A method for sampling debris laser ranging data to generate range rates for orbit determination

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Abstract

This paper presents a method for generating extra information from the data collected from space debris tracking facilities. Typically only angles and range tracking data are produced. Piecewise least squares Chebyshev polynomial fitting is used to obtain fitted observations and associated rates of change. In the angles-only measurement case, for each observation this yields 4-dimensional information of the required 6 to define a state vector. In the active laser ranging measurement case the full 6-dimensional information can be derived so that a state vector can be produced for each fitted observation. This has important implications in orbit determination, data association methods and data quality metrics. Brief accuracy checks are presented but a full statistical accuracy assessment is deferred to a future publication.

Introduction

Part of the Space Environment Research Centre's objectives is to provide a conjunction and threat warning service to satellite operators and to perform all-on-all conjunction assessments for Earth-orbiting objects. To achieve this goal, a new space object catalogue is being populated using precision observations from passive optical telescopes and active laser tracking systems located in Australia. The tracking systems are automated and are capable of laser ranging non-cooperative objects (objects without retro-reflectors) in low-Earth orbits as well as passive optical tracking in all orbits, subject to visibility.

New tracking sensors have been installed in Western Australia through a partnership between EOS Space Systems and Lockheed Martin with the support from Australian Department of Defence. Figure 1 shows a map of Australia depicting the sensor locations. EOS Space Systems existing debris tracking facility at Mount Stromlo is shown in the southeast image which is also home to the satellite laser ranging facility that is part of the International Laser Ranging Service active network (https://ilrs.cddis.eosdis.nasa.gov/network/stations/active/STL3_general.html).

Building and maintaining a precision observation database, requires careful processing of the data to ensure that the database does not get contaminated by erroneous observations. Tracking system health and calibration audits are a fundamental step. Accurate telescope mount modelling can be achieved if the tracking system has a stable mount, is well-aligned, a good spatial distribution of stars is used for model fitting, while avoiding highly correlated mount model terms and over-parameterisation. Automation is also a key feature of the systems at EOS Space Systems which allows abundant star observations to be collected which assists the fitting process. Another important aspect is data association and validation. This is to ensure that the object that was tracked is the correct one. Errors in the data can arise from many sources and multiple data accuracy checks are needed to reject outliers.

Figure 1: Map of Australia showing the current site locations. The existing site at Mount Stromlo is located in the southeast, and the new site is located in the northwest. The baseline separation is more than 3,500 km.

This paper is focussed on increasing the amount of information derived from the tracking data and presents a preliminary method to obtain the rate of change information from the observation passes. In what follows, the accuracy of the fitted observations is checked against accurate Consolidated Prediction Format (CPF) predictions for: (1) angles-only data collected during a passive debris tracking campaign in 2016; and, (2) angles and range data collected during a previous debris campaign in 2013. The results presented are for Lageos 1 and Lageos 2 due to the reliability of the reference orbits for error checking. Statistical accuracy tests of the method are delayed until a new method for outlier detection is employed in the post-processing and improvements in the mount modelling have been integrated. These will be presented in a follow up paper.

Orbital state generation

A six-dimensional orbital state vector, i.e. position and velocity for example, requires six observables to be defined. If we consider the topocentric slant range vector, $\boldsymbol{\rho}_{SEZ}$:

$$\boldsymbol{\rho}_{SEZ} = \begin{bmatrix} -\rho\cos(el)\cos(\beta) \\ \rho\cos(el)\sin(\beta) \\ \rho\sin(el) \end{bmatrix}, \quad \dot{\boldsymbol{\rho}}_{SEZ} = \frac{d}{dt}\boldsymbol{\rho}_{SEZ},$$

where ρ is the range to the object, β is the azimuth angle, and el is the elevation angle, the geocentric inertial state vector, $\boldsymbol{r}_{ECI}, \boldsymbol{v}_{ECI}$, may be generated using the site location once the observables have been substituted. In this case they are $(\beta, \dot{\beta}, el, \dot{el}, \rho, \dot{\rho})$ where the observation rates of change enter through the rate of change of the slant range vector, $\dot{\boldsymbol{\rho}}_{SEZ}$.

In passive optical tracking the sensor information published may be azimuth and elevation. This satisfies two of the required six dimensions. If the rate of change information is available for each axis then this doubles to four, leaving two quantities $\rho, \dot{\rho}$ to define the object state. The preliminary method to generate the observation rate of change is presented in the next section.

Fitting azimuth and elevation observations using piecewise Chebyshev polynomials

Chebyshev polynomials are a set of orthogonal polynomials that are useful in many areas of approximation theory. They can be defined recursively and minimise Runge's phenomenon when interpolating, where the function derivative towards the endpoints increases without bound as the polynomial order is increased. This oscillatory error is a common problem with high order polynomial interpolation.

In this section we present the method for generating the fitted observations as well as their rates. Piecewise Chebyshev polynomial approximation is used to generate the fitted points for the azimuth and elevation using data collected as part of a passive campaign. During the process the rate of change is also calculated from the derivative of the fit function. The Chebyshev polynomials of the first kind are defined recursively as:

$$T_0(x) = 1, \qquad T_1(x) = x, \qquad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

A plot of the first five Chebyshev polynomials is shown in Figure 2. To create the approximating function we assume a three term Chebyshev polynomial expansion, i.e.

$$f(x) = \sum_{n=0}^{2} a_n T_n(x), \quad x \in [-1,1],$$

where the coefficients a_i are constant. The approximating function is fitted to the observation data at discrete time intervals using least squares. The transformation:

$$x = \frac{2t - (t_a + t_b)}{t_b - t_a}, \; t \in [t_a, t_b],$$

is necessary to normalise the time-series data for the Chebyshev polynomial fitting where $[t_a, t_b]$ is the time interval of the observation data being fitted. The corresponding velocity observation for each axis is given at the fit node by the derivative of the fit function. It is expressed in Chebyshev polynomials of the second kind, which are defined recursively as:

$$U_0(x) = 1, \qquad U_1(x) = 2x, \qquad U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x),$$

and the corresponding velocity observation is given by:

$$f'(x) = \sum_{n=1}^{2} a_n \, n \, U_{n-1}(x), \quad x \in [-1,1],$$

where the coefficients a_i were found in the fitting process.

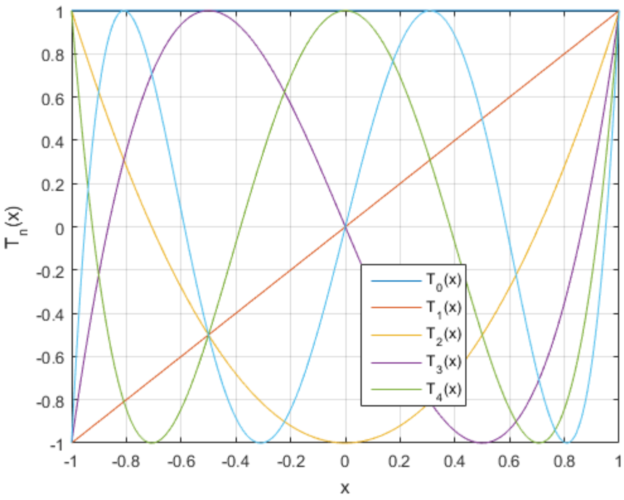

Figure 2: Plot of the first five Chebyshev polynomials of the first kind

Results for passive observation fitting

Figure 3 shows the results of applying the method to passive angles only Lageos 1 and Lageos 2 observations collected during a campaign in 2016. The root mean squared error is 0.9″ in both azimuth and elevation. Figure 4 shows the corresponding azimuth and elevation velocity errors. The root mean squared error was found to be 0.07″/s in the azimuth and 0.09″/s in the elevation. The results presented include low elevation and keyhole passes which suffer from larger pointing errors.

The data fitting gives lower variance than the raw data as well as generating the extra rate of change information. The standard deviation is also estimated from the fit which can be used to weight the observations in an orbit determination process.

Four dimensions of the six required to define a state vector for each observation are now available. To generate orbital states, the range and range rate are also required. Initial orbit determination using admissible regions has been investigated extensively in the literature, see [1-5] and references therein. An admissible region can be defined by restricting the two-body energy equation to create a 2-dimensional $(\rho, \dot{\rho})$ space which is sampled to find likely state vectors. Further constraints may be imposed to reduce this space, and the reader is referred to the aforementioned references for more information on methods that have been considered.

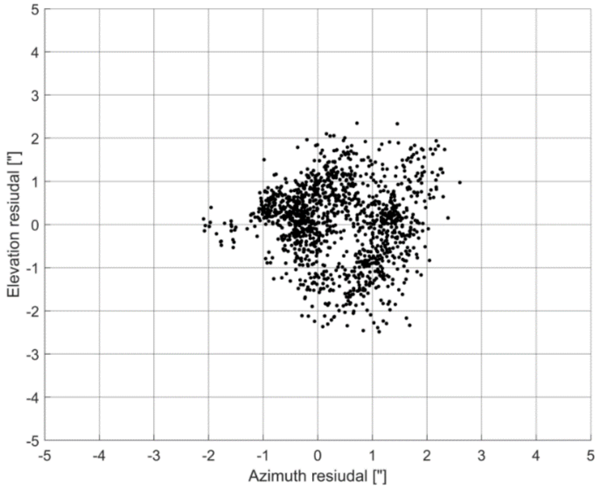

Figure 3: Plot of the azimuth and elevation residuals.

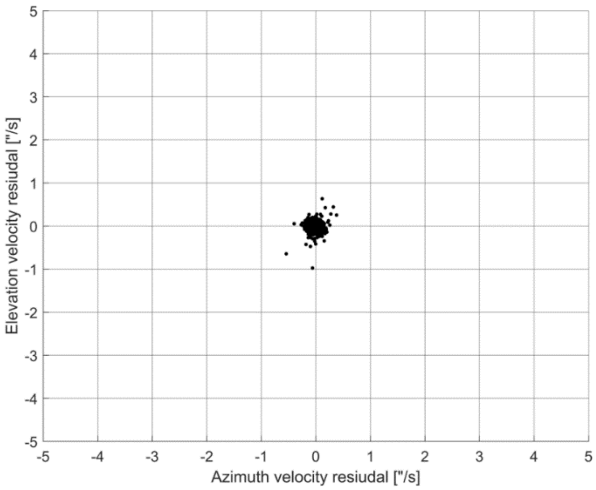

Figure 4: Plot of the azimuth and elevation velocity residuals.

When laser ranging, the range information provides one more dimension of information. Applying the method presented in this paper to the range data will then give full state information for each fitted observation. In the next section, the method is repeated on an azimuth, elevation and range pass for Lageos 1.

Fitting the azimuth, elevation and range observations

When laser tracking uncooperative space debris, accurate 3-dimensional azimuth, elevation, and range observations are collected. If we apply the fitting method to the data a state vector can be produced for each observation epoch. An example of the accuracy of the generated observations is provided. Observations for Lageos 1, collected during a debris tracking campaign in 2013, were fitted to produce state vectors. The angular observations were collected by the optical tracking camera system with the pointing accuracy limited to the mount model accuracy which is typically 1–3 arc-seconds. The range observations were given by the debris laser ranging system. In the post-processing of the data to generate the rates of change, no existing state vector or object-specific information was used.

Figure 5 and Figure 6 show the true Earth-centred Earth Fixed position and velocity errors for a single pass, respectively. The average Euclidean position error was found to be 62 metres, and the average velocity error was 1 m/s for the pass. No object information was assumed and the majority of the error arose from the angular data. Improvements in the mount modelling process will be implemented and the method will be tested rigorously.

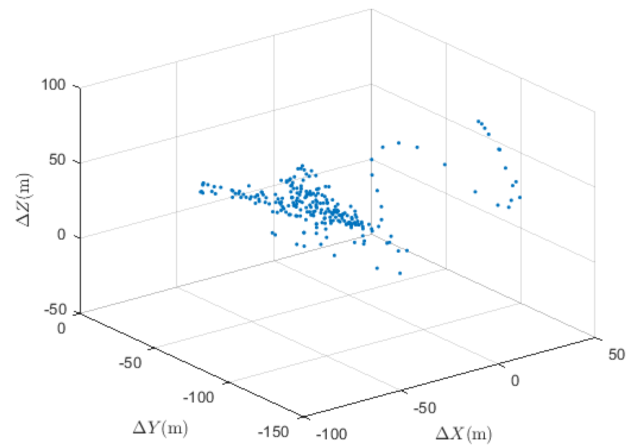

Figure 5: Error of the generated position vectors where the average error is 62 m.

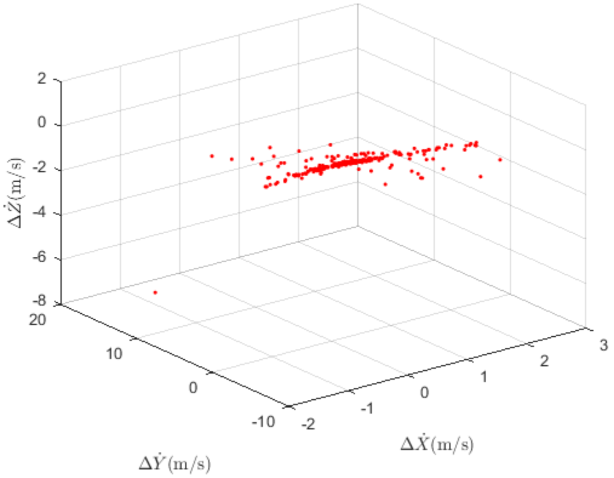

Figure 6: Error of the generated velocity vectors where the average error is 1 m/s.

Conclusions

The fitting method presented is promising and can be used to derive more information in routine uncooperative debris tracking. The method will be refined and then implemented in routine operations. In particular, the focus will be on reliable outlier detection and removal and data association techniques. Improvements in the mount modelling process will achieve more precise models which will improve the pointing accuracy.

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