Test of General Relativity using Lunar Laser Ranging data and the Planetary Ephemeris Program (PEP)

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Abstract: Since 1969, Lunar Laser Ranging (LLR) to the Apollo Cube Corner Retroreflectors (CCRs) has supplied almost all significant tests of General Relativity (GR). When first installed in the 1970s, the Apollo CCRs geometry contributed only a negligible fraction of the ranging error budget. Today, because of lunar librations, this contribution dominates the error budget, limiting the precision of the experimental tests of gravitational theories. The new MoonLIGHT-2 (Moon Laser Instrumentation for General relativity High-accuracy Tests) apparatus is a new-generation LLR payload made of a single large CCR unaffected by librations. Thanks to this new design, MoonLIGHT-2 can increase the precision of the measurement of the lunar geodetic precession up to a factor 100, compared to the Apollo CCRs. To optimize the MoonLIGHT-2 design and its lunar deployment, we performed an analysis of real LLR data both from station and dummy observations, using the Planetary Ephemeris Program software , developed by the Center for Astrophysics (CfA). The simulations suggest that MoonLIGHT-2 measurements are practically independent from Moon librations and that the absence of a sunshade does not have a relevant impact on the precision of GR tests.

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1. Introduction

Lunar Laser Ranging (LLR) provides accurate measurements of the lunar orbit through high-precision measurement of ranges between a laser station on the Earth and the Apollo Cube Corner Retroreflectors (CCRs) on the lunar surface. LLR has provided for decades the best tests of the validity of Einstein's theory of General Relativity with measurements of the weak and strong equivalence principle, the Parameterized Post Newtonian Parameter (PPN) β and γ, the time change of the Gravitational Constant, the Geodetic Precession (K_{GP}) and $1/r^2$ deviations ([1] and [2]). Over the years, LLR has benefited from a number of improvements in both observing technology and data modelling. These improvements has whichled to the current precision of ~2cm [3]. Unfortunately, the current geometry of the CCR array installed on Moon significantly limits further improvements in this precision. The main problem that affects the Apollo CCR array is the lunar librations in longitude that results from the eccentricity of the Moon's orbit around the Earth. Because of this phenomenon, one corner of the Apollo arrays is more distant from the Earth than the opposite corner by several centimeters, broadening the pulse coming back to the Earth. The broadening of the pulse is proportional to the array physical dimensions and the increase in the Moon-Earth distance, and it is about 30 cm (± 0.5 nanoseconds time flight increase) for the Apollo 15 array, and about 15cm (± 0.25 nanoseconds time flight increase) for the smaller Apollo 11 and Apollo 14 arrays.

The SCF group, in collaboration with the University of Maryland, developed a new design of lunar CCR whose performance is unaffected by either lunar librations or regolith motion, thanks to its very large thermal cycle. The design employs a series of single large CCR (130 mm of front face diameter), deployed separately on the lunar surface. This arrangement creates single short reflected pulses (see Figure 1) with a final precision better than 1 mm. We show in Table 1 the General Relativity (GR) tests that have been carried out using LLR.

Science measurements	1^{st} generation LLR accuracy (~cm)	2^{st} generation LLR accuracy (~mm)
EP	$\left\|{}^{\Delta a}/_{a}\right\| < 1.4 \cdot 10^{-13}$	10^{-14}
SEP	$\eta < 4.4 \cdot 10^{-4}$	$3 \cdot 10^{-5}$
β	$\|\beta - 1\| < 1.1 \cdot 10^{-4}$	10^{-5}
$\dot{G}/G$	$\|\dot{G} / G\| < 9 \cdot 10^{-13} yr^{-1}$	$5 \cdot 10^{-14}$
K_{GP}	$6.4 \cdot 10^{-3}$	$6.4 \cdot 10^{-4}$
$1/r^2$ deviations	$\|\alpha\| < 3 \cdot 10^{-11}$	10^{-12}

Table 1: *Improvement of the LLR accuracy (the EP is expresses in terms of variation of body acceleration a)*

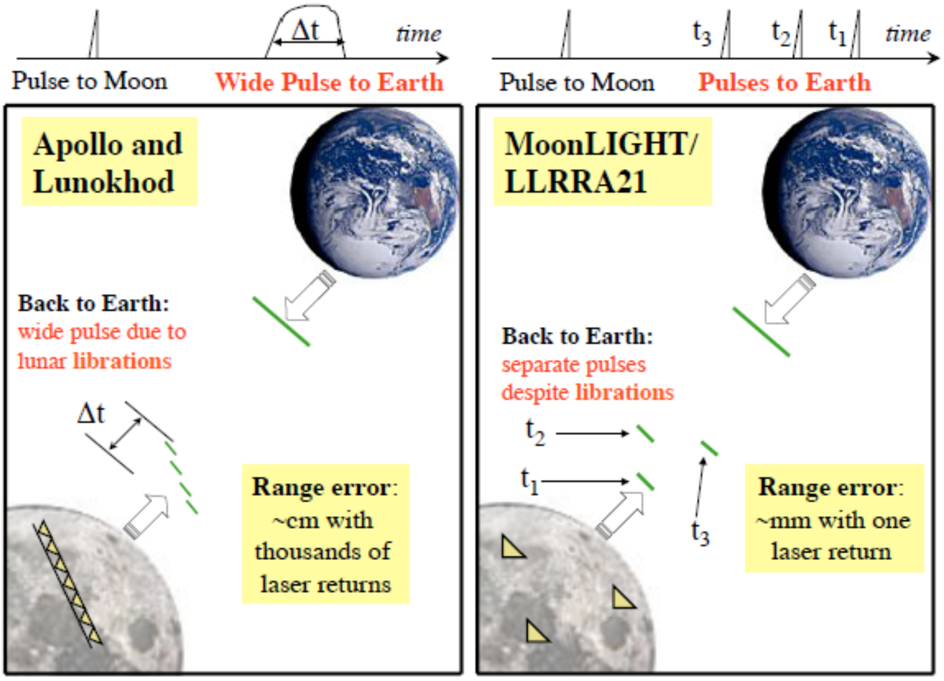

Figure 1: *Comparison between 1st and 2nd generation Lunar Ranging Arrays. The librations tilt the arrays on the left, but the single big CCRs are unaffected, on the right. So we have single short reflected pulses coming back using the MoonLIGHT-2 payloads.*

The original design of MoonLIGHT-2 had a sunshade (Figure 2). The sunshade was designed to block the direct sun into the CCR for most of the lunar day. It also reduced the exposure to dust that could accumulate on the front surface of the CCR, reducing the return signal. The simulation described in Section 2 shown that it was possible to eliminate the sunshade without compromising the instrument precision, allowing a significant optimization of the MoonLIGHT-2 weight.

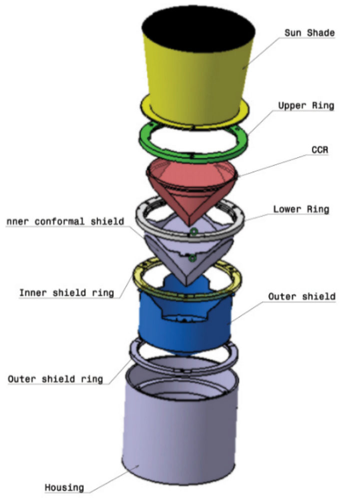

Figure 2: *MoonLIGHT-2 original design with Sunshade. After the PEP simulation, described in session 2 we choose the design without Sunshade.*

2. Simulations with Planetary Ephemeris Program (PEP)

We run a number of numerical simulations using PEP to develop and optimize the MoonLIGHT-2 design in time for its deployment in December 2015 with the MoonExpress lunar mission. PEP is a FORTRAN software package, developed by Center for Astrophysics (CfA) [4]. PEP includes a detailed mathematical model of the solar system, including the masses of all solar system bodies, with a large number of adjustable parameters.
The model parameter estimated are refined by minimizing the residual differences, in a weighted least-squares sense, between observations (O) and model predictions (C stands for ”Computation”), O-C where: ”Observed” is round-trip time of flight while ”Computed” is modeled by the PEP software. The main GR tests which are now being done in collaboration with CfA are the K_{GP}, β, η and $\dot{G}/G$ tests.
We run two different GR simulations using the Apollo CCR array and MoonLIGHT-2 CCR. For these simulations we used both real and dummy data from the real Apollo CCR. All the dummy data were computed by PEP after defining a Ground Laser Station, the CCR position on Moon surface and the accuracy of LLR. Table 2 and Table 3 shown all the inputs used to compute the dummy data with PEP for the two simulations.
In the first simulation, we used dummy data for MoonLIGHT-2 from the APOLLO ground Station and real data from the Apollo CCR. We simulate the GR test using two different Moon site separately for one MoonLIGHT-2 and then the two sites simultaneously. The aim of this simulation was to understand if the GR measurements with MoonLIGHT-2 were depended from the lunar position, and if it would have been possible to determine an optimal deployment site. For these simulations we computed the values of K_{GP}, β and $\dot{G}/G$.

SIMULATION #1: DEPLOYMENT SITE					
CCR Array	**Data Type**	**Time Span**	**Sites**	**Stations**	**Accuracy**
Apollo	Real	From 2002 To 2012	Apollo 11-14-15	-	-
MoonLIGHT	Dummy		65^0N, 40^0W	APOLLO	2.5cm
			87^0N, 40^0W		

Table 2: *Details about the best deployment site simulation with PEP.*

In the second simulation, we used dummy data from the Apollo and the MoonLIGHT-2 CCR for four different ground stations: APOLLO, CERGA, MLRS (McDonald Laser Ranging Station, USA) and MLRO (Matera Laser Ranging Observatory, Italy). For the MoonLIGHT-2 sites, we chose the real deployment sites from the mission MoonExpress, Astrobotic and Israel. In this analysis we simulated data until 2030 and use two types of MoonLIGHT-2 design, with and without sunshade. In the first case (design with a sunshade), the MoonLIGHT-2 reflectors were shielded and therefore available whenever conditions were suitable at the observation site. In the second case (no sunshade), the MoonLIGHT-2 reflectors were unavailable when illuminated, reducing the amount of collectable data. Also for this simulation, we computed the values of K_{GP}, β, η and $\dot{G}/G$.

SIMULATION #2: OPTIMAL DESIGN AND GR EXPECTED IMPROVEMENT					
CCR Array	**Data Type**	**Time Span**	**Sites**	**Stations**	**Accuracy**
Apollo	Dummy	From 2013 To 2030	Apollo 11-14-15	APOLLO	0.1cm
				CERGA	0.2cm
				MLRS	
				MLRO	
MoonLIGHT-2	Dummy	From 2016 To 2030	65^{0}N, 40^{0}W *(MoonExpress)*	APOLLO	0.5cm
				CERGA	1.0 cm
			50^{0}S, 35^{0}E *(Astrobotic)*	MLRS	
			45^{0}N, 27.2^{0}E *(Israel)*	MLRO	

Table 3: *Details about the optimal design and GR expected improvement simulation with PEP.*

3. Results and Discussion

The simulation using only one MoonLIGHT-2 are shown in the third column (Apollo + 1 MoonLIGHT-2) and gives similar results for both the Moon site. These results show that the GR simulations with MoonLIGHT-2 are poorly dependent from the Moon site position. The worst positions are at the poles since, the array will not be always visible from Ground Station because of the Moon libration. Any other position gives similar results for GR tests. The fourth column (Apollo + 2 MoonLIGHT-2) shows that in the case of low precision in the assessment of the geometry of the CCR array(for this simulation we use a precision of 2.5cm for the dummy data), we do not have any improvement in the GRs test even when adding additional CCR array to the Apollo CCR

	Only Apollo	**Apollo + 1 MoonLIGHT-2**	**Apollo + 2 MoonLIGHT-2**
$\boldsymbol{\beta}$	$2{,}0 \cdot 10^{-4}$	$2{,}0 \cdot 10^{-4}$	$2{,}0 \cdot 10^{-4}$
$\boldsymbol{KGP}$	$-8.6 \cdot 10^{-3}$	$-6.7 \cdot 10^{-4}$	$-2.0 \cdot 10^{-4}$
$\boldsymbol{\dot{G}/G}$	$9.2 \cdot 10^{-14}$	$3.6 \cdot 10^{-14}$	$3.1 \cdot 10^{-14}$

Table 4: *Results of Simulation #1: MoonLIGHT-2 best deployment site. All the results are expressed in terms of variation from constant value.*

The GR tests with the sunshade show a slightly better accuracy compared to the case without the sunshade. The improvement is due to the longer data acquisition interval available for the design with sunshade. Comparing these results with the case without Sunshade, we think that this improvement doesn't affect significantly the GR results. The minor decrease in the instrument performance in the absence of a sunshade is more than compensated by the optimization of the MoonLIGHT-2 weight for its deployment.

	2013	**2016**	**2018**	**2020**	**2022**	**2025**	**2030**	**Sunshade**
$\dot{G}/G$	$1.6 \cdot 10^{-14}$	$7.7 \cdot 10^{-15}$	$5.4 \cdot 10^{-15}$	$3.8 \cdot 10^{-15}$	$2.7 \cdot 10^{-15}$	$1.7 \cdot 10^{-15}$	$1.1 \cdot 10^{-15}$	YES
	-	$7.6 \cdot 10^{-15}$	$5.2 \cdot 10^{-15}$	$3.6 \cdot 10^{-15}$	$2.6 \cdot 10^{-15}$	$1.6 \cdot 10^{-15}$	$1.0 \cdot 10^{-15}$	NO
η	$2.6 \cdot 10^{-3}$	$1.6 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$	$8.2 \cdot 10^{-4}$	$7.4 \cdot 10^{-4}$	$5.9 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$	YES
	-	$1.5 \cdot 10^{-3}$	$9.8 \cdot 10^{-4}$	$7.2 \cdot 10^{-4}$	$6.3 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$	$4.2 \cdot 10^{-4}$	NO
KGP	$3.4 \cdot 10^{-4}$	$2.0 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$	$1.0 \cdot 10^{-4}$	$7.8 \cdot 10^{-5}$	$6.3 \cdot 10^{-5}$	YES
	-	$1.9 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$9.5 \cdot 10^{-5}$	$8.0 \cdot 10^{-5}$	$6.5 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$	NO
β	$6.4 \cdot 10^{-4}$	$4.2 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	YES
	-	$3.9 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$1.0 \cdot 10^{-4}$	NO

Table 5: *Results of Simulation #2: Optimal design and GR expected improvement. Be careful, all the results are expressed in terms of variation from constant value.*

Comparing the expected GR results at 2030 in Table 5 with the actual best accuracy in GR test in Table 1, we can see the expected improvement in GR test accuracy using the new generation lunar retroreflector MoonLIGHT-2 CCR with the Apollo CCR.

Figure 3, Figure 4 and Figure 5, show the accuracy improvement until 2030 of β, $\dot{G}/G$ and K_{GP}. For these simulations, the accuracy improvement in the GR tests using the new generation lunar retroreflectors is about one/half order of magnitude.

All the results of this simulation show a significant accuracy improvement in the GR simulation results using MoonLIGHT-2 together with Apollo. This improvement is due to the better LLR data accuracy provided by the the new retroreflector; this data become predominant respect to Apollo data as increase the data acquired and will be more significant after the 2030.

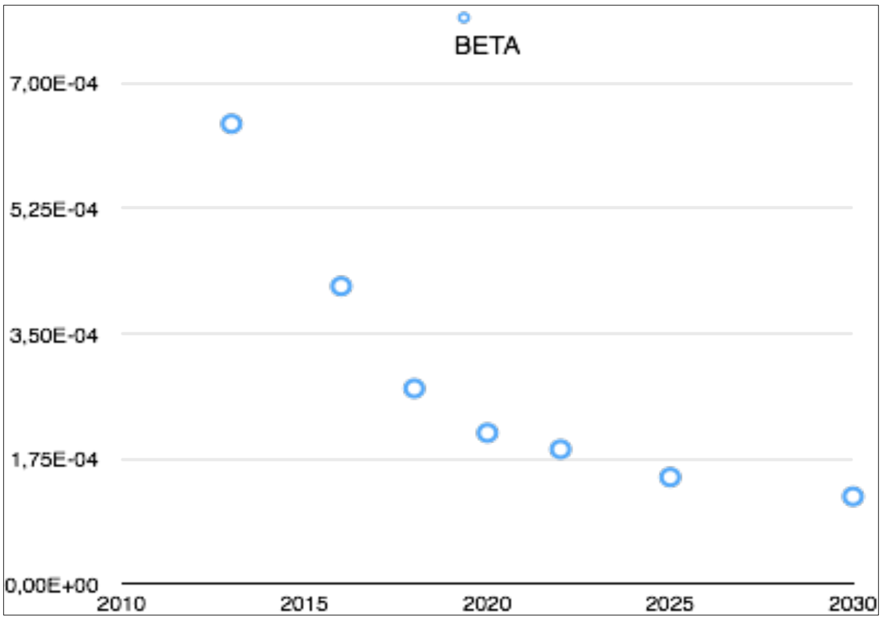

Figure 3: *β improvement during a long time simulation using MoonLIGHT-2 with Apollo*

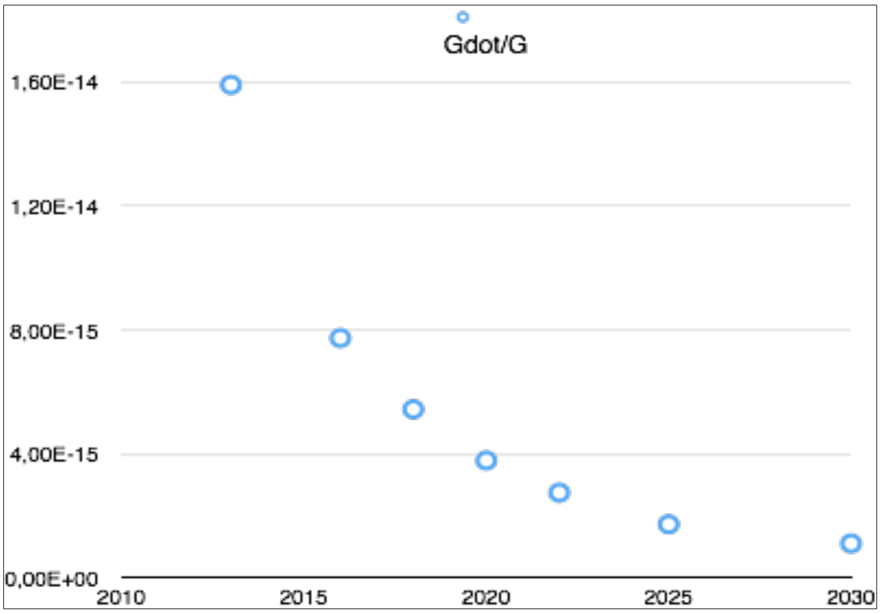

***Figure 4**: $\dot{G}/G$ improvement during a long time simulation using MoonLIGHT-2 with Apollo CCR.*

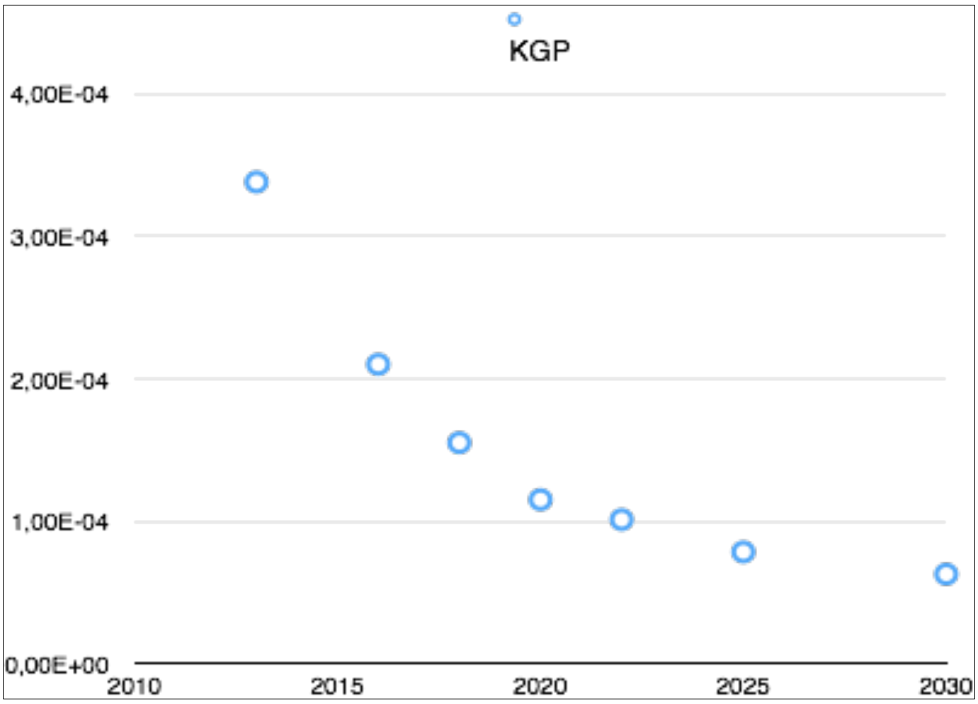

Figure 5: *KGP improvement during a long time simulation using MoonLIGHT-2 with Apollo CCR.*

4. Conclusions

Although Apollo retroreflectors will continue to operate and provide new science results, their geometry is now limiting the precision of the single photoelectron returns. The next generation retroreflector, MoonLIGHT-2, will support improvements in ranging precision, by one order of magnitude, depending on the method of deployment.

With the preliminary simulations described in this work we show that:

- The GR tests with MoonLIGHT-2 will be not dependent from the MoonLIGHT-2 deployment site with the exceptions of the poles (because of the lunar libration, the array is not always visible from Earth).
- There are not great differences in the GR tests using a MoonLIGHT-2 design with or without Sunshade. So we choose the design without the sunshade that will provide an important weight optimization (about 1kg) with similar results in GR tests.
- The expected improvement in the GR with MoonLIGHT-2 is about one/half order of magnitude during the 10 years of analysis for most GR tests.

The ultimate scientific objective of MoonLIght-2 is to provide constraints on the theories that are proposed to determine the properties of Dark Matter and Dark Energy, and other gravitational theories [5]. This improved precision will be useful to identify the theoretical directions that will further the development of an understanding of these mysterious phenomena that lie beyond our current understanding.

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