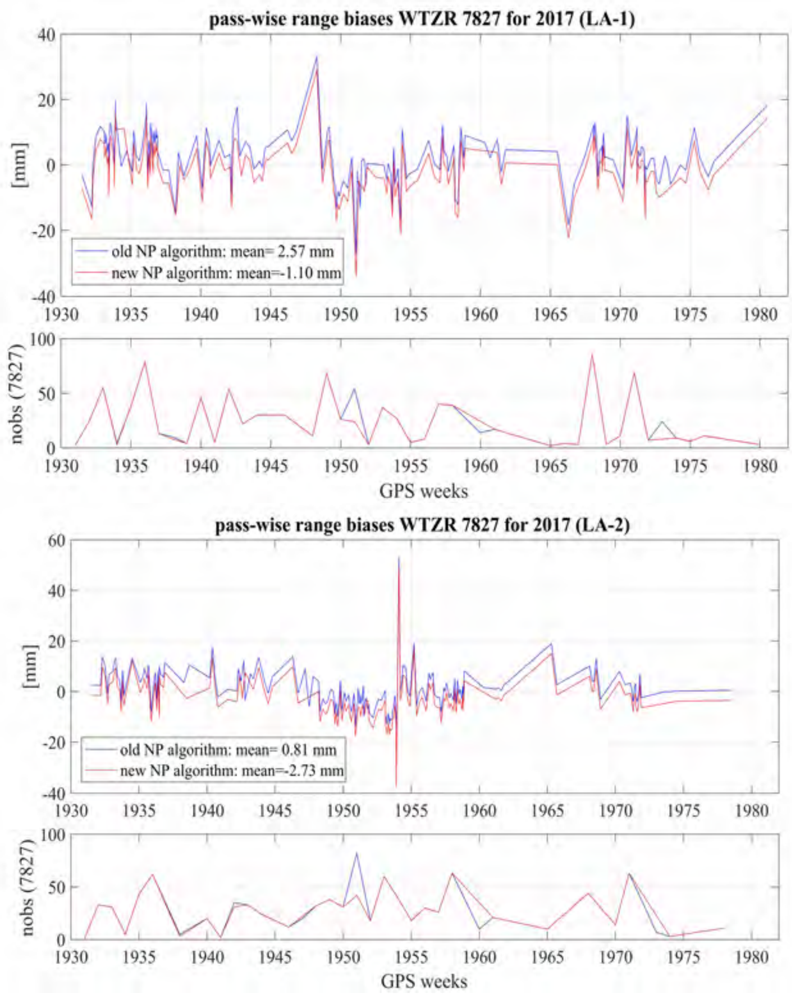

Figure 7: Rangebias estimation for LAGEOS1 (top) and LAGEOS2 (bottom) for the two normal point algorithms under investigation. The biases obtained from the algorithms differ by about 3.5mm for both satellites.

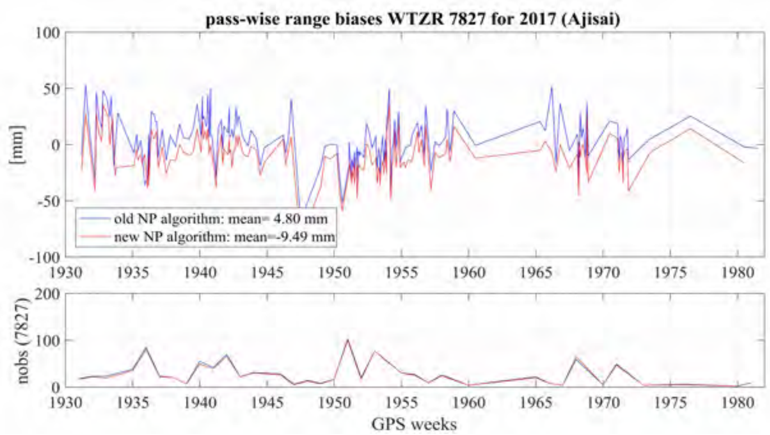

Figure 8: Rangebias estimation for AJISAI showing the largest discrepancy between the S2S and WF normal point algorithms among the reflector arrays under consideration.

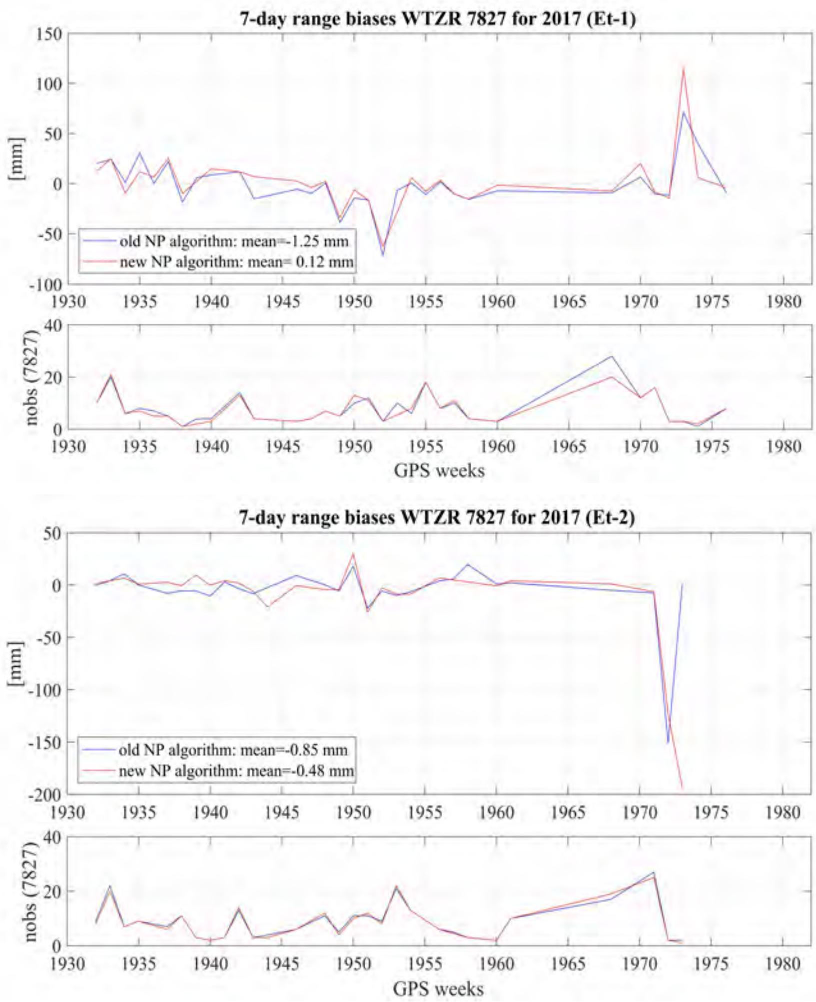

Figure 9: Rangebias estimation for ETALON1 (top) and ETALON2 (bottom) for the normal point algorithms under investigation. In both cases the WF normal point approach shows the least systematic error. It is remarkable that the results of both reflector arrays agree at the millimeter level. The bias outlier at the end of the data set is due to sparse observations.

4. Improved Normal Point Statistics and Conclusion

Figure 10 shows the residual statistics in terms of normal point rms and normal point residual for the S2S and WF normal point algorithms obtained for all three different reflector arrays under consideration. Whereas the S2S yields a statistical distribution elongated along the normal point rms axis, the statistics for the WF approach are in general much more confined and are much closer to a gaussian distribution. The latter fact suggests that the WF normal point algorithm performs superiour with respect to the S2S approach and therefore might be selected by the ILRS as the standard normal point algorithm for single photon SLR systems.

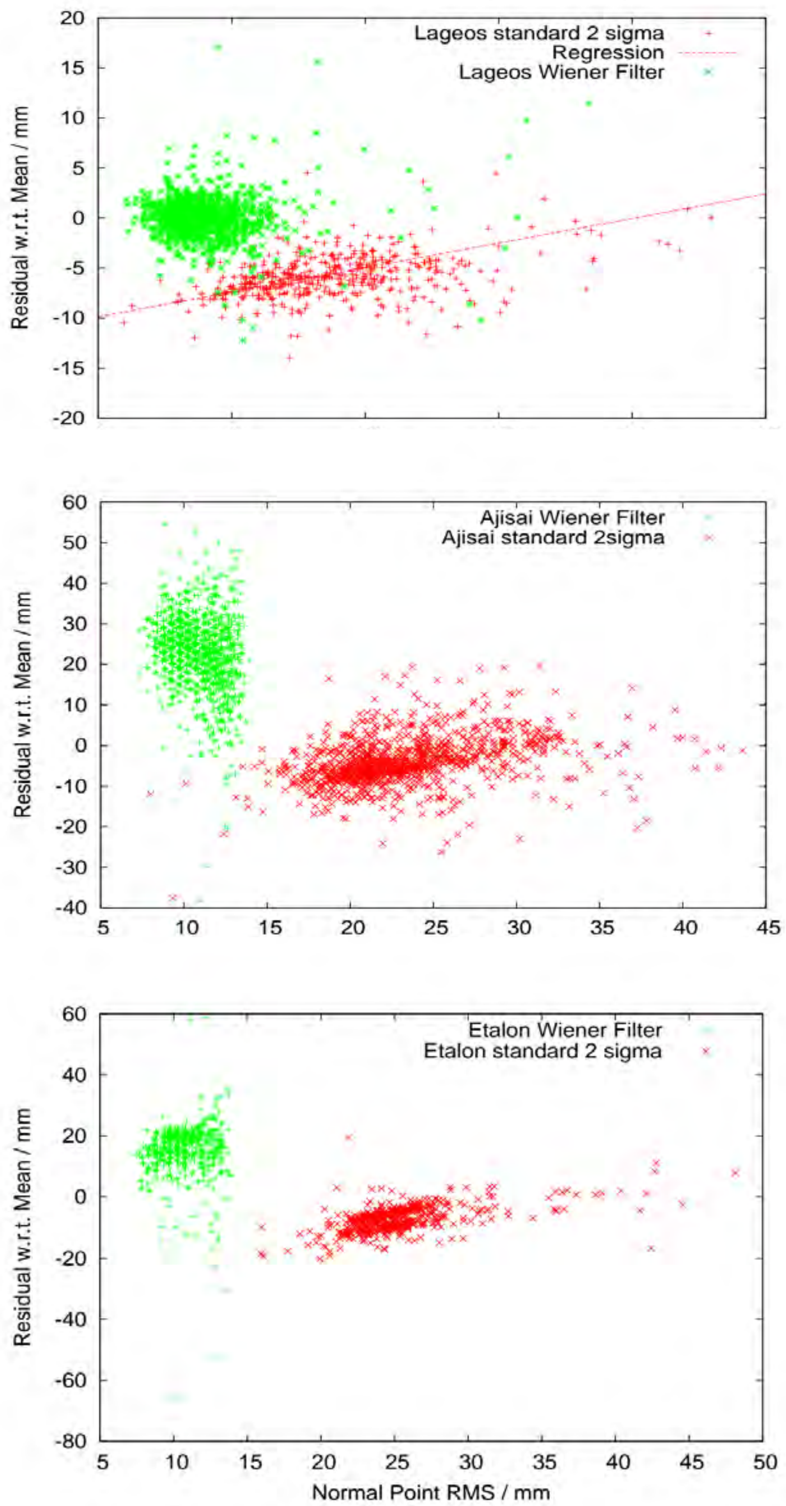

Figure 10: Normal point residual statistics for S2S and WF normal point algorithms and all three satellite reflector arrays under consideration.

References:

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