Lunar Laser Ranging

Putting Gravity to the Test

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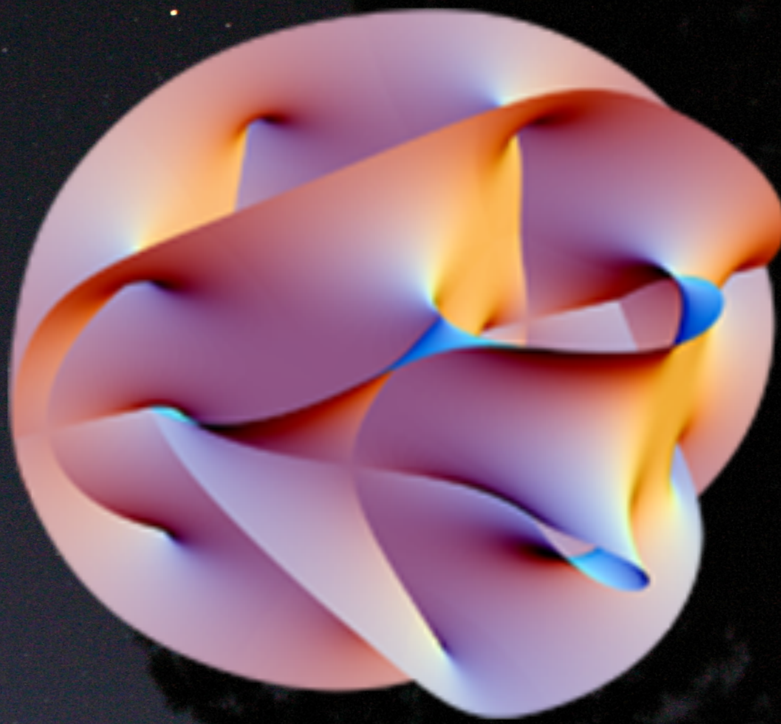
Gravity Cries for Help

- General Relativity (GR) and Quantum Mechanics are fundamentally incompatible
  - gravity relatively poorly tested
- New physics of the dark sector could be misunderstanding of large-scale gravity
  - GR used as metric backdrop for cosmic expansion
- Scalar fields introduced by string-inspired and other modifications to GR produce potentially measurable effects
  - violation of the equivalence principle
  - time variation of fund. “constants”



Relativistic Observables in the Lunar Range

- By measuring the *shape* of the moon's orbit, LLR provides a comprehensive probe of gravity, currently boasting the best tests of:
  - Equivalence Principle (mainly strong version, but check on weak)
    - $\Delta a/a \approx 10^{-13}$; SEP to 4×10^{-4}
  - time-rate-of-change of *G*
    - fractional change $< 10^{-12}$ per year
  - gravitomagnetism (origin of "frame-dragging")
    - to 0.2% (from motions of point masses—not systemic rotation)
  - geodetic precession
    - to ≈ 0.5%
  - $1/r^2$ force law
    - to 10^{-10} times the strength of gravity at 10^8 m scales

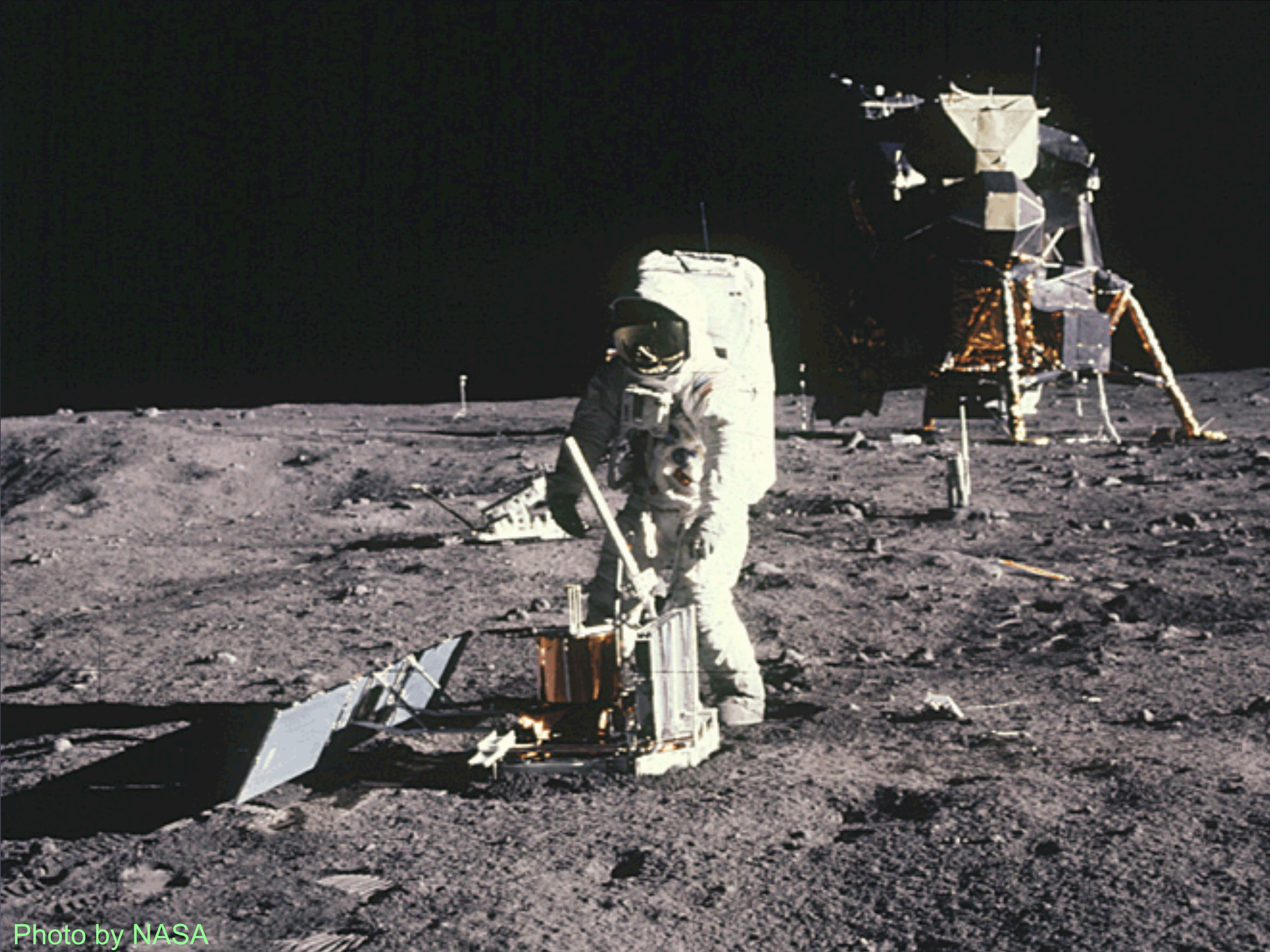


Photo by NASA

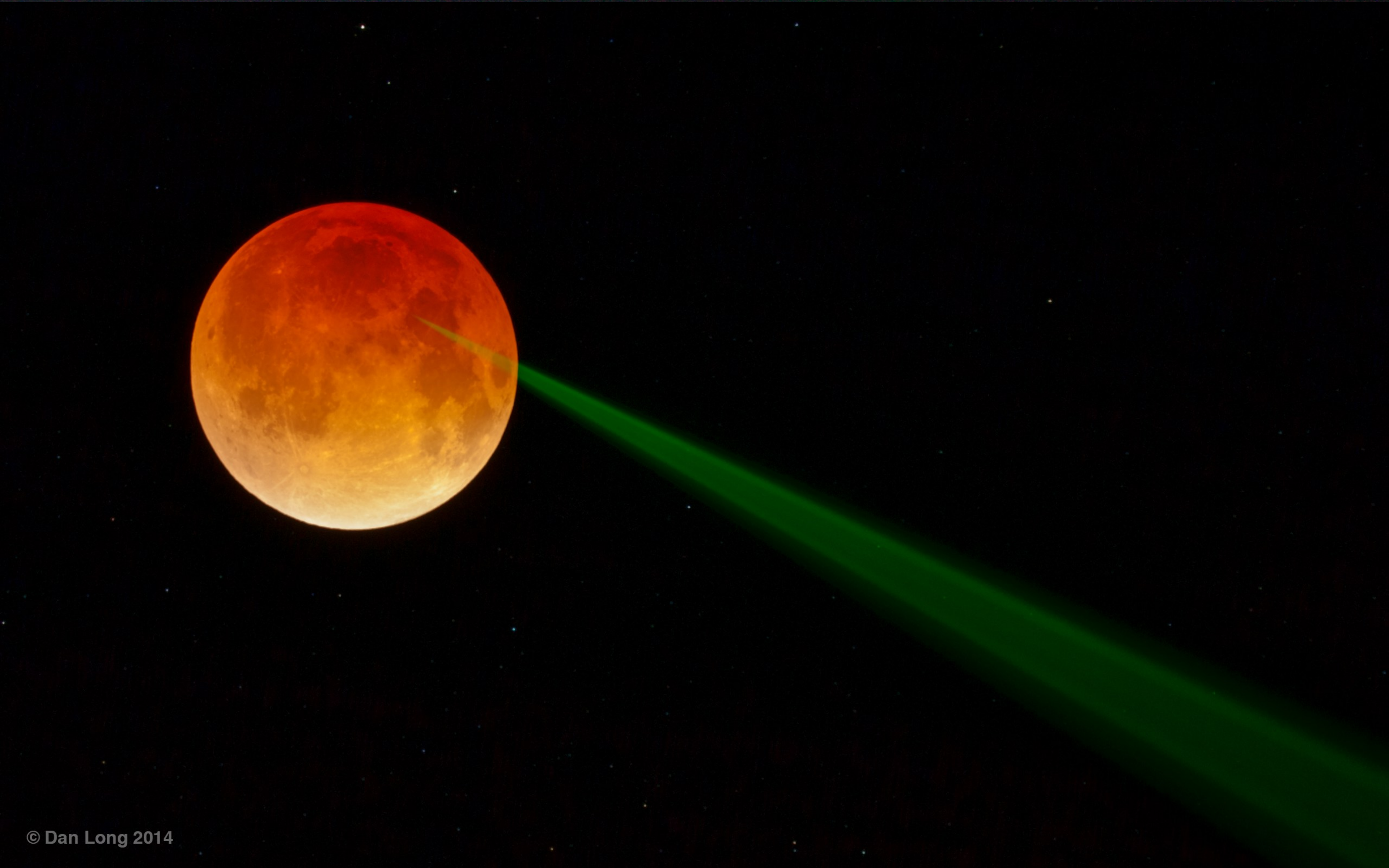
The Reflector Positions

- Three Apollo missions left reflectors
  - Apollo 11: 100-element
  - Apollo 14: 100-element
  - Apollo 15: 300-element
- Two French-built, Soviet-landed reflectors were placed on rovers
  - Luna 17: Lunokhod 1 rover
  - Luna 21: Lunokhod 2 rover
  - similar in size to A11, A14
- Signal loss is huge:
  - $\approx 10^{-8}$ of photons launched find reflector (atmospheric seeing)
  - $\approx 10^{-8}$ of returned photons find telescope (corner cube diffraction)
  - $>10^{17}$ loss considering other optical/ detection losses

Big Bang Theory: Making it Look Easy

photo by Dan Long

2014.04.15 Eclipse



The Full Parameterized Post Newtonian (PPN) Metric

- Generalized metric abandoning many fundamental assumptions
  - GR is a special case
  - Allows violations of conservations, Lorentz invariance, etc.

$$\begin{aligned} g_{00} \;=\; & -1 + 2U - 2\beta U^2 - 2\xi\phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\phi_1 \\ & + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\phi_2 + 2(1 + \zeta_3)\phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\phi_4 \\ & - (\zeta_1 - 2\xi)A - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i \\ & + \mathcal{O}(\epsilon^3) \end{aligned}$$

$$\begin{aligned} g_{0i} \;=\; & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\ & - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}) \end{aligned}$$

$$g_{ij} \;=\; (1 + 2\gamma U + \mathcal{O}(\epsilon^2))\delta_{ij}$$

Simplified (Conservative) PPN Equations of Motion (EoM)

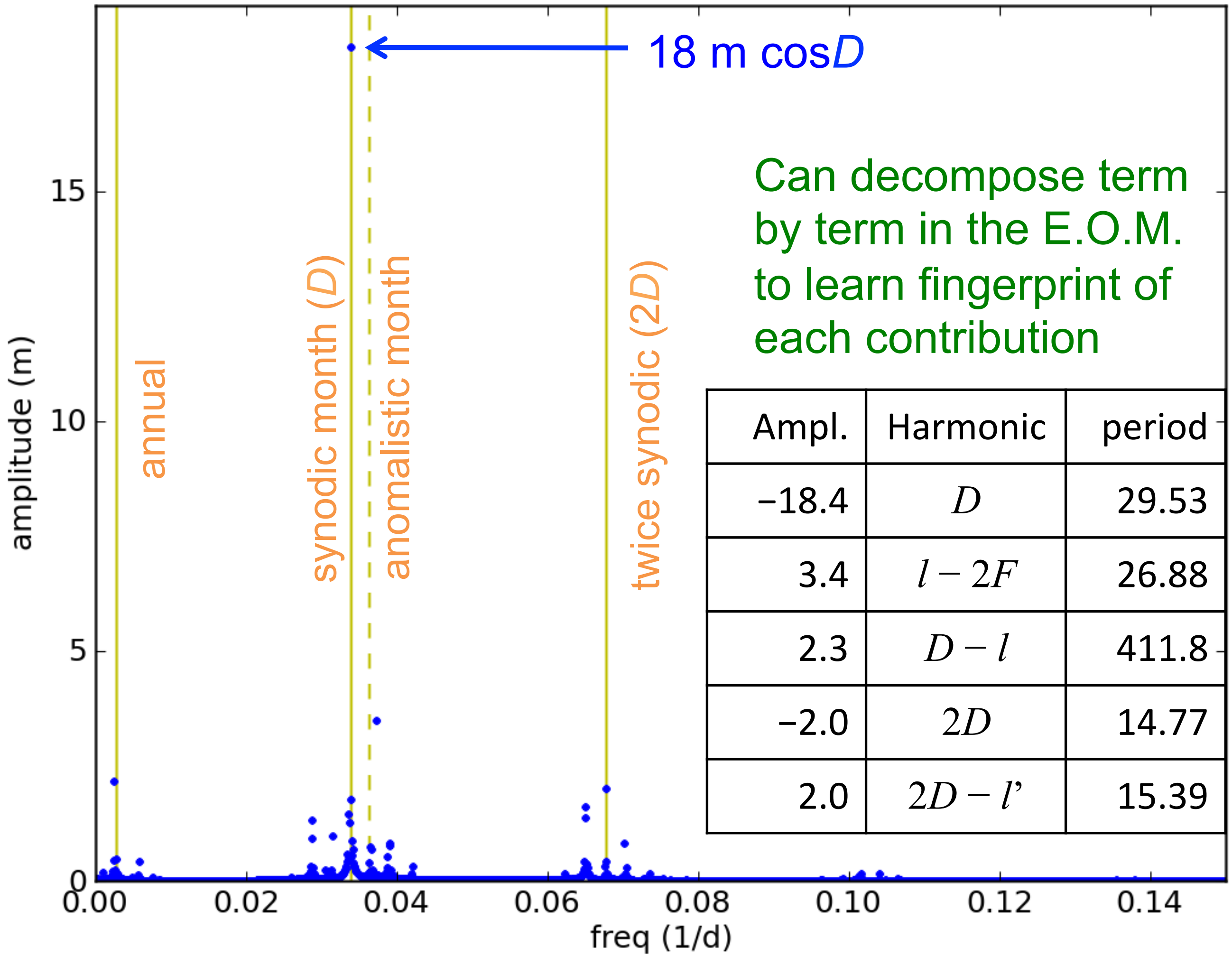
Newtonian piece

$$
\begin{aligned}
\ddot{\mathbf{r}}_{i_{\mathrm{point\ \ mass}}} = & \sum_{j\neq i} \frac{\mu_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \left\{ 1 - \frac{2(\beta+\gamma)}{c^2} \sum_{k\neq i} \frac{\mu_k}{r_{ik}} \right. \\
& - \frac{2\beta - 1}{c^2} \sum_{k\neq j} \frac{\mu_k}{r_{jk}} + \gamma \left(\frac{v_i}{c}\right)^2 + (1+\gamma)\left(\frac{v_j}{c}\right)^2 \\
& - \frac{2(1+\gamma)}{c^2} \dot{\mathbf{r}}_i \cdot \dot{\mathbf{r}}_j - \frac{3}{2c^2} \left[\frac{(\mathbf{r}_i - \mathbf{r}_j)\cdot \dot{\mathbf{r}}_j}{r_{ij}} \right]^2 \\
& \left. + \frac{1}{2c^2} (\mathbf{r}_j - \mathbf{r}_i) \cdot \ddot{\mathbf{r}}_j \right\} + \frac{1}{c^2} \sum_{j\neq i} \frac{\mu_j}{r_{ij}^3} \\
& \times \{ [\mathbf{r}_i - \mathbf{r}_j] \cdot [(2+2\gamma)\dot{\mathbf{r}}_i - (1+2\gamma)\dot{\mathbf{r}}_j] \} (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) \\
& + \frac{3+4\gamma}{2c^2} \sum_{j\neq i} \frac{\mu_j \ddot{\mathbf{r}}_j}{r_{ij}}
\end{aligned}
$$

Note PPN params. γ and β (both 1 in GR)

gravitomagnetic pieces

Post-Newtonian Spectrum: GR's Fingerprint



Can decompose term by term in the E.O.M. to learn fingerprint of each contribution

Ampl.	Harmonic	period
−18.4	D	29.53
3.4	$l - 2F$	26.88
2.3	$D - l$	411.8
−2.0	$2D$	14.77
2.0	$2D - l'$	15.39

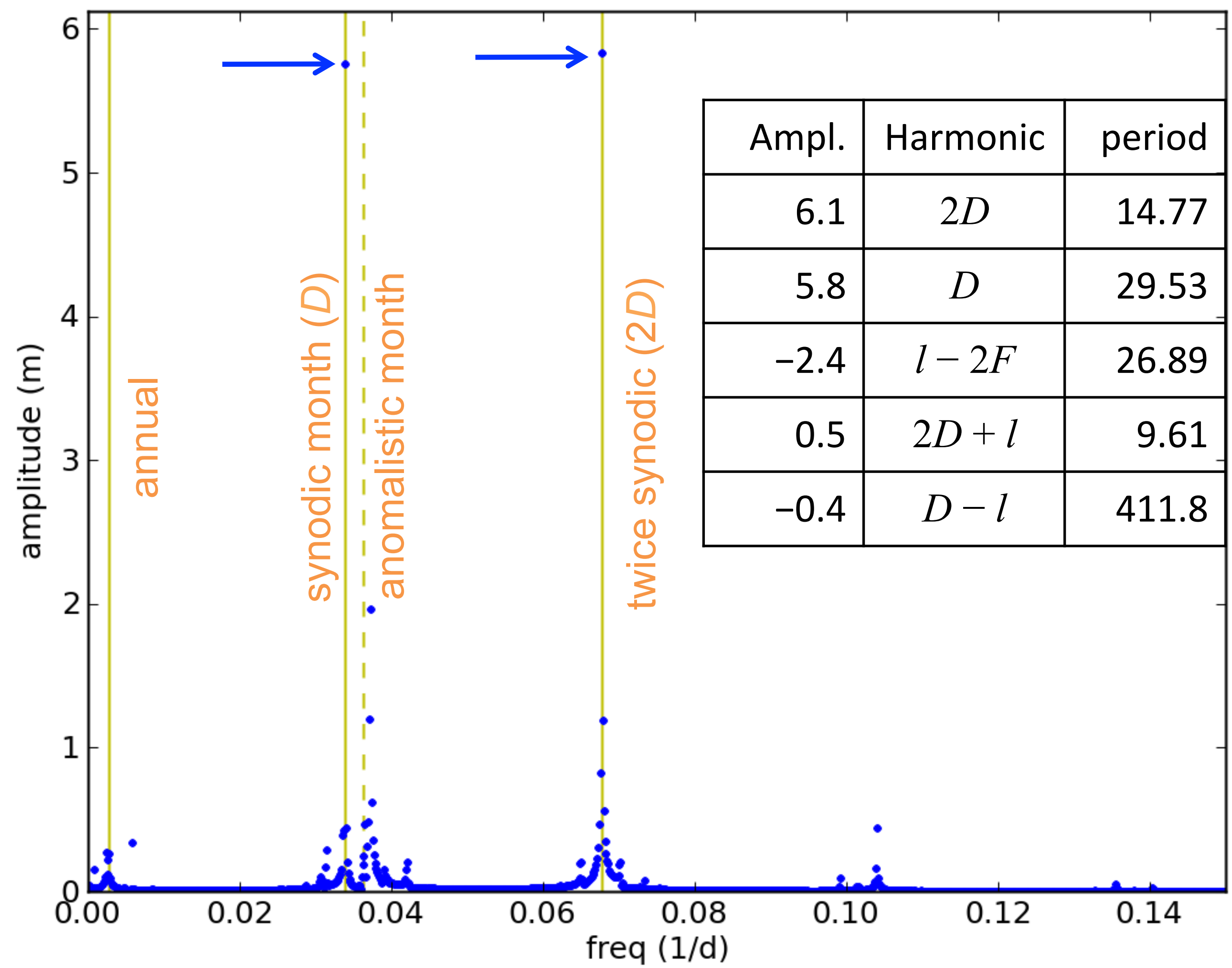
Example: Gravitomagnetic Effect

- The gravitomagnetic terms in the equation of motion can be collected into one and cast as a Lorentz acceleration:

$$\mathbf{a}_i = -\frac{\mu_j (2 + 2\gamma)}{c^2 r_{ij}^3} \mathbf{v}_i \times (\mathbf{v}_j \times \mathbf{r}_{ij})$$

- Combine $\boldsymbol{r}_{ij}$ with prefactor to get $4\boldsymbol{v}_i \times (\boldsymbol{v}_j \times \boldsymbol{g}_{ij})$; $\boldsymbol{g}_{ij}$ is grav. accel.
- The $\boldsymbol{v}_j \times \boldsymbol{g}_{ij}$ term acts like a magnetic field
  - a mass in motion (mass current) produces a circulating gravitomagnetic field
  - another mass moving through this field feels a sideways (Lorentz) force
- Gravitomagnetism is necessary for GR frame independence
- If Earth has velocity **V**, and moon is **V+u**, two terms of consequence emerge:
  - One proportional to V^2 with 6.1 meter cos2*D* signal
  - One proportional to Vu with 5.8 meter cos*D* signal

Gravitomagnetism Spectrum



Ampl.	Harmonic	period
6.1	$2D$	14.77
5.8	D	29.53
−2.4	$l-2F$	26.89
0.5	$2D+l$	9.61
−0.4	$D-l$	411.8

LLR Best Measure of Gravmag

- LLR determines cos*D* to 4 mm precision and cos2*D* to < 8 mm
  - Constitutes a ≈ 0.1% confirmation of effect
- Full simultaneous parameter fit shows 0.2%
  - Soffel, Klioner, Müller & Biskupek, *PRD* **78**, 024033 (2008)
- The same exact $v \times v \times g$ term can be used to derive the precession of a gyroscope in the presence of a rotating mass current
  - captures the full "frame dragging" effect sought by GP-B (meas. to 19%; 1% goal): this is *not* different physics
  - see Murphy, Nordtvedt, & Turyshev, *PRL* **98**, 071102 (2007) and Murphy, *Space Science Rev*. 148, 217 (2009)

Gravitomagnetism

- A moving mass produces a gravitomagnetic field, which then couples to other masses through a Lorentz-like force

  $\boldsymbol{F} = m\boldsymbol{v} \times \boldsymbol{B}$

- Like *any* magnetic field, gravitomagnetism carries with it a strong frame dependence
  - as in electromagnetism, the magnetic fi... ...necessary complement to the electric fi...
- True that magnetic fi... ...ot be transformed a...
  - a rot... ...mple
  - but th... ...magnetic fields does not make these magne... ...than transformable varieties
- The earth ... the Solar System Barycenter (SSB) frame produces a gravitomag...etic field through which the moon moves
  - resulting in deflections of the lunar orbit
  - this field could be killed by shifting to the geocenter frame, but this is a poor choice of frame for analysis (not asymptotically flat)

WARNING:

NOT GEODETIC PRECESSION

But isn't this just transformation fluff?

- Since LLR is "performed" in the geocenter frame, where the gravitomagnetic field of the moving earth is nulled, can LLR *really* measure this physics?
- Actual process:
  - measure proper times in earth frame of photon transmit and receive
  - transform these to SSB times using $dt/d\tau = 1 + ½v^2 - \Delta\phi$
  - perform least-squares fit of data to equation of motion
- In other words, we don't apply *phenomenological* distortions to the orbit in moving to the SSB and then "magically" find we need them to fit our data (this would indeed be vacuous)
  - it is far more subtle: the simple time transformation is the only action
  - there are small cos*D* effects in the v^2 term, but at the few-mm level
- LLR needs the physics of gravitomagnetism to work correctly
  - if another experiment found an anomaly in gravitomagnetism at the 0.2% level, LLR would stand in conflict and require resolution
  - a conflict would indicate we don't even understand time transformation sufficiently

Equivalence Principle Flavors

- Weak EP
  - Composition difference: e.g., iron in earth vs. silicates in moon
  - Probes all interactions but gravity itself
    - Currently tested by LLR to $\Delta a/a < 10^{-13}$
    - Comparable to best lab tests by Eöt-Wash group at UW
      - but better choices of mass pairs offer stronger WEP test than LLR
- Strong EP
  - Applies to gravitational "energy" itself
    - Earth self-energy has equivalent mass ($E = mc^2$)
      - Amounts to 4.6×10^{-10} of earth's total mass-energy
    - Does this mass have $M_G/M_I = 1.00000$?
  - Another way to look at it: gravity pulls on gravity
    - This gets at *nonlinear* aspect of gravity (PPN β)
  - LLR provides the best way to test the SEP
    - pulsar timing is closest competitor

The Strong Equivalence Principle

- Earth's energy of assembly amounts to 4.6×10^{-10} of its total mass-energy

$$M_{S.E.} = \frac{G}{c^2}\int\int\frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{|\mathbf{r}_1-\mathbf{r}_2|}d^3\mathbf{r}_1 d^3\mathbf{r}_2 \approx \frac{GM^2}{Rc^2}$$

The ratio of gravitational to inertial mass for this self energy is

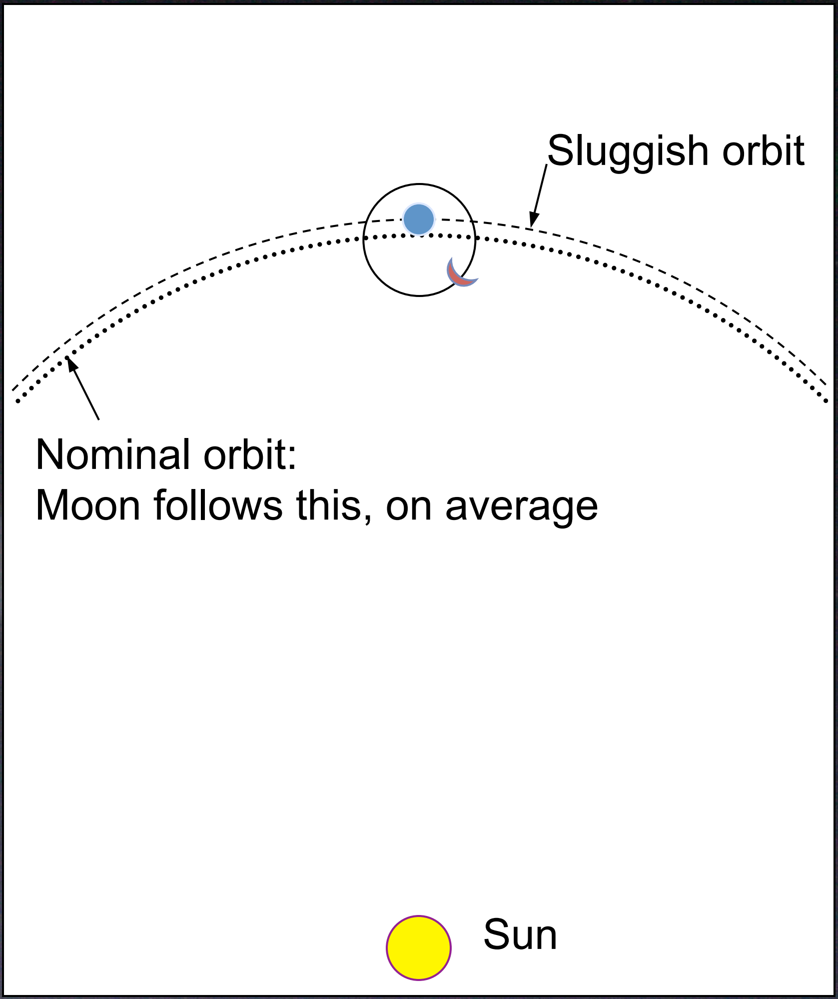
$$\frac{M_G}{M_I} = 1-(4\beta-3-\gamma)\frac{M_{S.E.}}{M} \equiv 1-\eta\frac{M_{S.E.}}{M}$$

The resulting range signal is then

$$\Delta r = 13.1\eta\cos D \quad \text{meters}$$

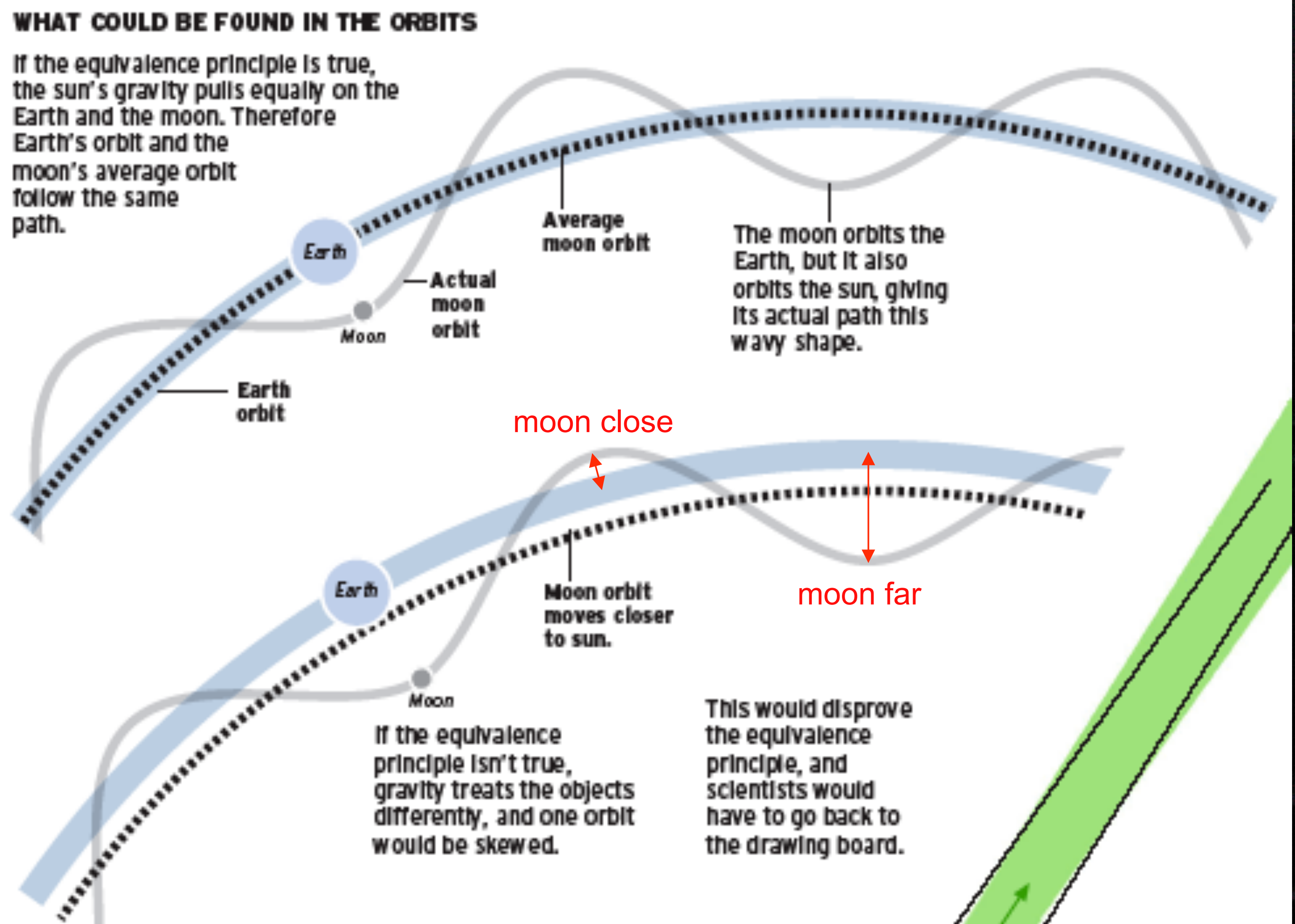
Currently η is limited by LLR to be $\leq 4.5\times10^{-4}$

Equivalence Principle Signal



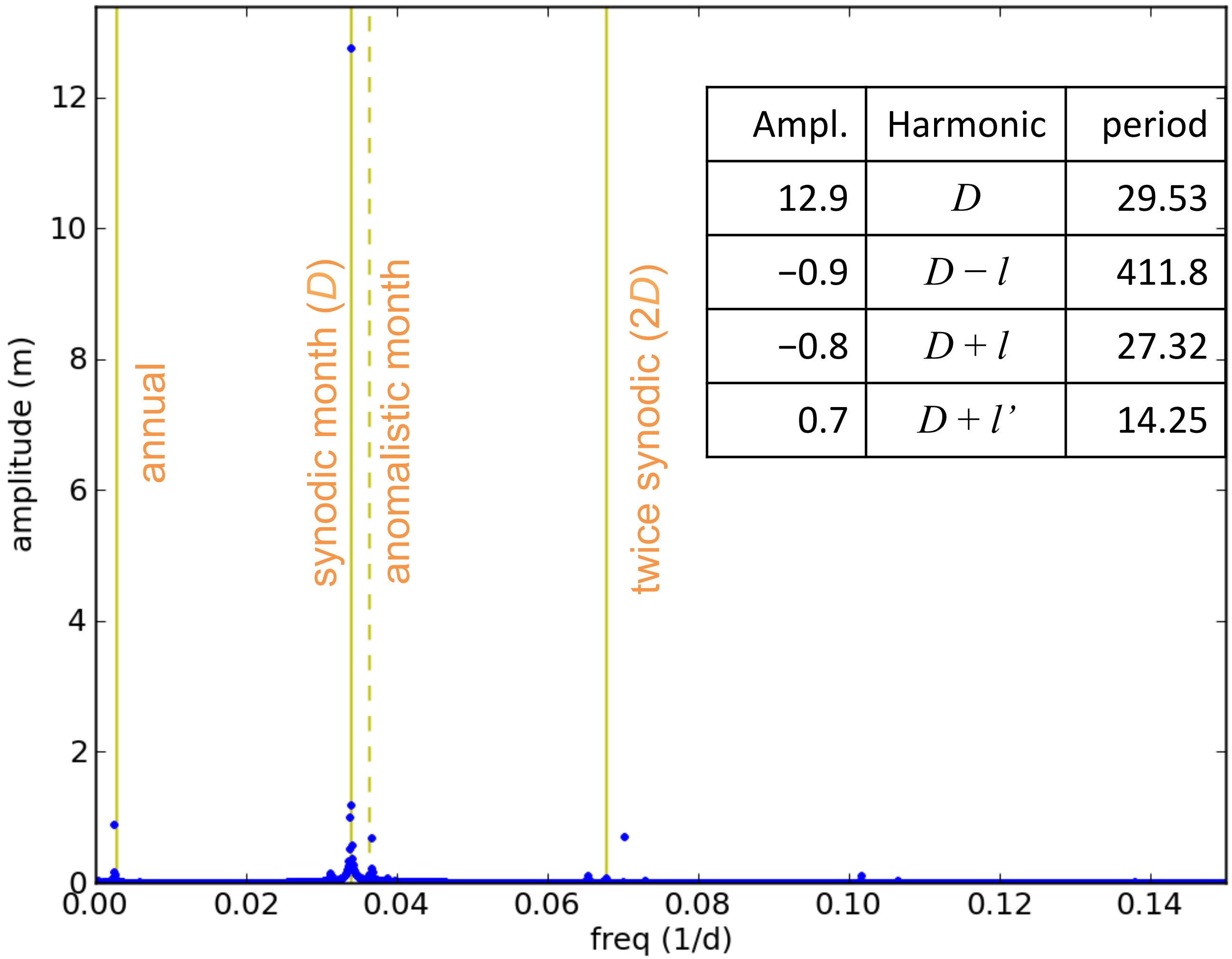
- If the Equivalence Principle (EP) is Violated:
  - In effect, gravitational mass and inertial mass are not equal
  - Earth and Moon would fall at different rates toward the sun
  - Would appear as a *polarization* of the lunar orbit
  - Range signal has form of cos*D* (*D* is lunar phase angle: 0° = new; 180° = full)
- If, for example, Earth has greater inertial mass than gravitational mass (while the moon does not):
  - Earth is sluggish to move
  - Alternatively, pulled weakly by gravity
  - Takes orbit of larger radius (than does Moon)
  - Appears that Moon's orbit is *shifted* toward sun: cos*D* signal

EP Signal, Illustrated



Graphic excerpt from San Diego Union Tribune

Strong EP Violation Spectrum if $\eta = 1$

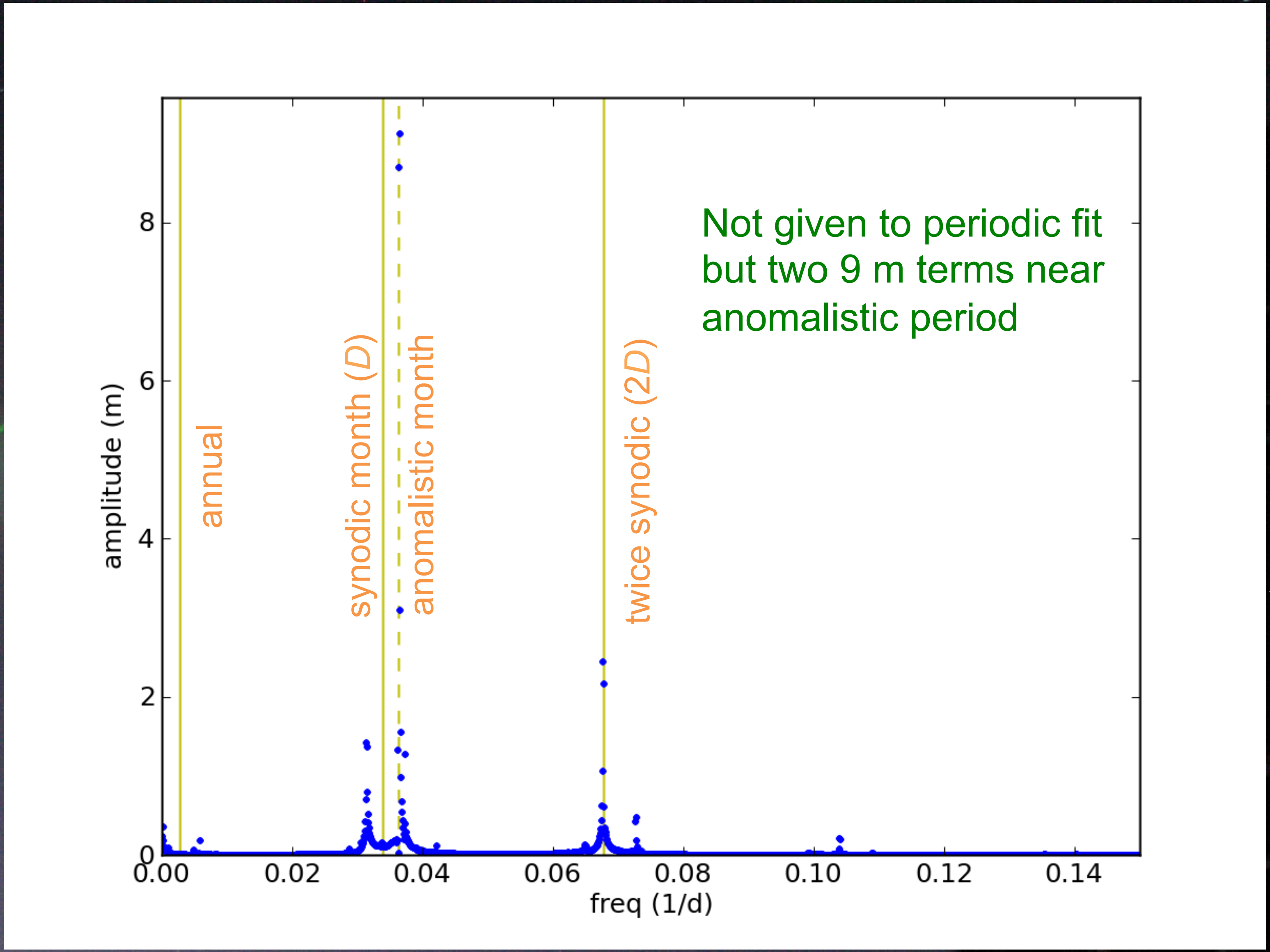


Ampl.	Harmonic	period
12.9	D	29.53
−0.9	$D - l$	411.8
−0.8	$D + l$	27.32
0.7	$D + l'$	14.25

Measuring *G*-dot & Significance

- Quadratic sensitivity:
  - If *G* changes with time, Kepler's law is broken
  - Range signal (semi-major axis) and period (phase) no longer run in lock-step
  - The *rate* of phase slippage grows linearly in time
  - The phase offset grows *quadratically* in time
  - LLR sensitivity now limits change to $\leq 10^{-12}$/yr variation
  - Less than 1% change over age of Universe
- Extra-dimension–motivated explanations of Universal acceleration (AdS/CFT) result in evolution of *G* and equation-of-state parameter *w*
  - Steinhardt & Wesley (2010) claim that factor-of-two improvements in *G*-dot and *w*' over today's limits will rule out AdS/CFT as mechanism for acceleration at $>3\sigma$

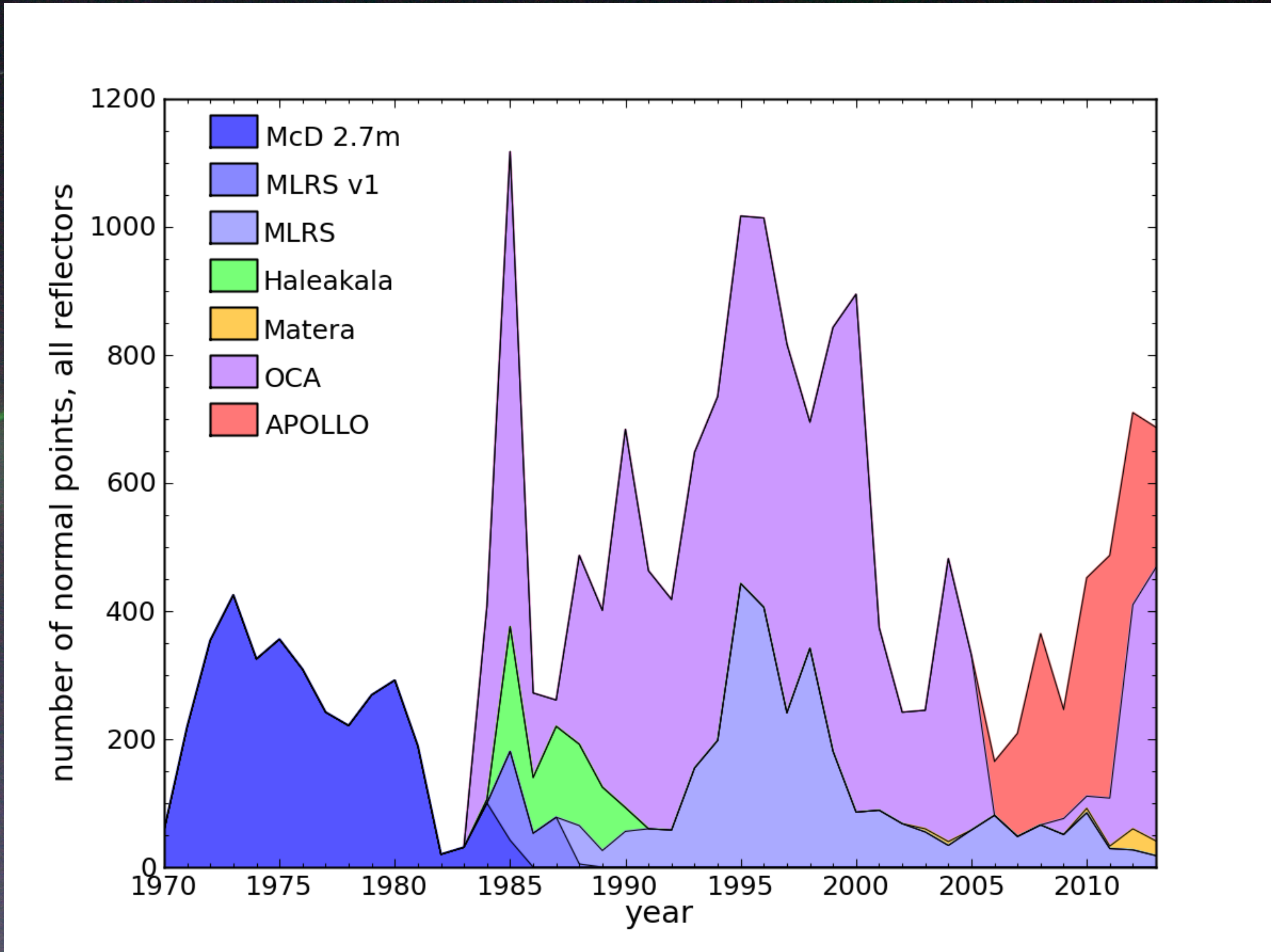
G-dot Spectrum at 10^{-10}/yr



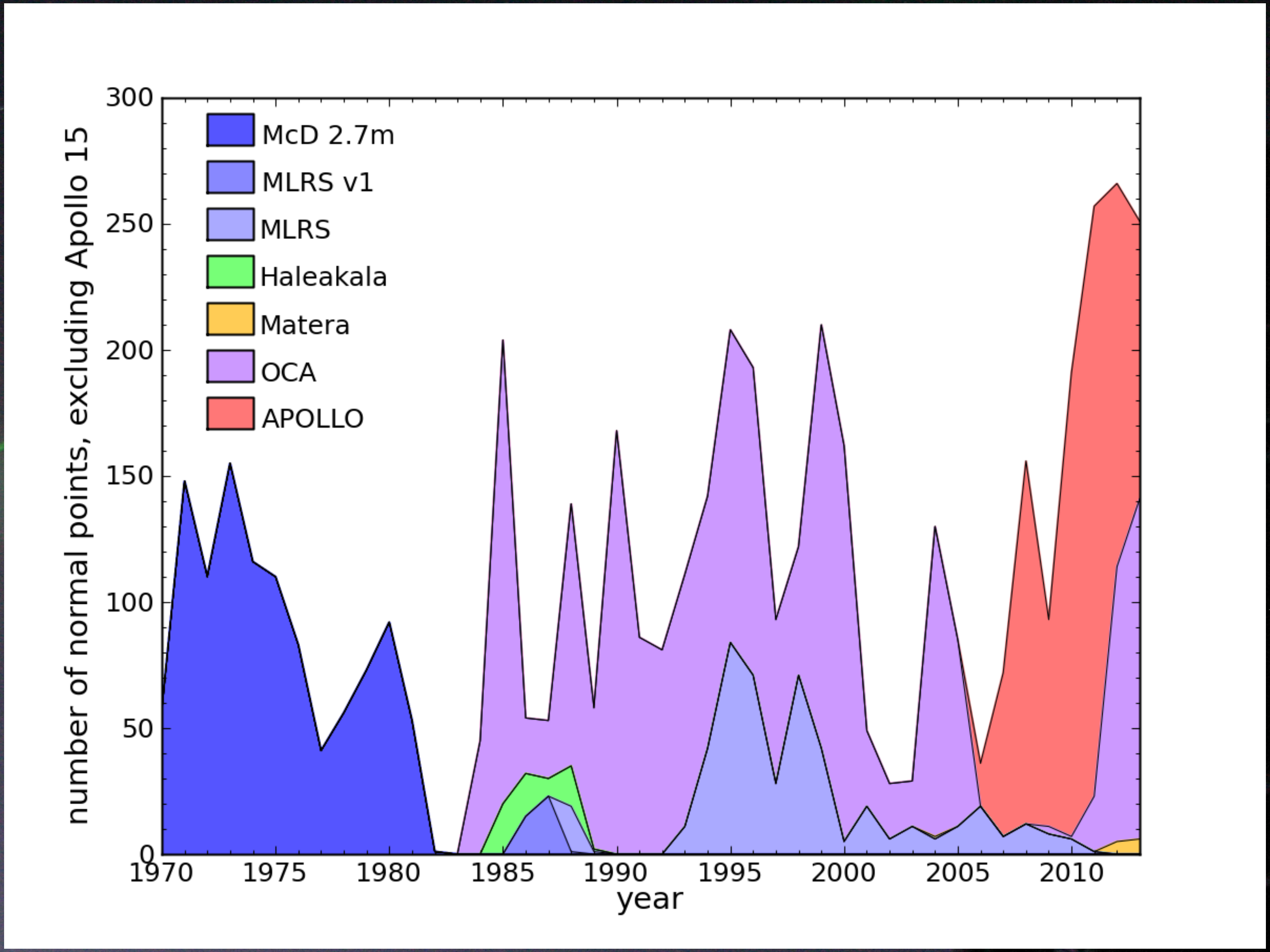
Non-gravitational Science from LLR

- Lunar Interior
  - liquid core of ~350 km radius
  - dissipation at core-mantle boundary
  - see work by Williams, Rambaux
- Coordinate Systems and Earth Orientation
  - contributes along with VLBI, GPS
  - see talk by Jürgen Müller in Session 13
- Dusty Reflectors?
  - overall 10× signal loss; sharp (10×) full moon effect; eclipse recovery speak to absorption at corner cube and thermal effect
  - ~50% dust layer could account for observations

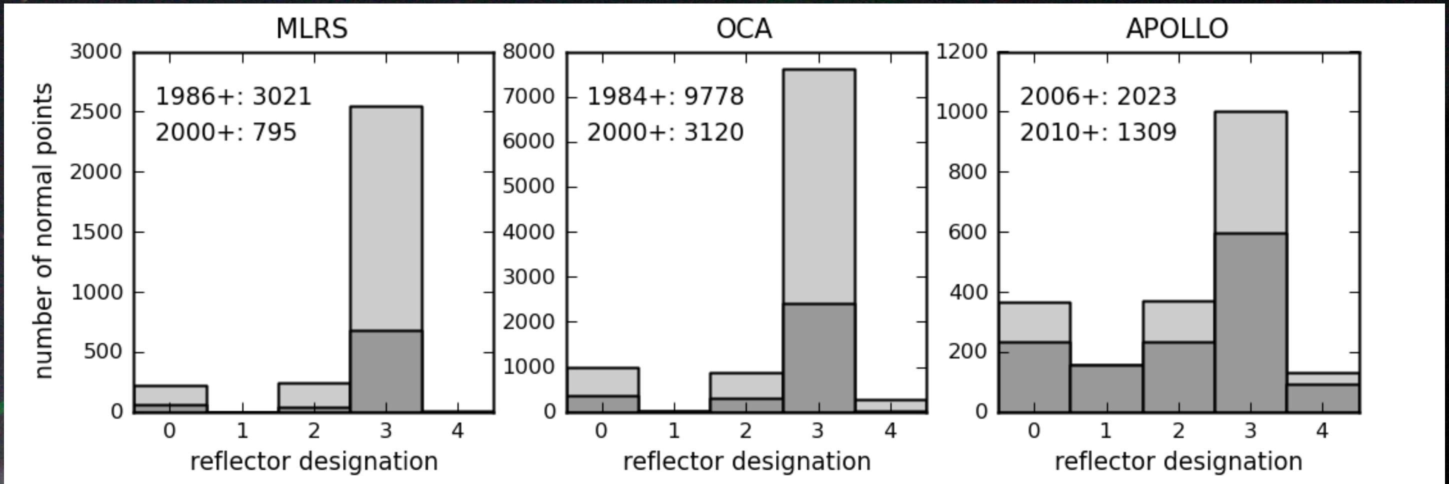
Historical Normal Point Contributions

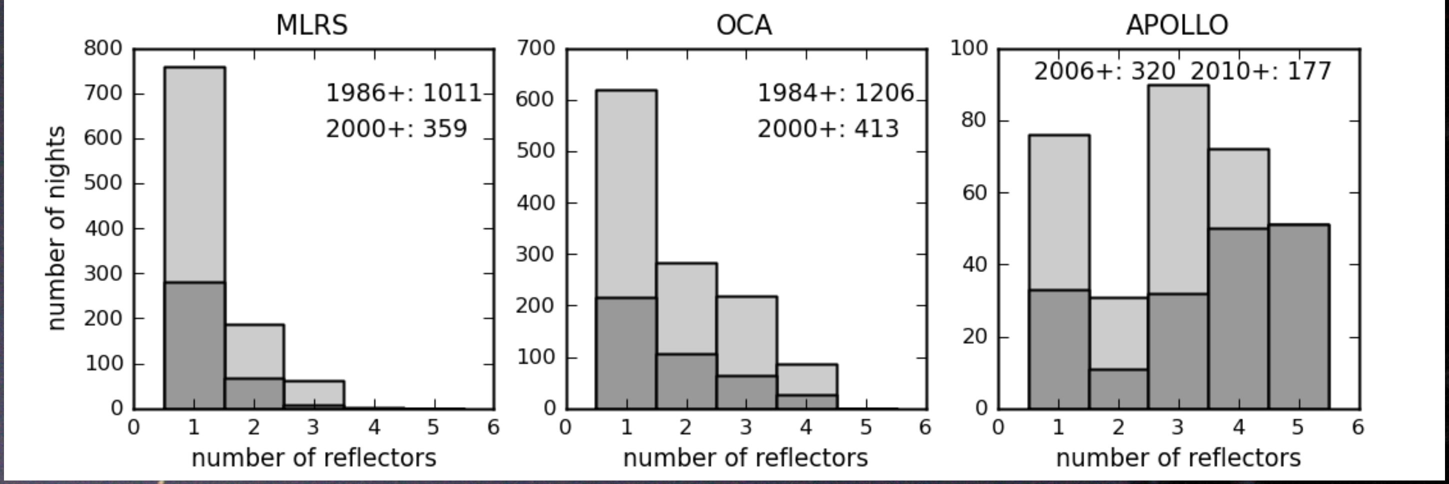


Normal Points excluding Apollo 15



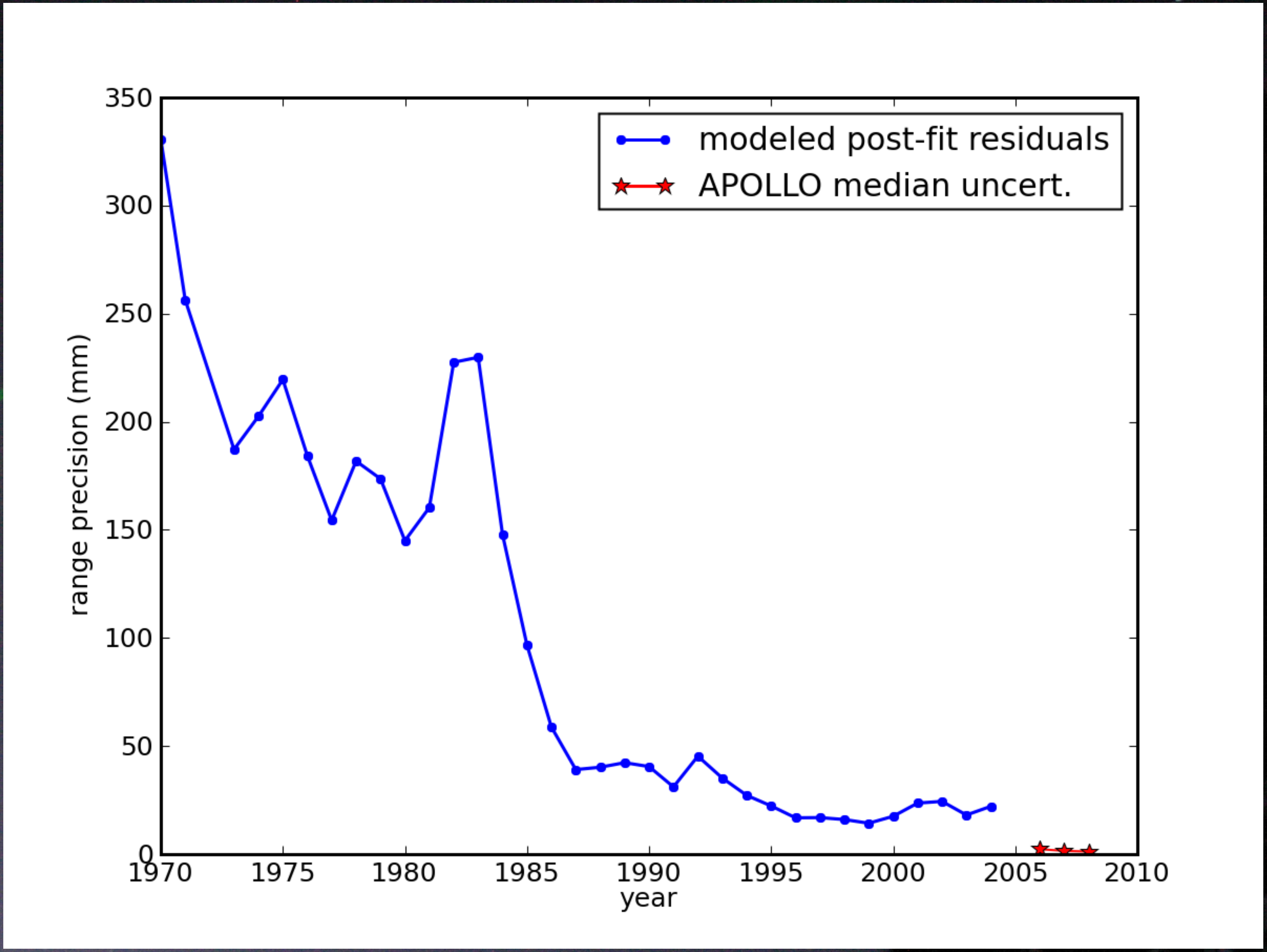
Distribution of Normal Points





LLR Through the Decades

Previously
200 meters



Key Step: Model Development

- Extracting science from LLR data requires a model that includes *all the physics* that can influence the Earth-Moon range
  - N-body relativistic gravity in solar system
  - body figure torques
  - site displacement phenomena
- The best LLR models currently produce ≥ 15 mm residuals
  - JPL; Hannover; PEP (Harvard/CfA); Paris have working models
- Many few-millimeter effects may not yet be included (varies by model)
  - crustal loading phenomena from atmosphere, ocean, hydrology
  - geocenter motion (center of mass with respect to geometry)
  - tidal model needs improvement
  - atmospheric propagation delay model needs updating
  - Earth orientation models could better incorporate LLR data
  - multipole representations of Earth and Moon mass distributions need improvement

Summary & Next Steps

- GR effects on lunar orbit are at 10 m scale
  - GM/Rc^2 is 10^{-8} in solar neighborhood, times 4×10^8 m orbit
- Present model capabilities achieve ~15 mm residuals and sub-cm narrow-band constraints
  - thus ~0.1% tests of GR
- Prospect of millimeter-quality data motivates push to model improvement