Determining parameters of Moon's orbital and rotational motion from LLR observations using GRAIL and IERS-recommended models

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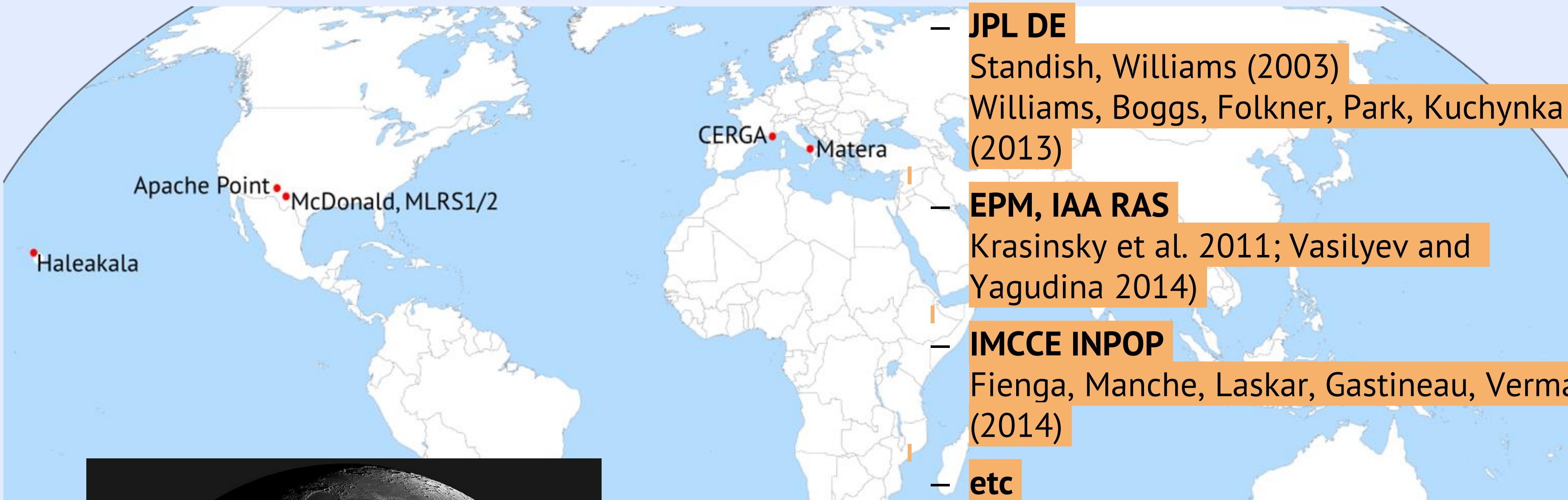
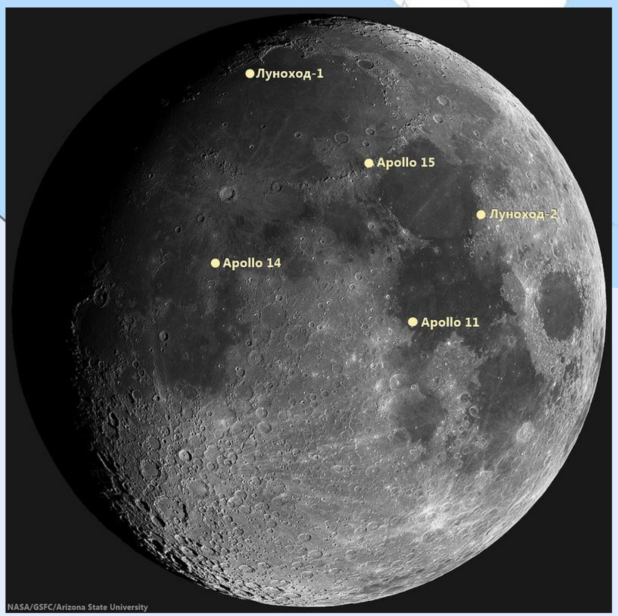
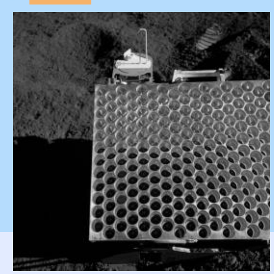
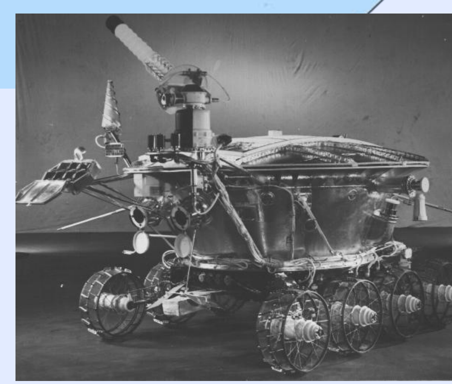
Lunar laser location and ephemerides



- **JPL DE**
  Standish, Williams (2003)
  Williams, Boggs, Folkner, Park, Kuchynka (2013)
- **EPM, IAA RAS**
  Krasinsky et al. 2011; Vasilyev and Yagudina 2014)
- **IMCCE INPOP**
  Fienga, Manche, Laskar, Gastineau, Verma (2014)
- **etc**







Model of orbital motion(I)

- Einstein-Infeld-Hoffmann relativistic equations of motion

$$\vec{a}_A = \sum_{B\neq A} \frac{Gm_B \vec{n}_{BA}}{r_{AB}^2} + \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B\vec{n}_{BA}}{r_{AB}^2}\Big[v_A^2 + 2v_B^2 - 4(\vec{v}_A\cdot\vec{v}_B) - \frac{3}{2}(\vec{n}_{AB}\cdot\vec{v}_B)^2 - 4\sum_{C\neq A}\frac{Gm_C}{r_{AC}} - \sum_{C\neq B}\frac{Gm_C}{r_{BC}} + \frac{1}{2}((\vec{x}_B-\vec{x}_A)\cdot\vec{a}_B)\Big] + \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}\left[\vec{n}_{AB}\cdot(4\vec{v}_A - 3\vec{v}_B)\right](\vec{v}_A-\vec{v}_B) + \frac{7}{2c^2}\sum_{B\neq A}\frac{Gm_B\vec{a}_B}{r_{AB}} + O(c^{-4})$$

- IERS 2010 recommended conventional Geopotential model based on **EGM2008**, truncated to 6

$$\frac{\boldsymbol{f}_{\mathrm{fig-pm}}}{m} = \mu\mathrm{Re}\left[\sum_{n=0}^{n_{\max}} R^n \sum_{m=0}^{n}(\bar{C}_{\mathrm{nm}} - i\bar{S}_{\mathrm{nm}})\nabla\bar{V}_{\mathrm{nm}}(r,\lambda,\phi)\right]$$

$$\bar{V}_{\mathrm{nm}}(r,\lambda,\phi) = N_{\mathrm{nm}}\frac{\cos m\lambda + i\sin m\lambda}{r^{n+1}}P_n^m(\sin\phi)$$

$$N_{\mathrm{nm}} = \sqrt{\frac{(n-m)!(2n+1)!(2-\delta_{0m})}{(n+m)!}}$$

- Earth acceleration in Moon's gravity field: **GL660b** (result of GRAIL mission) with some corrections.
- Accelerations from Earth, Sun, Venus, Jupiter, Mercury and Mars in Moon's gravity field

Table 6.2: Low-degree coefficients of the conventional geopotential model

Coefficient	Value at 2000.0	Reference	Rate / yr^{-1}	Reference
$\bar{C}_{20}$ (zero-tide)	-0.48416948×10^{-3}	Cheng *et al.*, 2010	11.6×10^{-12}	Nerem *et al.*, 1993
$\bar{C}_{30}$	0.9571612×10^{-6}	EGM2008	4.9×10^{-12}	Cheng *et al.*, 1997
$\bar{C}_{40}$	0.5399659×10^{-6}	EGM2008	4.7×10^{-12}	Cheng *et al.*, 1997

Model of orbital motion (II)

Tidal perturbations on Earth act on Moon's orbit.

IERS2010 model: perturbed Geopotential (2nd degree).

$$\Delta\bar{C}_{\mathrm{nm,E}} - i\Delta\bar{S}_{\mathrm{nm,E}} = \frac{k_{\mathrm{nm}}}{2n+1}\sum_{j=M,S}\frac{\mu_j}{\mu_E}\left(\frac{R_E}{r_j}\right)^{n+1}\bar{P}_{\mathrm{nm}}(\sin\Phi_j)e^{-im\lambda_j}$$

$$\Delta\bar{C}_{20}^{(\mathrm{fd})} = \mathrm{Re}\sum_f (A_0\delta k_f H_f)e^{i\theta_f}$$

$$\begin{cases}\Delta\bar{C}_{21}^{(\mathrm{fd})} - i\Delta\bar{S}_{21}^{(\mathrm{fd})} = -i\sum_f (A_1\delta k_f H_f)\,e^{i\theta_f}\\ \Delta\bar{C}_{22}^{(\mathrm{fd})} - i\Delta\bar{S}_{22}^{(\mathrm{fd})} = \sum_f (A_2\delta k_f H_f)\,e^{i\theta_f}\end{cases}$$

$$A_0 = \frac{1}{R_E\sqrt{4\pi}}, \quad A_m = \frac{(-1)^m}{R_E\sqrt{8\pi}}, \qquad (m=1,2).$$

$$\Delta\bar{C}_{\mathrm{nm}}^{(\mathrm{ocean})} - i\Delta\bar{S}_{\mathrm{nm}}^{(\mathrm{ocean})} = \sum_f \sum_{+}^{-}(\mathcal{C}_{\mathrm{f,nm}}^{\pm} \mp i\mathcal{S}_{\mathrm{f,nm}}^{\pm})e^{\pm i\theta_f}$$

DE430 model: acceleration with delay on rotation and orbit

$$\frac{\Delta \boldsymbol{f}}{m} = \frac{3\mu_E}{2}\left(\frac{R_E}{r}\right)^5\left[\frac{k_{20}}{r_0^{*5}}\left(\left(2z_0^{*2}\boldsymbol{z} + \rho_0^{*2}\boldsymbol{\rho}\right) - 5\frac{(zz_0^*)^2 + \frac{1}{2}(\rho\rho_0)^2}{r^2}\boldsymbol{r} + r_0^{*2}\boldsymbol{r}\right)\right.$$
$$+\frac{k_{21}}{r_1^{*5}}\left(2\left((\boldsymbol{\rho}\cdot\boldsymbol{\rho}_1^*)\boldsymbol{z}_1^* + zz_1^*\boldsymbol{\rho}_1^*\right) - \frac{10zz_1^*(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_1^*)\boldsymbol{r}}{r^2}\right)$$
$$\left.+\frac{k_{22}}{r_2^{*5}}\left(2(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_2^*)\boldsymbol{\rho}_2^* - \rho_2^{*2}\boldsymbol{\rho} - 5\frac{(\boldsymbol{\rho}\cdot\boldsymbol{\rho}_2^*)^2 - \frac{1}{2}(\rho\rho_2^*)^2}{r^2}\boldsymbol{r}\right)\right]$$

$$\boldsymbol{r}_m^* = R_z(\dot{\theta}_E\tau_{\mathrm{mR}})\boldsymbol{r}_{\mathrm{body}}(t-\tau_{\mathrm{mO}})$$

$$\boldsymbol{r} = \boldsymbol{\rho} + \boldsymbol{z}, \ \boldsymbol{r}_m^* = \boldsymbol{\rho}_m^* + \boldsymbol{z}_m^*$$

DE model of physical libration

Moon with fluid core

– Core rotation in lunar frame

$$\dot{\boldsymbol{\omega}}_c = \left(\frac{I_c}{m}\right)^{-1}\left[-\boldsymbol{\omega}\times\frac{I_c}{m}\boldsymbol{\omega}_c - \frac{\boldsymbol{N}_{\rm cmb}}{m}\right]$$

– liquid core contribution to inertia tensor

$$\frac{I_c}{m} = \alpha_c\frac{C_T}{m}\begin{bmatrix}1-f_c & 0 & 0\\ 0 & 1-f_c & 0\\ 0 & 0 & 1\end{bmatrix}, \qquad \frac{C_T}{m} = \frac{2(1+\beta)}{2\beta-\gamma+\beta\gamma}R_M^2\tilde{J}_2$$

$$\frac{I}{m} = \frac{2R_M^2\tilde{J}_2}{2\beta-\gamma+\beta\gamma}\begin{bmatrix}1-\beta\gamma & 0 & 0\\ 0 & 1+\gamma & 0\\ 0 & 0 & 1+\beta\end{bmatrix} - \frac{I_c}{m}$$
$$- k_2\frac{\mu_E}{\mu_M}\left(\frac{R_M}{r}\right)^5\begin{bmatrix}x^2-\frac{1}{3}r^2 & xy & xz\\ xy & y^2-\frac{1}{3}r^2 & yz\\ xz & yz & z^2-\frac{1}{3}r^2\end{bmatrix}$$
$$+ k_2\frac{R_M^5}{3\mu_M}\begin{bmatrix}2(\omega_x\dot{\omega}_x-\frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}}) & \omega_x\dot{\omega}_y+\dot{\omega}_x\omega_y & \omega_x\dot{\omega}_z+\dot{\omega}_x\omega_z\\ \omega_x\dot{\omega}_y+\dot{\omega}_x\omega_y & 2(\omega_y\dot{\omega}_y-\frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}}) & \omega_y\dot{\omega}_z+\dot{\omega}_y\omega_z\\ \omega_x\dot{\omega}_z+\dot{\omega}_x\omega_z & \omega_y\dot{\omega}_z+\dot{\omega}_y\omega_z & 2(\omega_z\dot{\omega}_z-\frac{1}{3}\boldsymbol{\omega}\cdot\dot{\boldsymbol{\omega}})\end{bmatrix}$$

$$C_{20} = \frac{1}{R_M^2}\left[\frac{1}{2}\left(\frac{I_{11}^*}{m}+\frac{I_{22}^*}{m}\right)-\frac{I_{33}^*}{m}\right]$$
$$C_{22} = \frac{1}{4R_M^2}\left[\frac{I_{22}^*}{m}-\frac{I_{11}^*}{m}\right]$$
$$C_{21} = -\frac{1}{R_M^2}\frac{I_{13}^*}{m}$$
$$S_{21} = -\frac{1}{R_M^2}\frac{I_{32}^*}{m}$$
$$S_{22} = -\frac{1}{2R_M^2}\frac{I_{21}^*}{m}$$
$$I^* = I + I_c$$

– CMB moment

$$\frac{\boldsymbol{N}_{\rm cmb}}{m} = \frac{C_T}{m}\left[\frac{k_v}{C_T}(\boldsymbol{\omega}_c-\boldsymbol{\omega}) + \alpha_c f_c(\hat{\boldsymbol{z}}\cdot\boldsymbol{\omega}_c)(\hat{\boldsymbol{z}}\times\boldsymbol{\omega}_c)\right]$$

– FC oblateness moment

Corrections of LLR-observations

- Equations for light travel time:

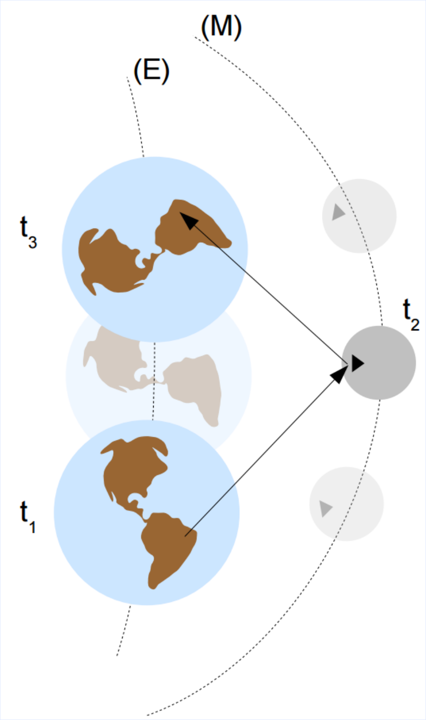
$$\begin{cases} t_2 - t_1 = \frac{|\boldsymbol{l}_{\mathrm{BCRS}}(t_2) - \boldsymbol{s}_{\mathrm{BCRS}}(t_1)|}{c} + \Delta_{\mathrm{grav}}(t_1, t_2) + \Delta_{\mathrm{atm}}(t_1, t_2) \\ t_3 - t_2 = \frac{|\boldsymbol{s}_{\mathrm{BCRS}}(t_3) - \boldsymbol{l}_{\mathrm{BCRS}}(t_2)|}{c} + \Delta_{\mathrm{grav}}(t_3, t_2) + \Delta_{\mathrm{atm}}(t_3, t_2) \end{cases}$$

- Signal relativistic delay (Kopeikin, 1990);
- Tropospheric delay (Mendes, Pavlis, 2004);
- UTC to TDB transformation, integrated with EPM ephemerides;

$$\begin{aligned} TDB - TT = {} & \frac{L_G - L_B}{1 - L_B}(TDB - T_0) + \frac{1 - L_G}{1 - L_B} TDB_0 \\ & + \frac{1 - L_G}{1 - L_B} \int_{T_0 + TDB_0}^{TDB} \frac{1}{c^2}\left(\frac{v_E^2}{2} + w_{0E} + w_{LE}\right) dt + \frac{1}{c^2} \mathbf{v}_E \cdot (\mathbf{r}_S - \mathbf{r}_E) \\ & - \frac{1 - L_G}{1 - L_B} \int_{T_0 + TDB_0}^{TDB} \frac{1}{c^4}\left(-\frac{v_e^4}{8} - \frac{3}{2} v_E^2 w_{0E} + 4 \mathbf{v}_E \cdot \mathbf{w}_{AE} + \frac{1}{2} w_{0E}^2 + \Delta_E\right) dt \\ & + \frac{1}{c^4}\left(3 w_{0E} + \frac{v_E^2}{2}\right) \mathbf{v}_E \cdot (\mathbf{r}_S - \mathbf{r}_E) \end{aligned}$$

$$\begin{aligned} C = {} & t_3 - t_1 + \mathrm{TTminusTDB}(t_3) - \mathrm{TTminusTDB}(t_1) \\ & + \frac{\dot{\boldsymbol{r}}_E(t_1) \cdot \boldsymbol{s}_{\mathrm{GCRS}}(t_1)}{c^2} - \frac{\dot{\boldsymbol{r}}_E(t_3) \cdot \boldsymbol{s}_{\mathrm{GCRS}}(t_3)}{c^2} + b \end{aligned}$$

- Transformation of TDB to TT;
- Unmodeled shifts.



(picture by H. Manche)

Corrections of LLR-observations (II)

- CRS➛TRS according to IAU2000/2006 and EOP from IERS C04 *(+ KEOF)* series*;*
- Station positions shifts: solid (Dehant, Matthews, 2000), ocean (FES2004) and polar tides;
- Relativistic transformation of positions of stations and reflectors to BCRS;
- Lunar solid tides from Earth and Sun.

$$\boldsymbol{s}_{\mathrm{GCRS}} = R_{\mathrm{T2C}}\,\boldsymbol{s}_{\mathrm{TRS}} + \boldsymbol{\Delta}_{\mathrm{pole}} + \boldsymbol{\Delta}_{\mathrm{solid}} + \boldsymbol{\Delta}_{\mathrm{ocean}}$$

$$\boldsymbol{s}_{\mathrm{BCRS}} = \boldsymbol{r}_E + \boldsymbol{s}_{\mathrm{GCRS}}\left(1 - \frac{U_E}{c^2}\right) - \frac{1}{2}\left(\frac{\dot{\boldsymbol{r}}_E \cdot \boldsymbol{s}_{\mathrm{GCRS}}}{c^2}\right)\dot{\boldsymbol{r}}_E$$

$$\boldsymbol{\Delta}_{\mathrm{solidmoon}} = \frac{\mu_A l^4}{\mu_M r_{\mathrm{MA}}^3}\Bigg[3l_2\left(\hat{\boldsymbol{r}}_{\mathrm{MA}} \cdot \hat{\boldsymbol{l}}\right)\hat{\boldsymbol{r}}_{\mathrm{MA}} + \left(3\left(\frac{h_2}{2} - l_2\right)\left(\hat{\boldsymbol{r}}_{\mathrm{MA}} \cdot \hat{\boldsymbol{l}}\right)^2 - \frac{h_2}{2}\right)\hat{\boldsymbol{l}}\Bigg]$$

$$\boldsymbol{l}_{\mathrm{BCRS}} = \boldsymbol{r}_M + \boldsymbol{l}_{\mathrm{LCRS}}\left(1 - \frac{U_M}{c^2}\right) - \frac{1}{2}\left(\frac{\dot{\boldsymbol{r}}_M \cdot \boldsymbol{l}_{\mathrm{LCRS}}}{c^2}\right)\dot{\boldsymbol{r}}_M$$

$$\boldsymbol{l}_{\mathrm{LCRS}} = R_{\mathrm{L2C}}\,\boldsymbol{l}_{\mathrm{PA}} + \boldsymbol{\Delta}^{(E)}_{\mathrm{solidmoon}} + \boldsymbol{\Delta}^{(S)}_{\mathrm{solidmoon}}$$

Unmodeled effects

Longitude libration in MER

$$\Delta\Lambda = A_1 \cos l' + A_2 \cos(2l - 2D) + A_3 \cos(2F - 2L)$$

l' – Sun mean anomaly, *l* – Moon mean anomaly, *D* – elongation of Moon from Sun, *F* – argument of latitude.

MER ➛ PA:

$$R_{\mathrm{libr}}(\Lambda) = R_x(-\delta_x)R_y(-\delta_y)R_z(\Lambda)R_y(\delta_y)R_x(\delta_x) \approx R_z(\Lambda)$$

Eccentricity rate (extra de/dt)

From analytic theory (Chapront-Touzé, Chapront 1988):

$$\mathrm{d}A/\mathrm{d}e \approx -\frac{20905.4}{e}\cos l - \frac{3699.1}{e}\cos(2D-l) \approx -380791\cos l - 67379\cos(2D-l)$$

Effects in Moon's inner structure

C_{32}, S_{32}, C_{33} are estimated, not taken from GL660b.

Implementations of JPL and IAA

	JPL	IAA RAS
Earth rotation	Modified IAU1980 with additional estimated parameters, JPL KEOF series	IAU2000/2006 (SOFA), EOP C04 series (KEOF before 1984)
Geopotential	EGM2008 (?) with modified C_{20}	The same + IERS conventional model (EGM2008-based)
Tidal orbit perturbations	Simplified with 3 fixed and 2 estimated parameters	The same + additional solution with IERS2010 (up to 2 degree)
Moon gravitational potential	Solution close to GL660b; C21 = S21 = S22 = 0 with additional periodical variations. C32, S32, C33 are estimated.	The same
Solar system	DE430	EPM

Estimated parameters

Initial parameters (15)

- Moon position and velocity $(x, y, z, \dot{x}, \dot{y}, \dot{z})$
- Initial libration angles (φ, θ, ψ) and their rates
- Angular velocity of fluid core (ω_c)

Dynamical parameters (9)

- Sum of GM of Earth and Moon
- Inertia parameters $\beta = (C - A)/B$, $\gamma = (B - A)/C$
- Tidal delay τ
- Moon's potential coefficients C_{32}, S_{32}, C_{33}
- Oblateness of fluid core f_c, friction parameter k_v/C

Reduction parameters (72)

- Positions of 5 reflectors and 7 sites
- Velocities of CERGA and Apache/McDonald/MLRS1/MLRS2
- h_2 of Moon
- Amplitudes of additional terms in libration ψ: $\cos l'$, $\cos(2l\text{-}2D)$, $\cos(2F\text{-}2l)$
- 28 station biases

All parameters of dynamical model

Notation	parameter	type	notes
μ_S	standard gravitational parameter of the Sun	fixed	fixed to DE430 value in this work; may differ in the EPM ephemeris
μ_E/μ_M	Earth-Moon mass ratio	fixed	determined from spacecraft observations; fixed to DE430 value in this work, may differ in EPM
$\mu_E+\mu_M$	standard gravitational parameter of the E-M system	fit	
$\bar{C}_{\mathrm{nm,E}}$, $\bar{S}_{\mathrm{nm,E}}$	spherical harmonic coefficients of Earth's gravitational potential	fixed	up to $n_{\max}=6$, taken from model based on EGM2008, see section 6.1 of Conventions; DE tidal model comes with an altered $\bar{C}_{20,\mathrm{E}}$
k_{20}, k_{21}, k_{22}	potential degree-2 Love numbers of Earth zonal, diurnal, and semi-diurnal tides	fixed	in DE tidal model: $k_{20}=0.335$, $k_{21}=0.320$, $k_{22}=0.282$; IERS tidal model is more complex
$\tau_{0\mathrm{O}}$, $\tau_{1\mathrm{O}}$, $\tau_{2\mathrm{O}}$	orbital delays of Earth zonal, diurnal, and semi-diurnal tides	fixed/absent	only in DE tidal model: $\tau_{0\mathrm{O}}=0.0780$ d, $\tau_{1\mathrm{O}}=-0.44$ d, $\tau_{2\mathrm{O}}=-0.113$ d
$\tau_{1\mathrm{R}}$, $\tau_{2\mathrm{R}}$	rotational delays of Earth diurnal, semi-diurnal tides	fit/absent	present only in DE tidal model
l_2, k_2	degree-2 lunar Shida number and Love number	fixed	taken from GRAIL results
h_2	degree-2 lunar radial displacement Love number	fit	
$\bar{C}_{20}$	undistorted normalized main zonal lunar harmonic	fixed	taken from GRAIL (solution GL660b)
β, γ	ratios between undistorted main moments of itertia	fit	
$\bar{C}_{21}$, $\bar{S}_{21}$, $\bar{S}_{22}$	other degree-2 harmonics	fixed	zero; $\bar{S}_{21}$ taken from GL660b in one solution
$\bar{C}_{32}$, $\bar{S}_{32}$, $\bar{S}_{33}$	some degree-3 harmonics	fit	
$\bar{C}_{\mathrm{nm}}$, $\bar{S}_{\mathrm{nm}}$	other lunar harmonics	fixed	taken from GL660b up to degree 6
τ	lunar tidal delay	fit	
f_c	oblateness of the lunar core	fit	
k_v/C_T	CMB interaction	fit	
α_c	core polar moment / undistorted total polar moment	fixed	DE430 fixed value 0.0007
A_1, A_2, A_3	unmodeled longitude libration amplitudes	fit	
$\boldsymbol{l}_{\mathrm{PA}}$ ($\times 5$)	positions of five lunar retroreflectors	fit	
$\boldsymbol{r}_{\mathrm{EM}}$, $\dot{\boldsymbol{r}}_{\mathrm{EM}}$	position and velocity of the Moon w.r.t. Earth in the inertial frame at epoch	fit	
ϕ, θ, ψ, $\dot\phi$, $\dot\theta$, $\dot\psi$	Euler angles and their rates at epoch	fit	
$\boldsymbol{s}_{\mathrm{TRS}}$ ($\times 7$), $\dot{\boldsymbol{s}}_{\mathrm{TRS}}$ ($\times 4$)	positions and velocities of stations at their epochs	fixed/fit	see Table 4
$\boldsymbol{\omega_c}$	angular velocity of the lunar core at epoch	fit	
b ($\times 28$)	biases	fit	see Table 2
$\mathrm{d}e/\mathrm{d}t$	extra eccentricity rate	fit/absent	present in some solutions

Parameter	type	notes
McDonald position	fit	epoch 01.01.1991
MLRS1 position	fit	epoch 01.01.1991
MLRS2 position	fit	epoch 01.01.1991
McDonald, MLRS1, MLRS2 velocity	fit	
Apache position	fit	epoch 01.06.2009
Apache velocity	fixed	GNSS solution (P027): $(-1.35, 0.03, -0.04)^T$ cm/yr
CERGA position	fit	epoch 01.01.2000
CERGA velocity	fit	
Haleakala position	fit	epoch 01.04.1986
Haleakala velocity	fixed	GNSS solution: $(-1.30, 6.16, 3.21)^T$ cm/yr
Matera position	fit	epoch 01.01.2008
Matera velocity	fixed	GNSS solution: $(-1.85, 1.86, 1.47)^T$ cm/yr

Obtained solutions

All the solutions are based on the same set of observations, while differing slightly in dynamical models and determined parameters.

SOLUTION I	**DE tidal model**, de/dt is absent (close to the original DE430).
SOLUTION Ie	same as SOLUTION I, but with de/dt fit
SOLUTION II	**IERS tidal model**, de/dt is absent.
SOLUTION IIe	same as SOLUTION II, but with de/dt fit

One-way wrms in cm

Station	Data span	SOLUTION I			SOLUTION II		
		used	rej.	wrms	used	rej.	wrms
McDonald	1970-1985	3545	59	**19.9**	3545	59	**20.1**
MLRS1	1983-1988	587	44	**11.0**	588	43	**11.3**
MLRS2	1988-2013	3210	443	**3.5**	3206	447	**3.8**
Haleakala	1984-1990	748	22	**5.4**	750	20	**5.8**
CERGA (Ruby)	1984-1986	1109	79	**17.2**	1109	79	**17.5**
CERGA (YAG)	1987-2005	8272	52	**2.3**	8271	53	**2.4**
CERGA (MeO)	2009-2013	645	9	**2.2**	645	9	**2.7**
Apache	2006-2012	1546	27	**1.4**	1549	24	**1.5**
Matera	2003-2013	64	19	**3.8**	63	20	**3.3**

Some other results

Extra de/dt drift:
SOLUTION Ie (tidal model DE430):

extra de/dt = $(1.4 \pm 0.2) \times 10^{-12}$/yr
friction parameter $\mathbf{k_v/C_T} = (16.3 \pm 0.2) \cdot 10^{-9}$ /day
$\mathbf{C_{32}} = (14184.3 \pm 0.3) \cdot 10^{-9}$
$\mathbf{S_{32}} = (4931.8 \pm 0.6) \cdot 10^{-9}$
$\mathbf{C_{33}} = (11975 \pm 11) \cdot 10^{-9}$

SOLUTION IIe (tidal model IERS2010):

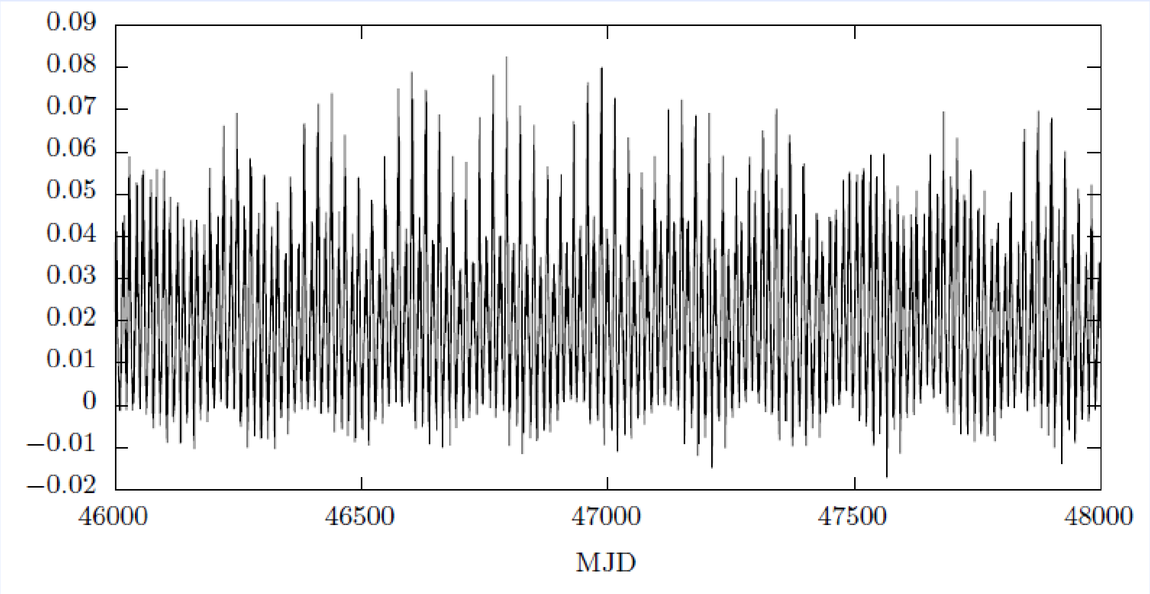
extra de/dt = $(-1.3 \pm 0.2) \times 10^{-12}$/yr
friction parameter $\mathbf{k_v/C_T} = (18.6 \pm 0.2) \cdot 10^{-9}$ /day
$\mathbf{C_{32}} = (14185.5 \pm 0.4) \cdot 10^{-9}$
$\mathbf{S_{32}} = (4937.4 \pm 0.7) \cdot 10^{-9}$
$\mathbf{C_{33}} = (11912 \pm 11) \cdot 10^{-9}$

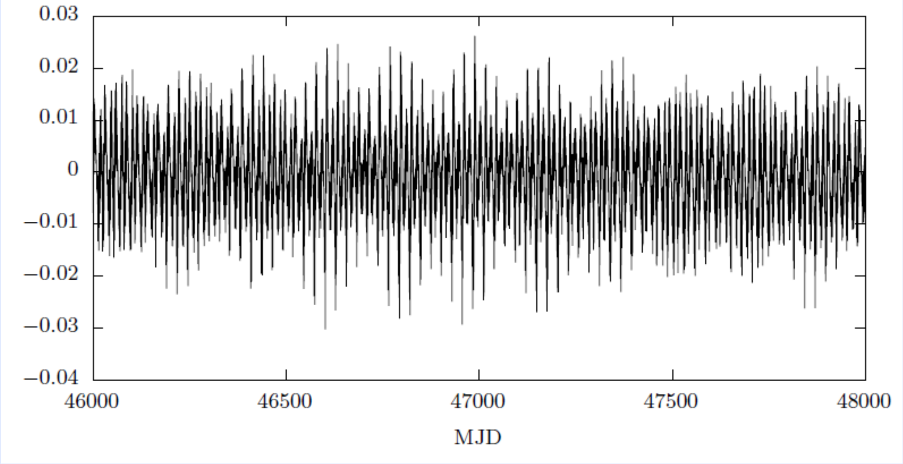
GRAIL values:

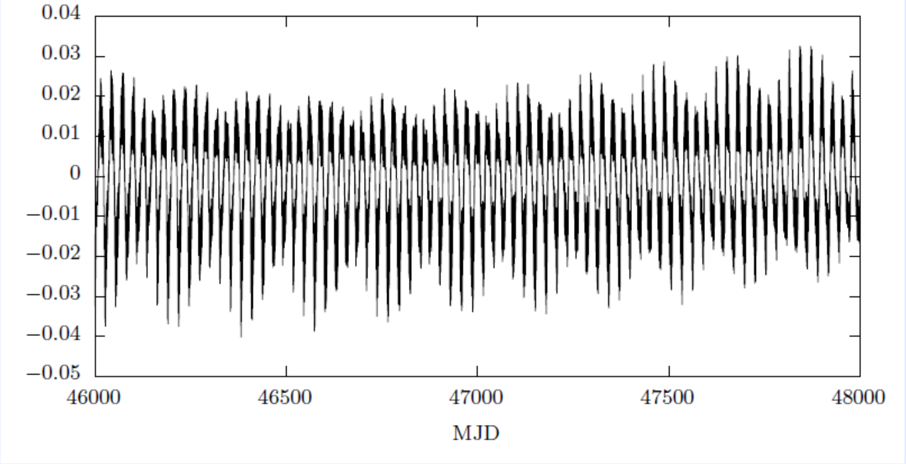
$\mathbf{C_{32}} = (14171.5 \times 10^{-9})$ diff < 0.1%
$\mathbf{S_{32}} = (4878.0 \times 10^{-9})$ diff 0.4-1.2%
$\mathbf{C_{33}} = (12275 \times 10^{-9})$ diff 3%

Tidal acceleration: SOLUTION I minus SOLUTION II

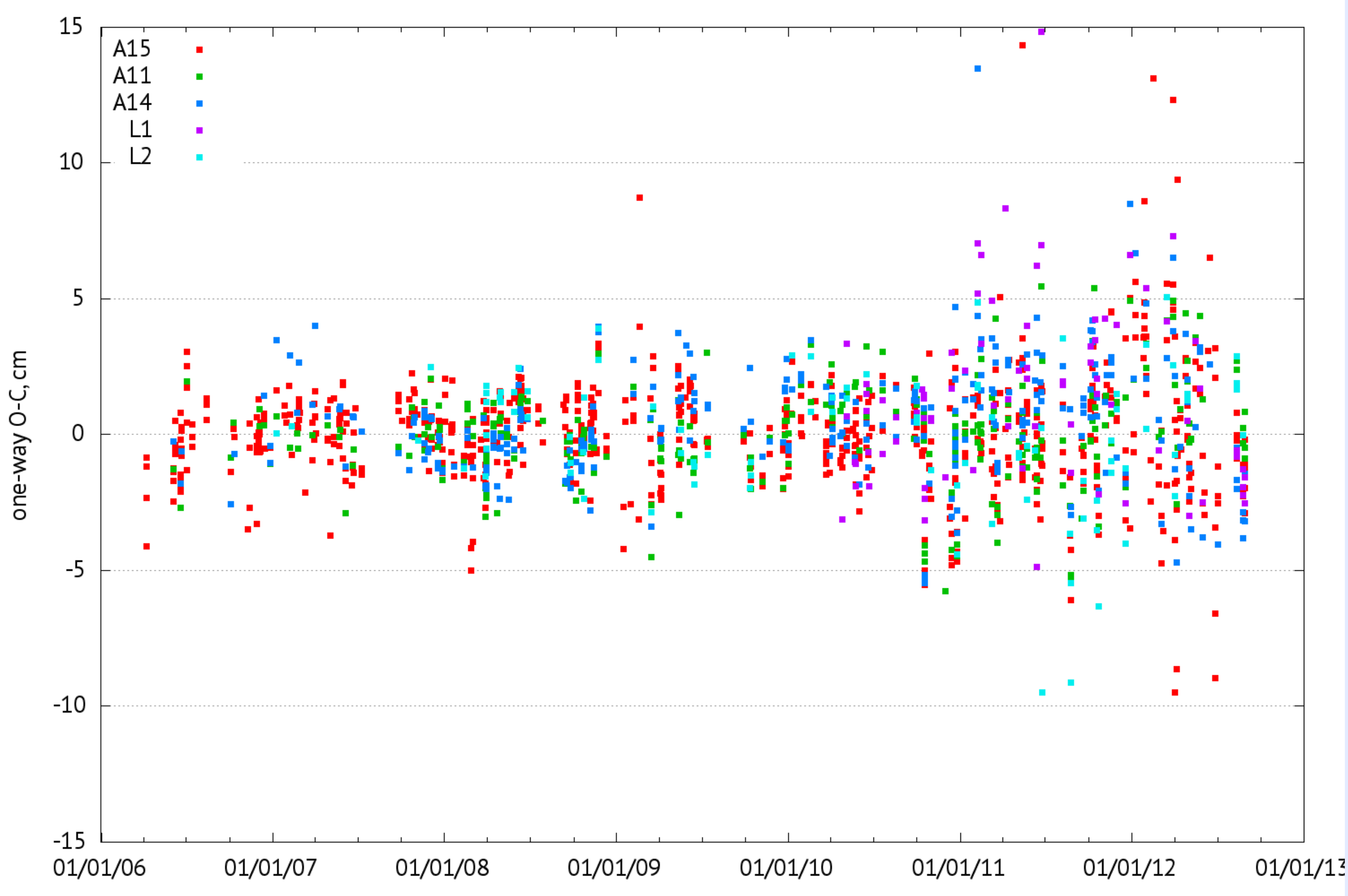
In lunar PA frame, mm/day^2, **X** (towards Earth), **Y** and **Z** components:



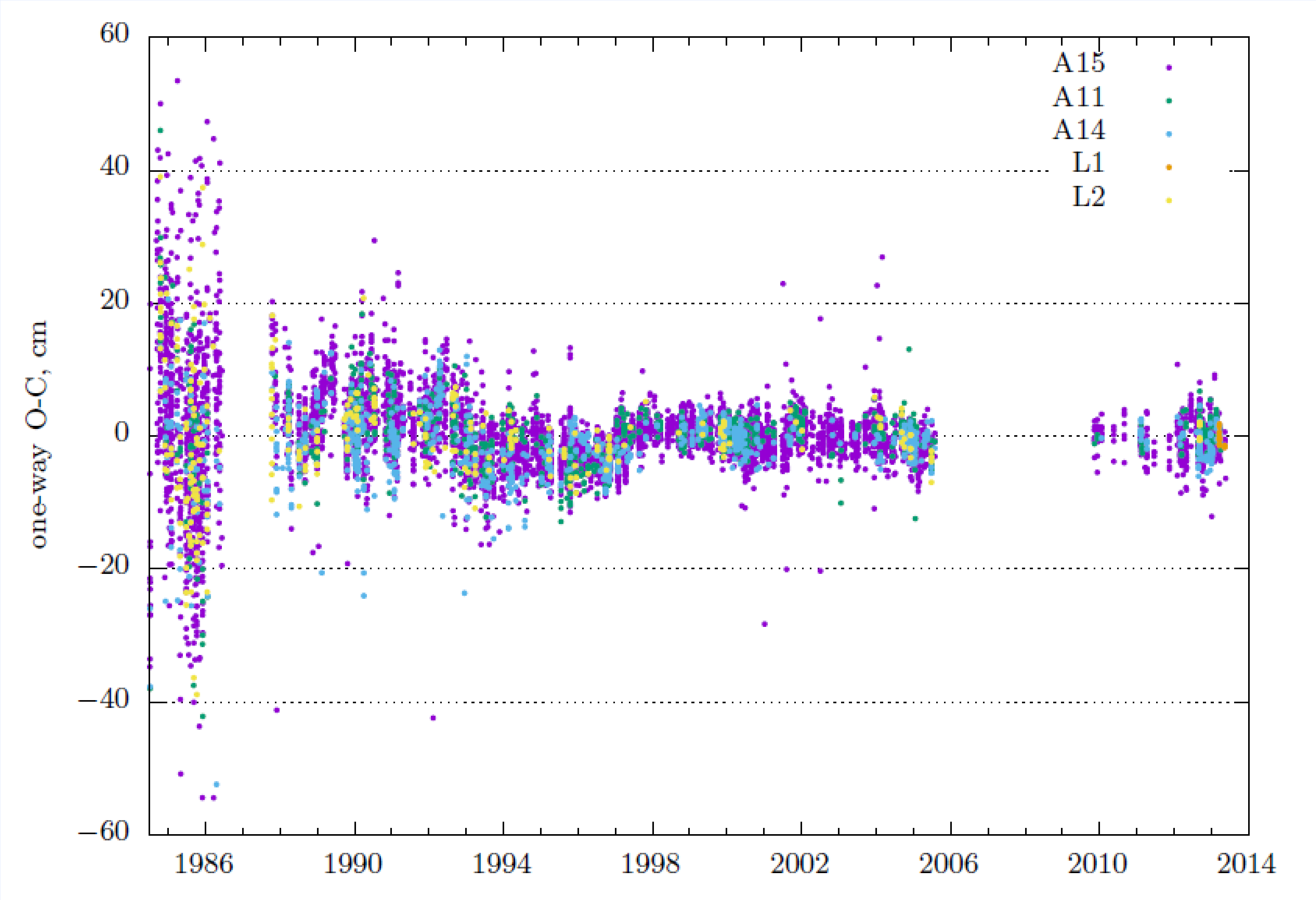




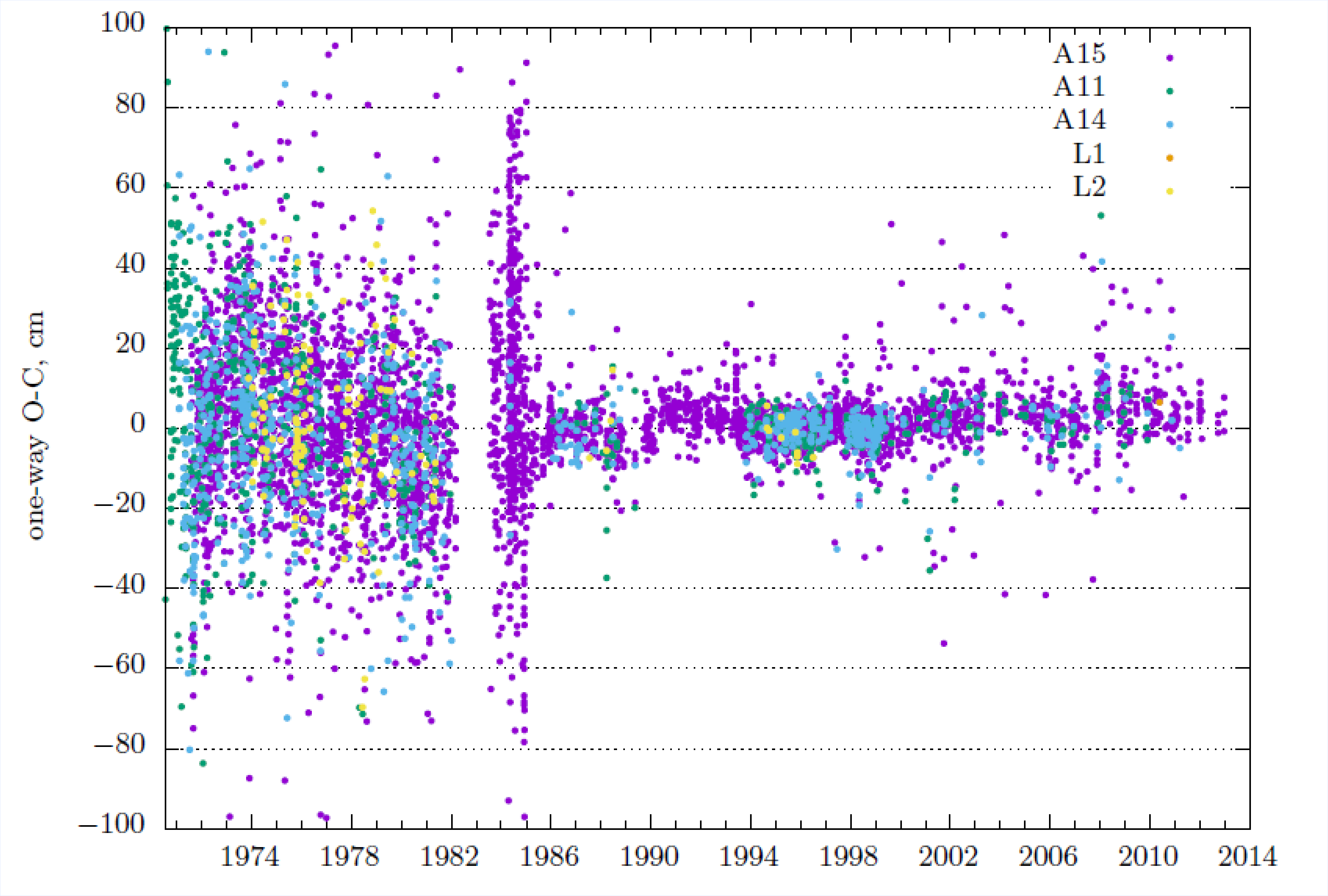
Apache residuals



CERGA residuals



McDonald, MLRS1, and MLRS2 postfit **residuals**



Summary

– Full implementation of **DE430 lunar model** was done and built into the **EPM** ephemeris software;

– The conventional EGM2008-based model of Geopotential is suitable for analyzing LLR observations;

– **IAU2000/2006** PN + **IERS C04 EOP** series are suitable for processing LLR observations (except of IERS C04 before 1984 – JPL KEOF EOP series used)

– IERS 2010 recommended model of **solid tides** for station displacements, model of **tropospheric delay** are suitable for processing LLR observations.

– Implemented IERS recommended Geopotential perturbations model is slightly worse for LLR compared to DE430 model of tidal acceleration of the orbit of the Moon (probably because of two additional parameters in DE model). Lunar and extra de/dt are very sensitive to the tidal model used (extra de/dt falls from 1.4×10^{-12} with the “DE model” to -1.3×10^{-12} with the “IERS model”)

– Towards Earth tidal acceleration is bigger in “IERS” compared to “DE”. Probably due to k_{21} in K_1 is smaller than average value used in DE430.
– Strong detection of $\mathbf{k_V/C_T}$ demonstrates that the Moon has a fluid core.
– Determined C_{32} differs from GL660b by <0.1%; derived C_{22} is also very close to GL660b;
– Determined S_{32} value differs from the GL660b value by 0.4–1.2%;
– Determined C_{33} value differs by about 3% from the GL660b value.

Further plans

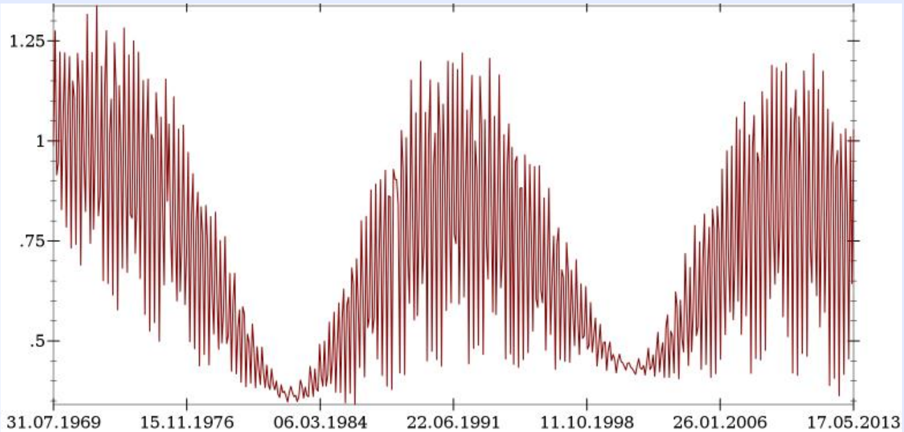
1. Study of different tropospheric models and ocean loading to observations fit.
2. Research the cause of the misalignment of the lunar PA frame in the model with the GRAIL's frame. (non-zero values of C_{21}, S_{21}, S_{22} in GL660b). Probably a better model is needed.
3. Explanation why GRAIL and LLR give different S_{32}, C_{33}.

1. Including new observations (next talk, Dr. E. Yagudina), particularly IR(?).

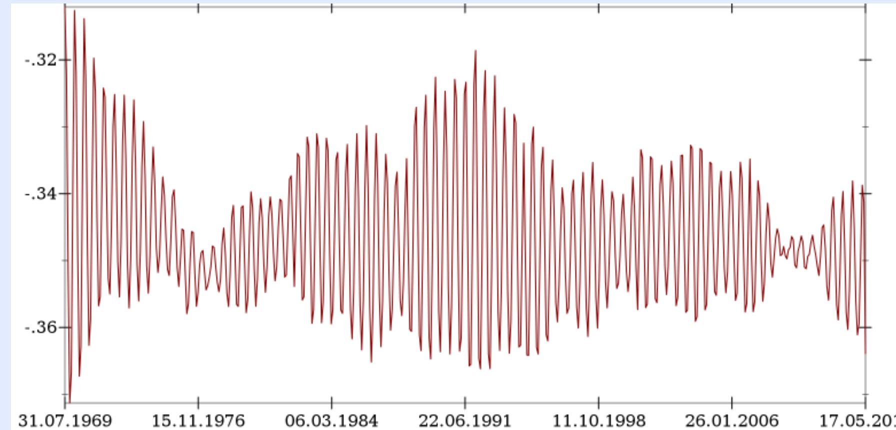
Detailed description of the work:
D.A. Pavlov, J. G. Williams, V. V. Suvorkin: **Determining parameters of Moon's orbital and rotational motion from LLR observations using GRAIL and IERS-recommended models**
Celestial Mechanics and Dynamical Astronomy 126(1), 61-88 (2016)

Thank you for your attention!

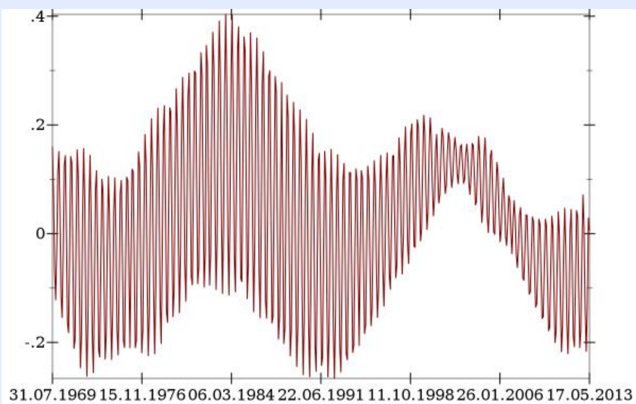
Difference to DE430



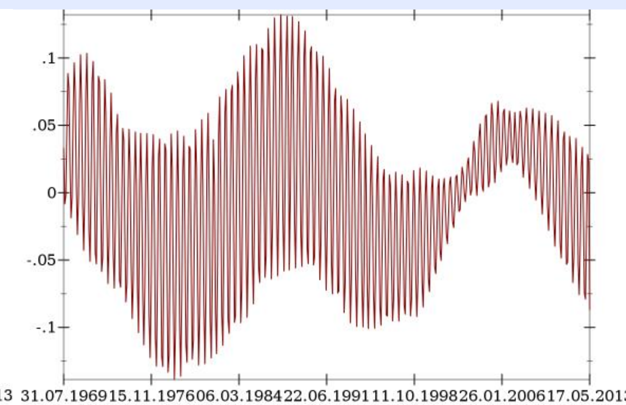
Moon position on orbit, m



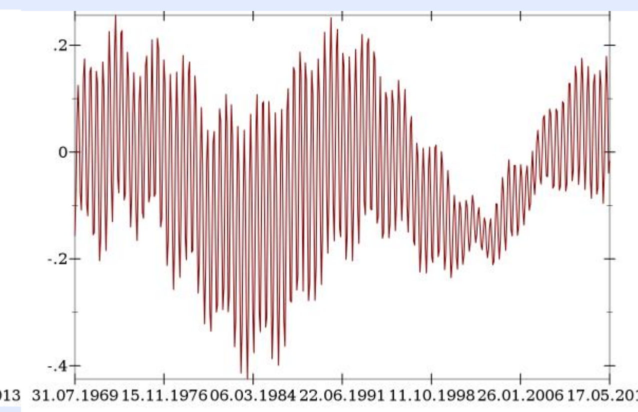
Distance from Earth to Moon, m



libration angle φ, m



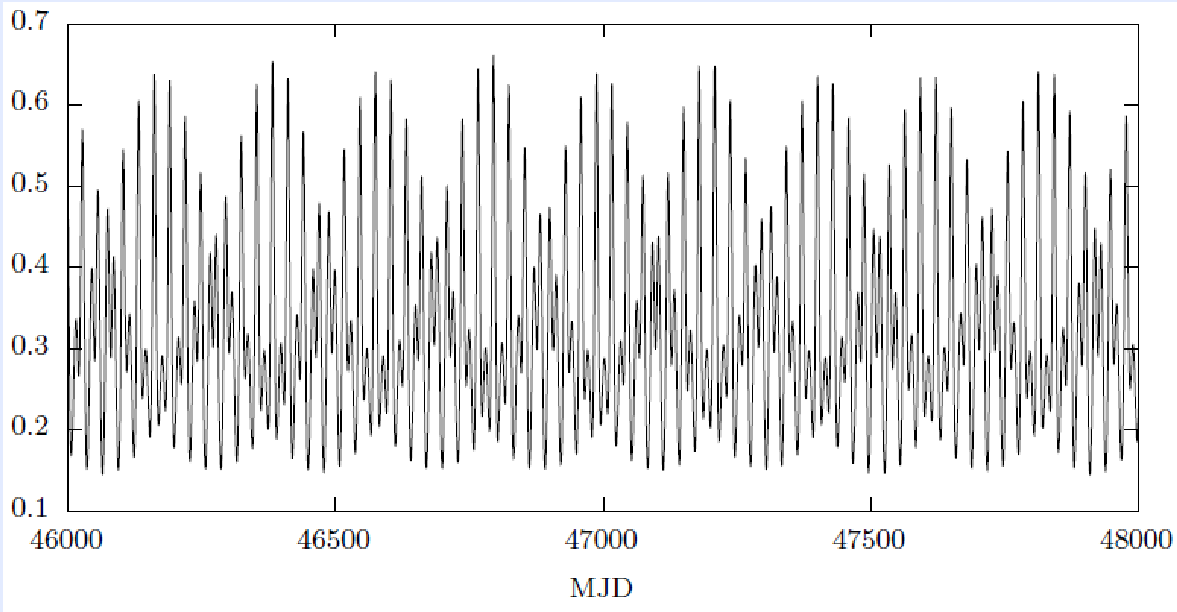
libration angle θ, m

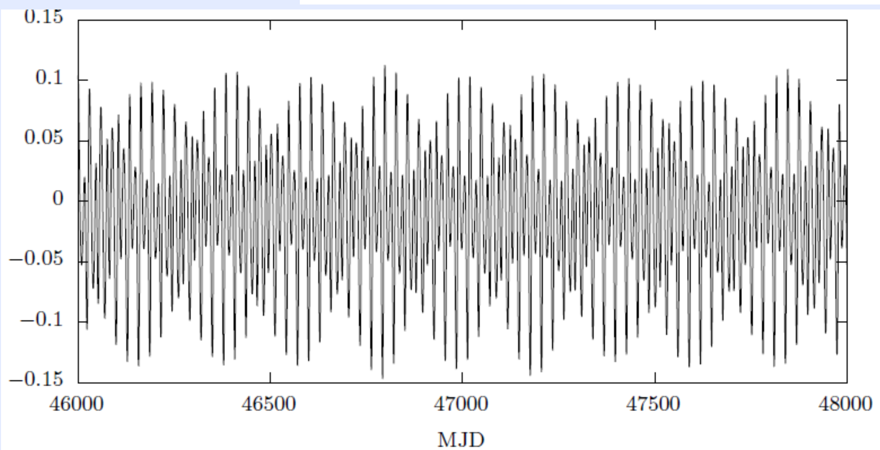


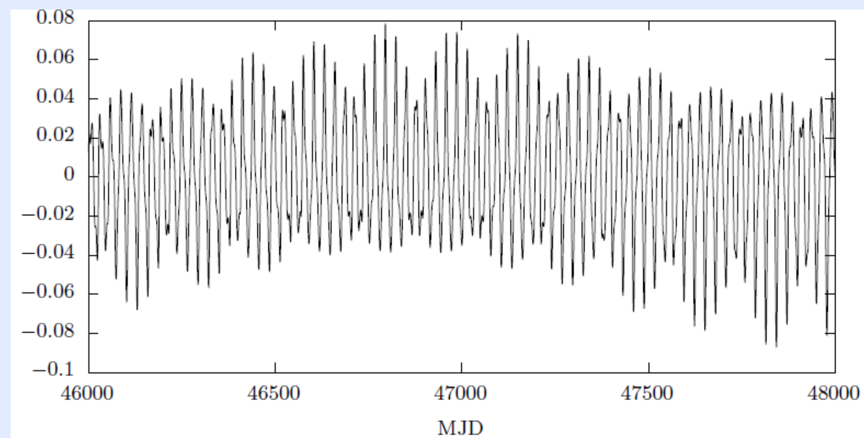
libration angle ψ, m

DE tidal acceleration

In lunar frame, mm/day^2, X (towards Earth), Y and Z components







Estimated parameters

Parm.	Solution I value	Solution II value	units
$\boldsymbol{r}_{\mathrm{EM}}$.x	-137136474.08 ± 0.05	-137136473.34 ± 0.06	m
$\boldsymbol{r}_{\mathrm{EM}}$.y	-311514604.04 ± 0.05	-311514604.28 ± 0.06	m
$\boldsymbol{r}_{\mathrm{EM}}$.z	-141738600.42 ± 0.05	-141738600.20 ± 0.05	m
$\dot{\boldsymbol{r}}_{\mathrm{EM}}$.x	962372276.17 ± 0.15	962372276.40 ± 0.16	μm/sec
$\dot{\boldsymbol{r}}_{\mathrm{EM}}$.y	-375608190.25 ± 0.14	-375608188.58 ± 0.15	μm/sec
$\dot{\boldsymbol{r}}_{\mathrm{EM}}$.z	-268439311.47 ± 0.06	-268439310.20 ± 0.07	μm/sec
$\dot{\boldsymbol{\omega}}_{c}$.x	$(-908 \pm 4) \cdot 10^{-6}$	$(-952 \pm 4) \cdot 10^{-6}$	rad/day
$\dot{\boldsymbol{\omega}}_{c}$.y	$(-6378 \pm 8) \cdot 10^{-6}$	$(-6439 \pm 8) \cdot 10^{-6}$	rad/day
$\dot{\boldsymbol{\omega}}_{c}$.z	$(229.63 \pm 0.05) \cdot 10^{-3}$	$(230.24 \pm 0.02) \cdot 10^{-3}$	rad/day
ϕ	$(-5823794 \pm 2) \cdot 10^{-8}$	$(-5823801 \pm 2) \cdot 10^{-8}$	rad
θ	$(39511623 \pm 1) \cdot 10^{-8}$	$(39511619 \pm 1) \cdot 10^{-8}$	rad
ψ	$(113574558 \pm 3) \cdot 10^{-8}$	$(113574573 \pm 3) \cdot 10^{-8}$	rad
$\dot{\phi}$	-74.538 ± 0.001	-74.544 ± 0.001	″/day
$\dot{\theta}$	-37.0264 ± 0.0004	-37.026 ± 0.004	″/day
$\dot{\psi}$	47501.855 ± 0.001	47501.860 ± 0.001	″/day
$\mu_E + \mu_M$	403503.2366 ± 0.0002	403503.2360 ± 0.0002	$\mathrm{km}^3/\mathrm{s}^2$
β	$(631027.9 \pm 0.5) \cdot 10^{-9}$	$(631027.9 \pm 0.5) \cdot 10^{-9}$	1
γ	$(227734.5 \pm 0.7) \cdot 10^{-9}$	$(227735.4 \pm 0.7) \cdot 10^{-9}$	1
τ	0.099 ± 0.001	0.072 ± 0.001	day
$\tau_{1\mathrm{R}}$	0.00783 ± 0.0003	N/A	day
$\tau_{2\mathrm{R}}$	0.002862 ± 0.00003	N/A	day

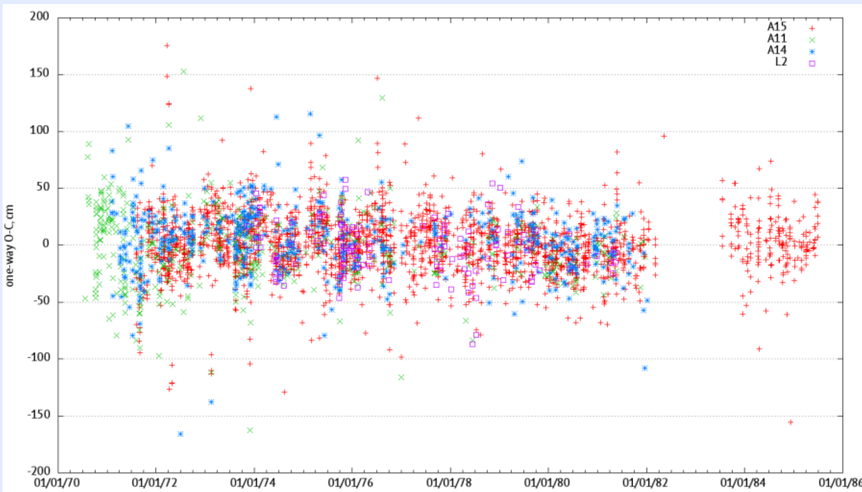
f_c	$(0.209 \pm 0.005) \cdot 10^{-3}$	$(0.219 \pm 0.005) \cdot 10^{-3}$	1
k_v/C_T	$(16.2 \pm 0.2) \cdot 10^{-9}$	$(19.9 \pm 0.2) \cdot 10^{-9}$	day^{-1}
h_2	0.042 ± 0.001	0.040 ± 0.001	1
A_1	4.6 ± 0.2	4.6 ± 0.2	mas
A_2	1.8 ± 0.2	1.0 ± 0.2	mas
A_3	-6.7 ± 0.5	-12.3 ± 0.5	mas
C_{32}	$(14185.1 \pm 0.4) \cdot 10^{-9}$	$(14186.4 \pm 0.4) \cdot 10^{-9}$	1
S_{32}	$(4930.5 \pm 0.7) \cdot 10^{-9}$	$(4935.0 \pm 0.8) \cdot 10^{-9}$	1
S_{33}	$(11965 \pm 11) \cdot 10^{-9}$	$(11927 \pm 12) \cdot 10^{-9}$	1
A11 x	1591966.95 ± 0.06	1591966.77 ± 0.07	m
A11 y	690699.52 ± 0.04	690699.47 ± 0.05	m
A11 z	21003.76 ± 0.02	21003.80 ± 0.02	m
A14 x	1652689.88 ± 0.07	1652689.71 ± 0.07	m
A14 y	-520997.52 ± 0.04	-520997.66 ± 0.05	m
A14 z	-109730.51 ± 0.02	-109730.48 ± 0.02	m
A15 x	1554678.62 ± 0.07	1554678.41 ± 0.07	m
A15 y	98095.60 ± 0.04	98095.46 ± 0.04	m
A15 z	765005.20 ± 0.03	765005.22 ± 0.04	m
L1 x	1114292.57 ± 0.06	1114292.36 ± 0.07	m
L1 y	-781298.49 ± 0.04	-781298.75 ± 0.04	m
L1 z	1076058.50 ± 0.04	1076058.37 ± 0.04	m
L2 x	1339363.70 ± 0.06	1339363.49 ± 0.07	m
L2 y	801872.00 ± 0.04	801871.94 ± 0.04	m
L2 z	756358.66 ± 0.04	756358.67 ± 0.04	m

Parameter	Solution I value	Solution II value	units
McD λ	$17.06520015(7 \pm 6)$	$17.06520013(2 \pm 2)$	hours
McD $r\cos\phi$	5492414.47 ± 0.03	5492414.45 ± 0.03	m
McD $r\sin\phi$	3235697.50 ± 0.02	3235697.47 ± 0.02	m
MLRS1 λ	$17.06560804(6 \pm 4)$	$17.06560805(1 \pm 4)$	hours
MLRS1 $r\cos\phi$	5492037.72 ± 0.02	5492037.65 ± 0.02	m
MLRS1 $r\sin\phi$	3236146.77 ± 0.01	3236146.77 ± 0.01	m
MLRS2 λ	$17.06565358(3 \pm 1)$	$17.06565358(6 \pm 2)$	hours
MLRS2 $r\cos\phi$	5491888.44 ± 0.01	5491888.44 ± 0.01	m
MLRS2 $r\sin\phi$	3236481.67 ± 0.01	3236481.66 ± 0.01	m
CERGA λ	$0.46143818(5 \pm 1)$	$0.46143818(3 \pm 1)$	hours
CERGA $r\cos\phi$	4615328.453 ± 0.002	4615328.488 ± 0.002	m
CERGA $r\sin\phi$	4389355.108 ± 0.003	4389355.106 ± 0.003	m

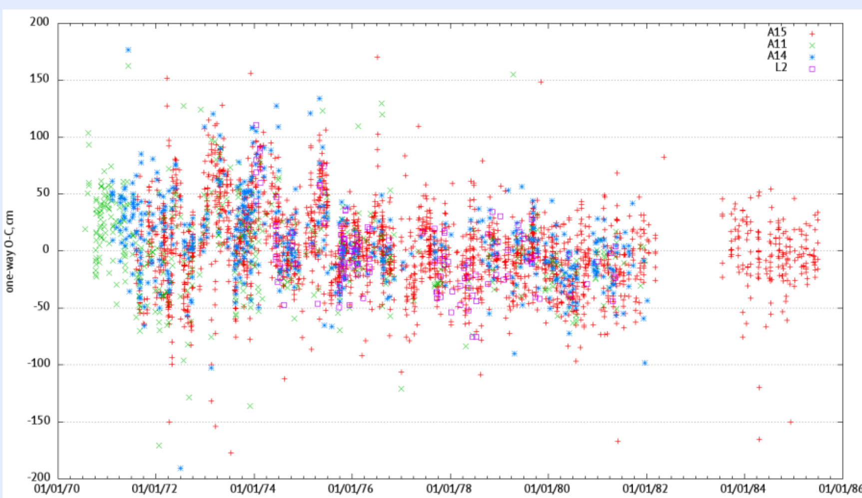
Haleakala λ	$13.58293969(6 \pm 1)$	$13.58293970(6 \pm 2)$	hours
Haleakala $r\cos\phi$	5971474.51 ± 0.01	5971474.53 ± 0.01	m
Haleakala $\sin\phi$	2242.188420 ± 0.01	2242.18845 ± 0.01	m
Apache λ	$16.94530512(0 \pm 1)$	$16.94530511(7 \pm 1)$	hours
Apache $r\cos\phi$	5370045.374 ± 0.003	5370045.379 ± 0.003	m
Apache $\sin\phi$	3435012.901 ± 0.002	3435012.911 ± 0.002	m
Matera λ	$1.1136409(0 \pm 6)$	$1.1136408(9 \pm 1)$	hours
Matera $r\cos\phi$	4846504.3 ± 0.2	4846504.23 ± 0.05	m
Matera $r\sin\phi$	4133249.59 ± 0.07	4133249.59 ± 0.02	m
McD $\dot{\lambda}$	-0.57 ± 0.1	-0.65 ± 0.01	mas/yr
McD $(r\cos\phi)^{\cdot}$	4.4 ± 0.2	2.9 ± 0.2	mm/yr
McD $(r\sin\phi)^{\cdot}$	1.7 ± 0.5	0.2 ± 0.5	mm/yr
CERGA $\dot{\lambda}$	0.923 ± 0.008	0.916 ± 0.009	mas/yr
CERGA $(r\cos\phi)^{\cdot}$	-15.7 ± 0.2	-16.9 ± 0.2	mm/yr
CERGA $(r\sin\phi)^{\cdot}$	14.5 ± 0.4	14.3 ± 0.5	mm/yr

IERS C04 and early LLR observations

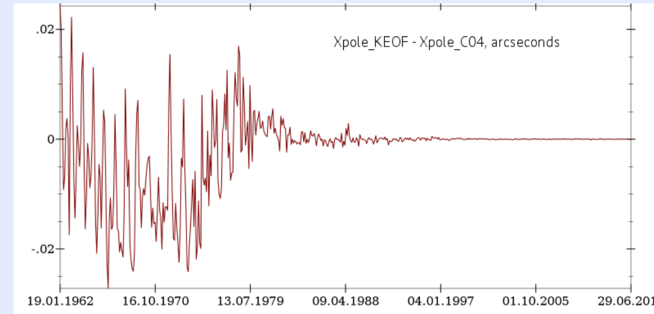
O-C для наблюдений McDonald (1970-1985):

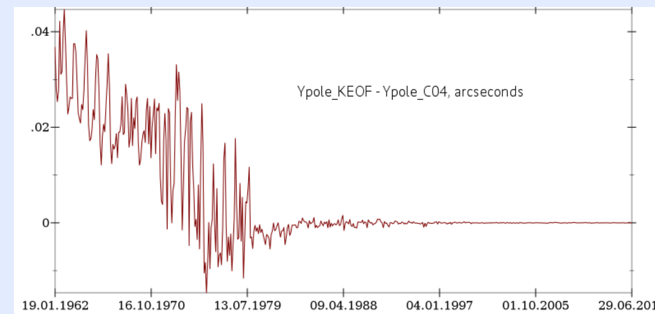


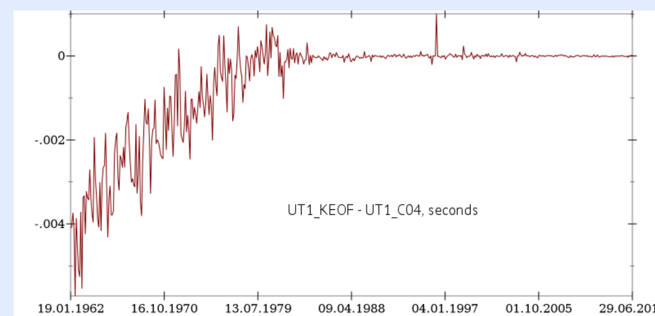
JPL KEOF
wrms = 21 см



IERS C04
wrms = 32 см







JPL KEOF vs IERS C04,
1962-2014

misc

Normal points of LLR:
http://polac.obspm.fr/llrdatae.html, http://physics.ucsd.edu/~tmurphy/apollo/norm_pts.html

Software used:
ERA (Ephemeris Research in Astronomy), version 8 – Racket programming platform + C lnguage numerical procedures

SOFA library (Hohenkerk 2012; http://www.iausofa.org) for IAU2000/2006 model, conversion of time scales, calculation of Delaunay arguments, and conversion between geocentric and geodetic coordinates.

For optical zenith delay (Mendes and Pavlis 2004) and mapping function (Mendes et al. 2002), **FCULZD HPA** and **FCUL A** routines were used. Station displacement due to solid tides (Mathews et al. 1997) was calculated with the **DEHANTTIDEINEL** package.

For numerical integration, an implementation of Gauss-Everhart algorithm from (Avdyushev 2010) was used, but rewritten from Fortran to C and modified to use extended precision floating-point numbers (80-bit) instead of double precision (64-bit).