# AN UPGRADED SGSLR LINK ANALYSIS WHICH INCLUDES THE EFFECTS OF ATMOSPHERIC SCINTILLATION AND TARGET SPECKLE

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# Link Equation and SGSLR Values

$$n_s = \frac{E_t}{h\nu}\eta_t \frac{2}{\pi(\theta_d R)^2}\exp\left[-2\left(\frac{\Delta\theta_p}{\theta_d}\right)^2\right]\left[\frac{1}{1+\left(\frac{\Delta\theta_j}{\theta_d}\right)^2}\right]\left(\frac{\sigma A_r}{4\pi R^2}\right)\eta_r\eta_c T_a^2 T_c^2$$

**Reference: J. Degnan, "Millimeter Accuracy Satellite Laser Ranging: A Review", in Contributions of Space Geodesy to Geodynamics: Technology Geodynamics, 25, pp. 133-162, 1993.**

| VARIABLE | SYMBOL | VALUE | SECTION |
|---|---|---|---|
| Laser Pulse Energy | $E_t$ | 1.5 mJ | 3.0 |
| Laser Repetition Rate | $f_L$ | 2 kHz | 3.0 |
| Transmit Optics Efficiency | $\eta_t$ | 0.766 | 2.1, 2.2 |
| Receive Optics Efficiency | $\eta_r$ | 0.542 | 2.1, 2.2 |
| Detector Counting Efficiency | $\eta_c$ | 0.28 | 2.2 |
| Spectral Filter Efficiency | $\eta_f$ | 0.7 | 2.2 |
| Effective Receive Aperture | $A_r$ | 0.187 $m^2$ | 2.8 |
| Tracking Pointing Bias | $\Delta\theta_p$ | 2 arcsec (Sigma Range Receiver) | 2.3 |
| Telescope RMS Pointing Jitter | $\Delta\theta_J$ | 2 arcsec | 2.3 |
| Full Transmitter Divergence | $2\theta_d$ | 28 arcsec (Starlette, LAGEOS)<br>14 arcsec (GNSS) | 3.0 |
| | | | |
| Coherence Length | $\rho_0$ | 2.5 cm (Worst Case: GGAO) | 2.6.2 |
| Zenith Log Amplitude Variance | $C_l^s(0)$ | 0.054 (Worst Case: GGAO) | 2.6.2 |

# Atmospheric Attenuation

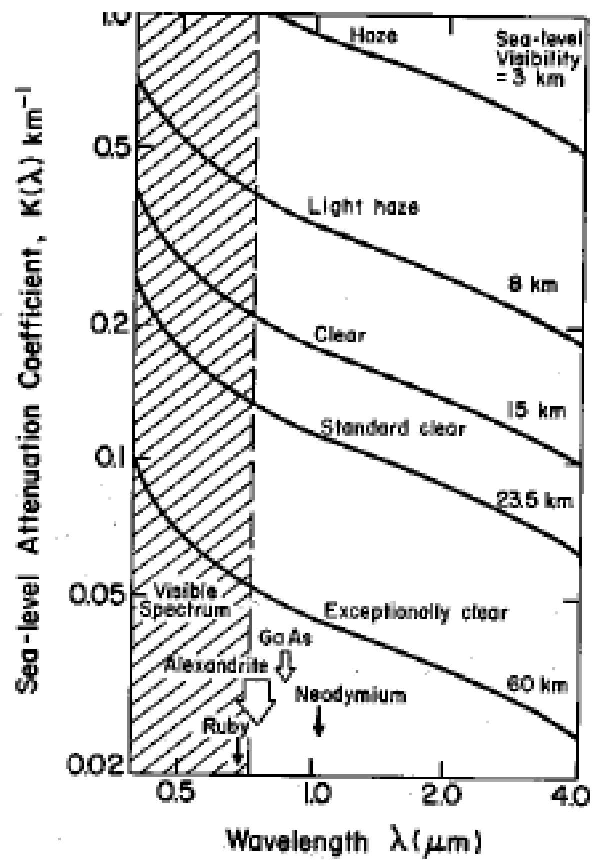
The atmospheric attenuation coefficient decreases approximately exponentially with altitude, h, according to the equation

$$\sigma_{atm}(\lambda, V, h) = \sigma_{atm}(\lambda, V, 0)\exp\left(-\frac{h}{h_v}\right)$$

where $V$ is the sea level visibility and $h_v$ = 1.2 km is a visibility scale height. Thus, the one way attenuation from a SLR station at elevation $h_g$ above sea level to a satellite outside the atmosphere is

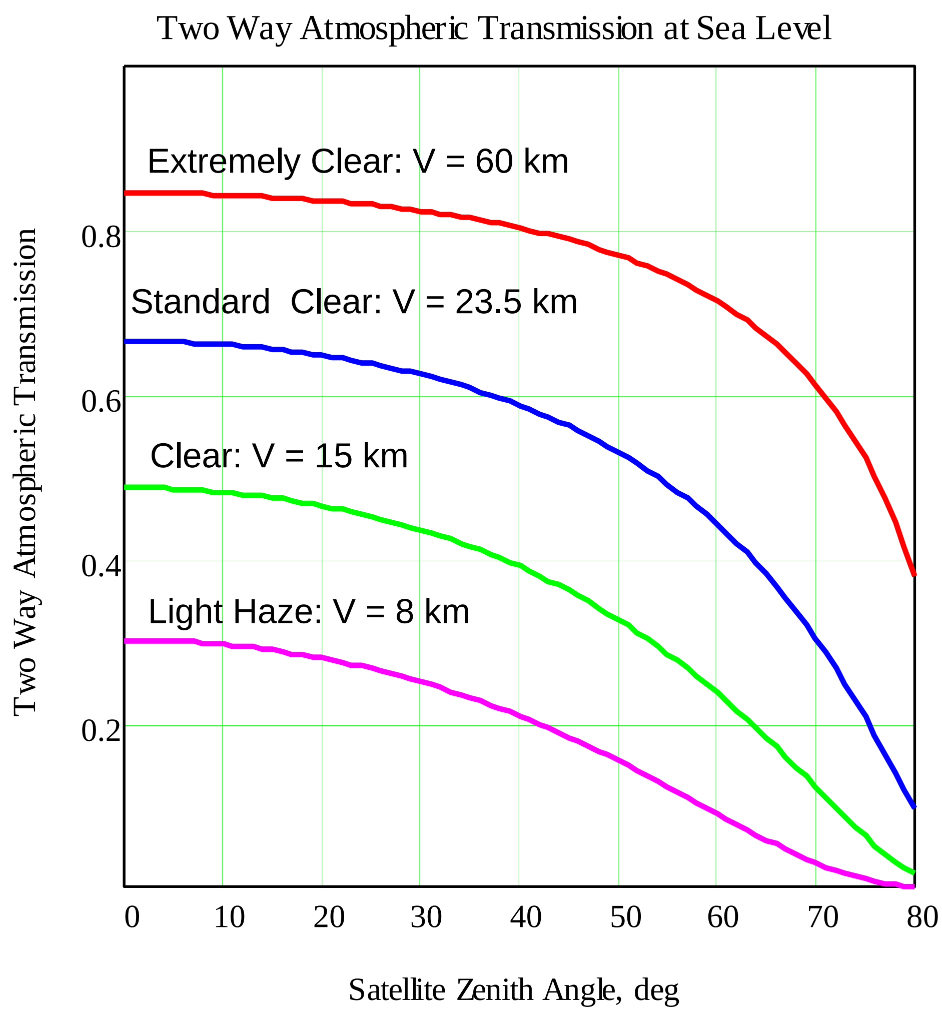
$$T_{atm}(\lambda, V, h_g) = \exp\left[-\sec\theta_{zen}\int_{h_g}^{\infty}\sigma_{atm}(\lambda, V, h)dh\right]$$

$$= \exp\left[-\sigma_{atm}(\lambda, V, 0)h_v\sec(\theta_{zen})\exp\left(-\frac{h_g}{h_v}\right)\right]$$




*Graph of Sea level attenuation coefficients obtained from R. J. Pressley, Handbook of Lasers, Chemical Rubber Co., Cleveland (1971).

# Two-Way Atmospheric Transmission at 532 nm (Worst Case: station at sea level)

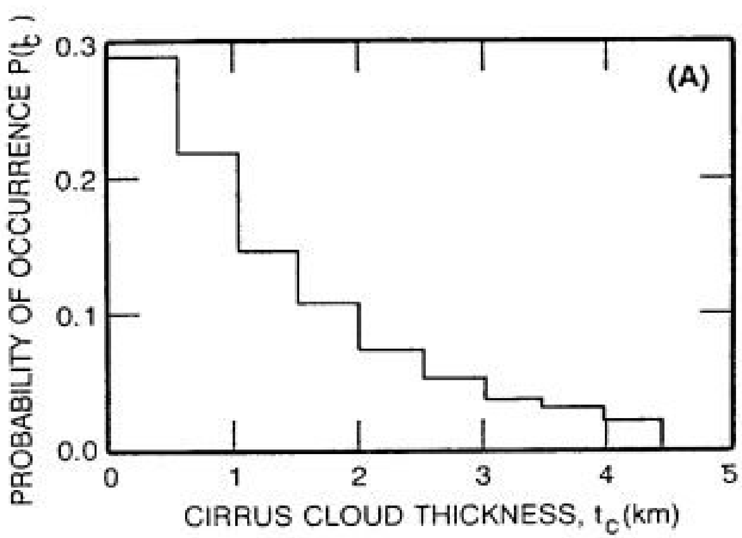


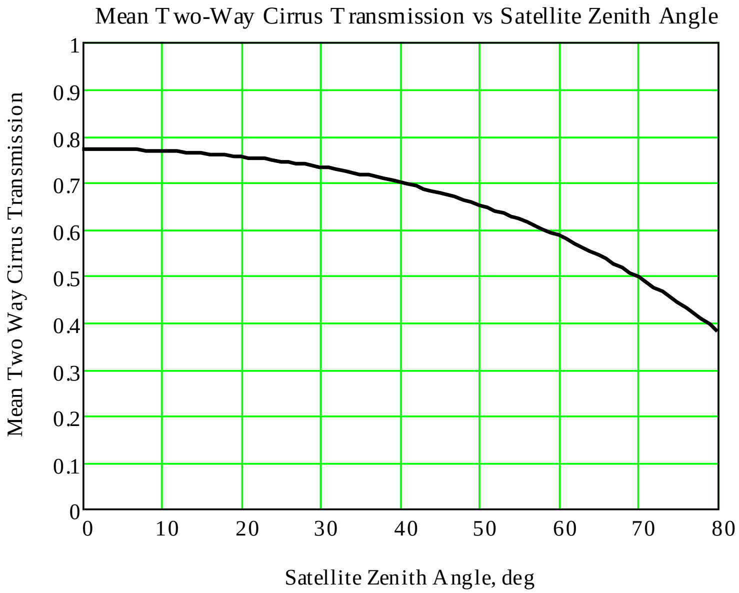
# Mean Cirrus Cloud Transmission

Experimentally, it is found that the one-way cirrus cloud transmission is given by

$$T_c = \exp\left[-0.14\left(t \sec\theta_{zen}\right)^2\right]$$

where t is the cirrus cloud thickness. Typically, cirrus clouds are present about 50% of the time above most locations. The probability of having a certain thickness is given by the plot on the left and the computation of the mean two-way cirrus transmissions on the right . The computation of the mean includes the assumption that there are no cirrus clouds (i.e. $T_c = 1$) 50% of the time.



# Atmospheric Turbulence

Atmospheric turbulence affects return signal strength in three ways:

1. Beam Wander – random translations of the spatial centroid of the beam generally caused by beam passage through large turbulent eddies
2. Beam Spread – short term growth in the effective beam divergence produced by smaller eddies in the beam path
3. Scintillation or "beam fading" – responsible for the familiar "twinkling" of starlight

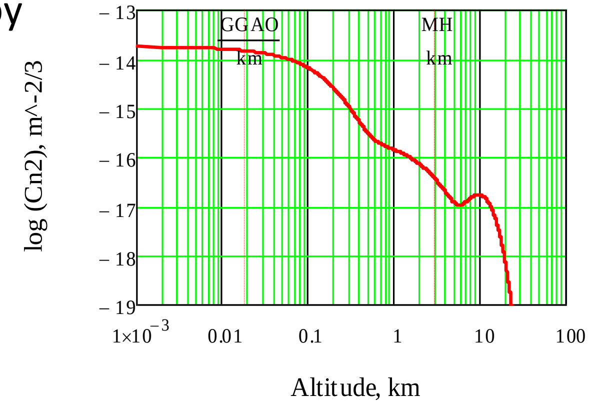
Effects 1 and 2 are often discussed together in terms of a "long term beam spread" defined as

$$\langle \theta_L \rangle = \theta_d \sqrt{1+\left(\frac{\omega_0}{\rho_0}\right)^2} = \frac{M^2\lambda}{\pi\omega_0}\sqrt{1+\left(\frac{\omega_0}{\rho_0}\right)^2} = \frac{M^2\lambda}{\pi}\sqrt{\frac{1}{\omega_0^2}+\frac{1}{\rho_0^2}} \approx \frac{M^2\lambda}{\pi\rho_0} \; for\, \omega_0 >> \rho_0$$

where $\theta_d$ is the transmitter beam half-divergence angle out of the telescope, $\omega_0 = M_T\omega_L$ is the Gaussian beam radius at the telescope exit aperture, and $\rho_0$ is the "transverse atmospheric coherence length" defined by

$$\rho_0 = \left\{1.46k^2 \int_{h_s}^{h_{\lim}} dh C_n^2(0) m^{-2/3}\right\}^{-3/5}$$

and $C_n^2(0)$ is the "optical strength variance"



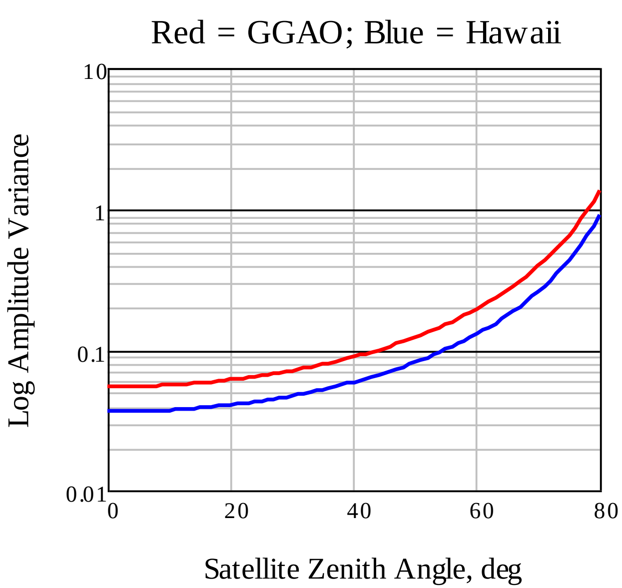
# Log Amplitude Variance and Uplink Scintillation PDF

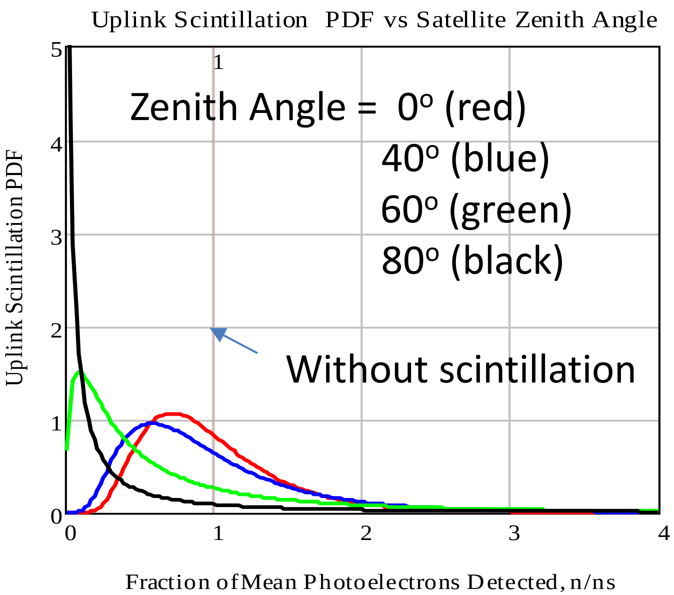
$$p_A(J,\theta_z) = \frac{1}{2J\sqrt{2\pi C_l^s(\theta_z)}} \exp\left\{ \frac{-\left[\frac{1}{2}\ln\left(\frac{J}{\langle J(\theta_z)\rangle}\right) + C_l^s(\theta_z)\right]^2}{2C_l^s(\theta_z)} \right\}$$

$$C_l^s(\theta_z) = 0.56k^{7/8} R \int_0^1 C_n^2(\xi R)|1-\xi|^{5/3}(\xi R)^{5/6} d\xi =$$

$$0.56k^{7/8}(\sec\theta_z)^{11/6} \int_{h_s}^{h_{\cdot \lim}} C_n^2(h)(h)^{5/6} dh = C_l^s(0)(\sec\theta_z)^{11/6}$$

Uplink ξ~0
Downlink ξ~1
k=2π/λ



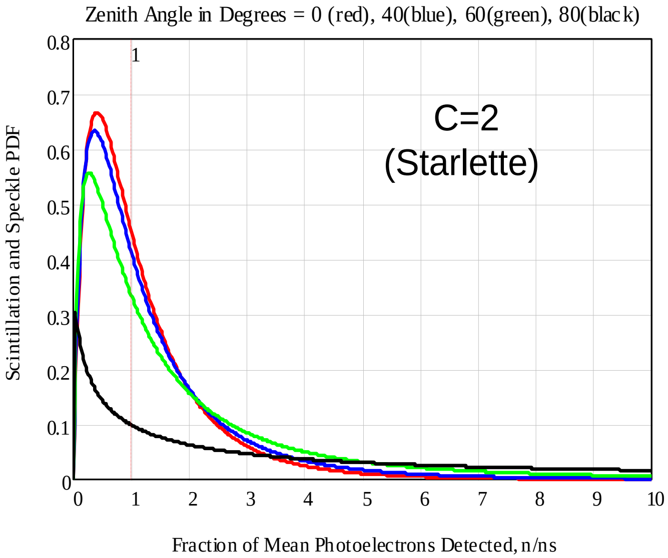



# Combined Scintillation and Speckle PDF

$$p_T(n,\theta_z) = \frac{1}{2y(C-1)!\sqrt{2\pi C_l^s(\theta_z)}} \int_0^\infty \left(\frac{yC}{x}\right)^C \exp\left(-\frac{yC}{x}\right) \exp\left[-\frac{\left(\frac{1}{2}\ln(x) + C_l^s(\theta_z)\right)^2}{2C_l^s(\theta_z)}\right] dx$$

C = nominal number of retroreflectors aperture averaged at the receiver.

X = irradiance/mean irradiance at satellite; y = fraction of mean photoelectrons detected in the absence of scintillation



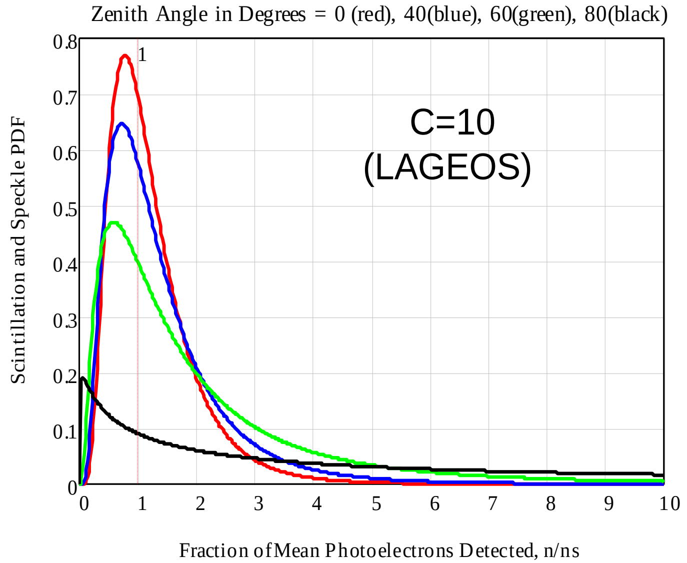




**Figure 12: Combined scintillation and speckle PDF as a function of y = n/$n_s$ and four satellite zenith angles - θz = 0(red), 40 (blue), 60 (green) and 80 degrees (black) . (a) Starlette (C=2) ; (b) LAGEOS (C =10).**

# Combined Effects of Uplink Scintillation and Target Speckle

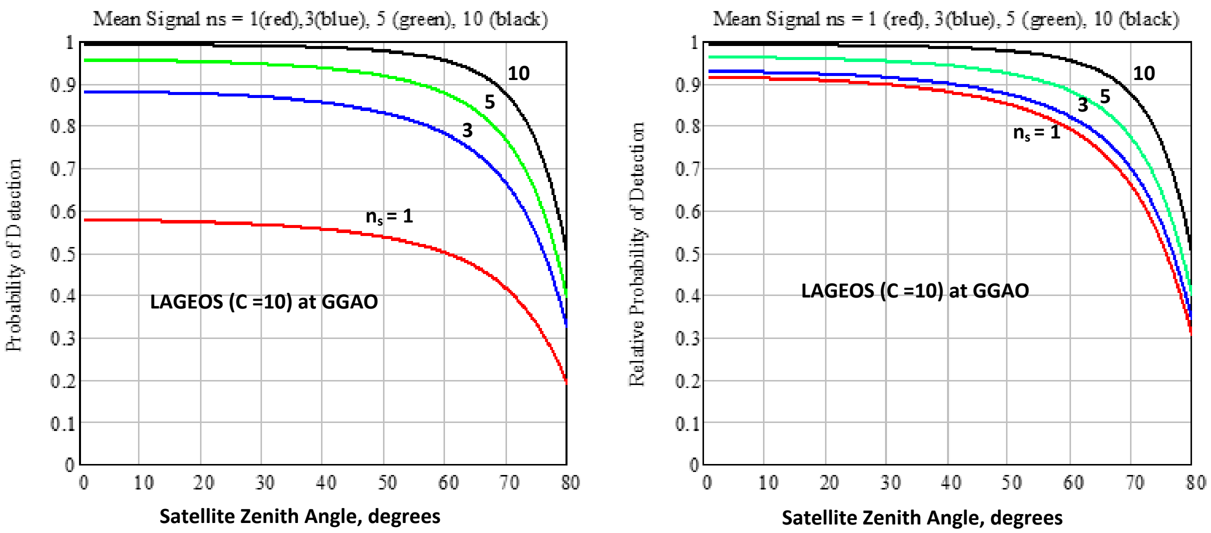



**Figure 13: (a) The probability of detecting the signal from LAGEOS (C=10) at GGAO (worst case) due to the combined effects of uplink atmospheric scintillation and target speckle if, in the absence of scintillation and speckle, the ground signal strength is $n_s$ = 1(red), 3 (blue), 5 (green), or 10 photoelectrons. (b) The probability of detection relative to the zero scintillation, zero speckle case. Figure 13(b) represents an additional loss in the link equation given by Eq. (1).**

# Detection Probability and Normal Point Precision

For a SLR system with a single photon detection threshold, the probability of detecting the satellite signal is

$$P_d = 1 - \exp(-n_s) \cong n_s$$

where the approximation holds for $n_s$<<1. Thus, the number of range measurements contributing to a satellite “normal point” is

$$N = P_d f_L \tau_{np} = \left(1 - e^{-n_s}\right) f_L \tau_{np}$$

where

$f_L$= the laser repetition rate = 2 kHz

$\tau_{np}$ = the normal point time interval

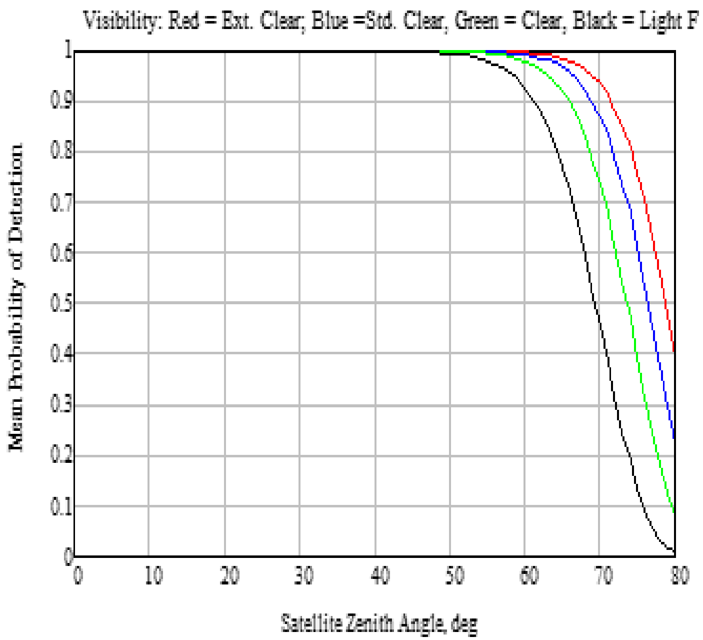
and the desired normal point precision is equal to

$$\sigma_{np} = \frac{\sigma_{ss}}{\sqrt{N}} = \frac{1}{\sqrt{N}} \sqrt{\sigma_L^2 + \sigma_D^2 + \sigma_{ET}^2 + \sigma_S^2} \approx 1mm$$

where $\sigma_{ss}$ is the satellite-dependent , Single Shot range precision obtained from the contributions of the laser (L), detector (D), Event Timer (ET), and Satellite (S). Thus, the integration time required to generate a normal point with normal point precision $\sigma_{np}$ is

$$\tau_{np} = \frac{N}{\left(1 - e^{-n_s}\right) f_L} = \frac{1}{\left(1 - e^{-n_s}\right) f_L} \left(\frac{\sigma_{ss}}{\sigma_{np}}\right)^2$$

# Starlette Link



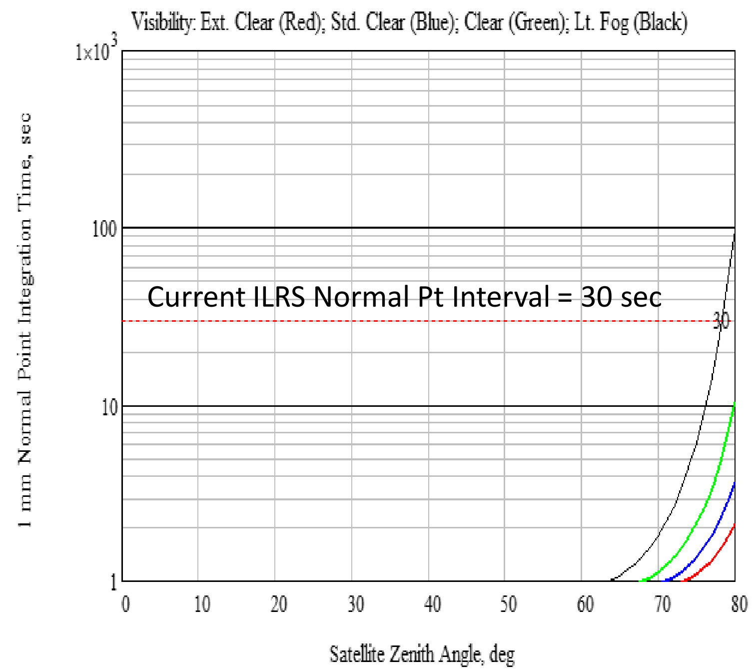




**Figure 17 : Starlette results for a 28 arcsec full width transmitter divergence from a worst case site near Mean Sea Level (e.g. GGAO) as a function of satellite zenith angle and atmospheric quality (Extremely Clear to Light Haze) : (a) Mean Probability of Detection per Pulse; (b) Time required to create 1 mm normal point in seconds. The horizontal dashed red line in Fig. 17(b) marks the 30 second integration time established by the ILRS for most LEO satellites.**

# LAGEOS Link

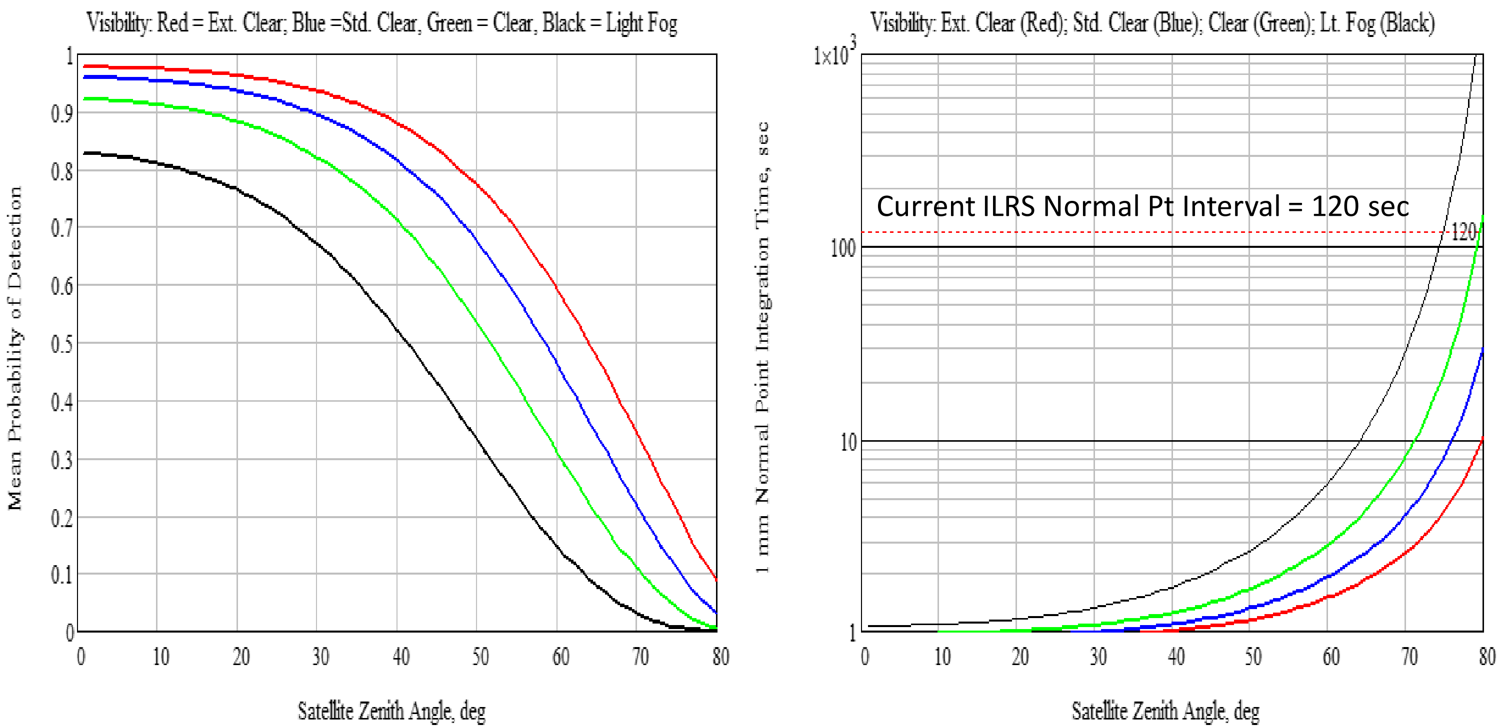



**Figure 18: LAGEOS results for a 28 arcsec full width transmitter divergence from a worst case site near Mean Sea Level (e.g. GGAO) as a function of satellite zenith angle and atmospheric quality (Extremely Clear to Light Haze) : (a)Mean Probability of Detection per Pulse; (b) Time required to create 1 mm normal point in seconds. The horizontal dashed red line in Fig. 18(b) marks the 120 second integration time established by the ILRS for the two LAGEOS satellites.**

# GNSS Link

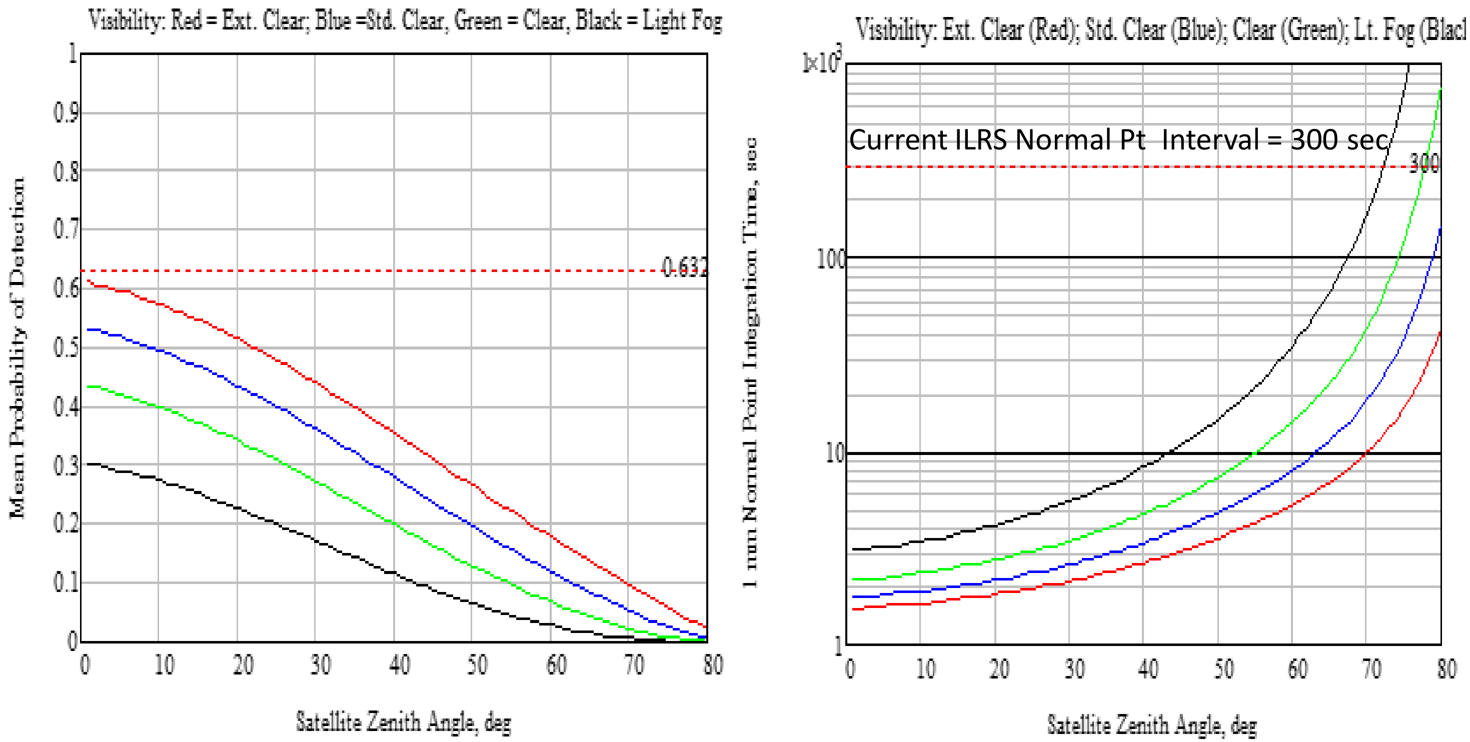



**Figure 19: GNSS results for a 14 arcsec full width transmitter divergence from a worst case site near Mean Sea Level (e.g. GGAO) as a function of satellite zenith angle and atmospheric quality (Extremely Clear to Light Haze) : (a)Mean Probability of Detection per Pulse; (b) Time required to create 1 mm normal point in seconds. The horizontal dashed red line in Fig. 17(b) marks the 300 second integration time established by the ILRS for GNSS satellites.**

# Summary

The SGSLR link analyses presented considered the following effects:

1. SLR System
   - laser energy (1.5 mJ) and fixed beam divergence (14 for GNSS or 28 arcsec for Starlette and LAGEOS)
   - detector PDE (~28%)
   - transmit (77%) and receive (54%) optical throughput efficiencies, spectral filter (70%) and obscurations (secondary mirror and transmit injection mirror)
   - telescope pointing bias and jitter (2 arcsec each during tracking with automated pointing correction –next talk)
2. Target
   - Optical cross-section (from ILRS tables/recommendations)
   - Target speckle effects
3. Atmosphere
   - Atmospheric transmission vs ground visibility - extremely clear (60 km), standard clear (23 km), clear (15 km) and light fog (8 km)
   - Mean cirrus cloud transmission
   - Worst case atmospheric turbulence effects (GGAO): short and long term beam wander, uplink scintillation (downlink is negligible)
   - Telescope aperture averaging of target speckle effects.

The wide variation in signal strength as a function of satellite zenith angle suggests reducing the beam divergence at low elevation angles in order to increase the data rate and reduce the normal point integration time.