Simulated comparative analysis of one- and two-way planetary laser ranging systems

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Abstract

Both one- and two-way laser ranging systems have the potential to provide increased science return for planetary missions. Although the one-way system requires less hardware, clock errors on both the space and ground segments will accumulate over time, degrading the quality of the range measurements. For the two-way system, which requires a more extensive hardware package on the space segment, the range measurements are only sensitive to clock errors integrated over the two-way light time.

We investigate the performance of both one- and two-way laser ranging systems by simulating their operation. We generate realizations of clock error time histories from Allan variance profiles, and use them to create range measurement error profiles. We subsequently perform the orbit determination process from this data to quantify the system's performance. For our simulations, we use two test cases: a lunar orbiter similar to LRO and a Phobos lander similar to the Phobos Laser Ranging concept. We include the estimation of clock parameters over a number of arc lengths for our simulations of the one-way range system and use a variety of state arc durations for the lunar orbiter simulations

We perform Monte Carlo simulations and generate true error distributions for both missions for various combinations of clock and state arc length. Thereby, we study the optimal data analysis strategies for such missions and quantify the relative capabilities of the one- and two-way laser range systems.

1 Introduction

The use of laser ranging has been proposed for use over interplanetary distances (Degnan, 2002), extending the use of Satellite Laser Ranging (SLR) and Lunar Laser Ranging (LLR) to Interplanetary Laser Ranging (ILR). Over such distances, an active space segment is required, reducing the signal-strength dependency to inverse square with distance. In a one-way laser range system, only a receiving system is required on the spacecraft and the observable is directly obtained from the uplink light-time. For two-way systems, a transmitter is additionally needed on the space segment, which is used to (asynchronously) fire laser pulses to the ground station(s) (Degnan, 2002). Using such a system, the two-way range observable is realized through pairing of the up- and downlink light-times (Birnbaum et al., 2010).

The primary difference in error budget between the one- and two-way systems stems from the different influence of clock noise on the observables. For the one-way system, clock noise in both the transmitting and the receiving system accumulates over time, in a similar manner that dynamical model errors accumulate in spacecraft orbit determination. For two-way range systems, this clock noise accumulation only occurs over a limited time, specifically the two-way light-time for the ground station clock noise and the retransmission time for the space segment clock noise.

This paper, including this introduction, is based heavily on the paper by Dirkx et al. (2015) and can be seen as a concise summary of the work presented there. Here, we focus on the key results, deferring many of the details to the main paper. We investigate the influence of signal timing errors on the performance of both one- and two-way laser ranging systems by quantifying the mapping of uncertainties in the clock stability to parameter estimation accuracy. Using these results, we compare the performance of one- and two-way laser ranging systems.

2 Simulation overview

For our simulations we use both a lunar orbiter and a Phobos lander as test cases. We analyze the performance of one- and two-way laser ranging t a lunar polar orbiter over a period of 1 month and that of a Phobos lander over a period of 1 year. For the lunar orbiter, we assume orbital characteristics similar to LRO. We do not discuss the full settings of our simulations here for the sake of brevity (see Dirkx et al. (2015) for details). We include an empirical force model uncertainty for the lunar orbiter to include the effects of mismodelled non-conservative forces and do not estimate any geophysical characteristics of the Moon (only the orbiter's state). For the Phobos lander, we estimate its C_{22} gravity field coefficient and a number of libration amplitudes in longitude W, right ascension α and declination δ, as well as its initial state and the Phobos-fixed lander position.

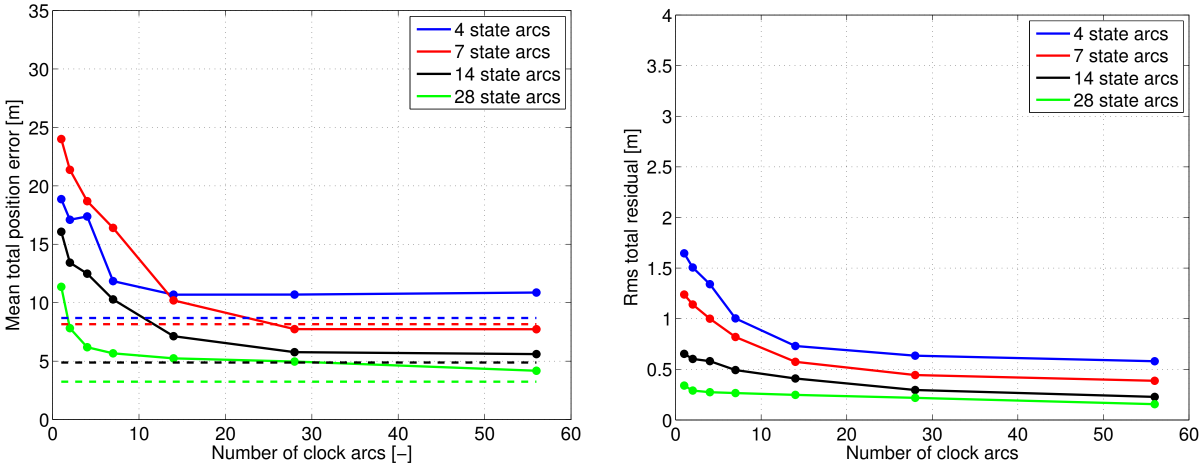
During the processing of one-way laser ranging data, the deterministic clock parameters of the space segment (and participating ground stations) may be estimated during the orbit determination process (Bauer et al., 2013). This approach is preferred to *a posteriori* clock calibration, where *post*-fit residuals are attributed to clock errors, since this would obfuscate any correlations between clock parameters and other estimated parameters.

3 Lunar Orbiter One- and two-way range comparison

To remove accumulated clock noise from the one-way range observations of the lunar orbiter, we include the estimation of clock parameters on a per-arc basis (Section 2). We perform the estimation for a range of values of the number of state arcs N_s and clock arcs N_c, which we do not impose as being equal to one another. Our full simulation spans a 28 day duration. All our numerical results are based on a Monte Carlo analysis with 25 independent simulations using different realizations of the observer clock noise processes.

Comparing the two-way range to the one-way range results in Fig. 1(a), it can be seen that the two-way range simulations mostly produce slightly better results than the one-way range simulations. The magnitude of the difference is quite small, with the one-way solution even slightly better for $N_s = 7$. This indicates that the clock-induced range errors are not the primary source of error in the parameter estimation, and the influence of errors in the dynamical model are dominant instead. For the one-way range simulations to reach an estimation quality that is similar to the two-way system, clock parameters must be estimated over sufficiently short arcs ($\lesssim$ 1 day) to remove sufficient clock-induced range errors.

Our simulations indicate that in an environment with even a reasonably low force model uncertainty (the lunar gravity field is exceptionally well mapped), the added value of a two-way system over a one-way system may be marginal, since it is the dynamics error,



(a) (b)

Figure 1: Estimation results for lunar orbiter one-way range observations as a function of number of state arcs and clock arcs, with errors in estimation dynamical model a) Mean position error over full 28-day period (dotted line indicates results for two-way system, for which N_c=0) b) Post-fit range residual. Circular dots denote data point for which results were generated by means of Monte Carlo analysis ($N = 25$).

not the clock-induced error, which dominates the position estimation error budget (under the asumptions we make). However, it must be noted that, depending on the type of (non-conservative) forces that dominate the dynamics error budget, a two-way system can improve the force modelling much better than a one-way system, since small unmodelled dynamical signatures can be extracted from the two-way observation residuals more easily.

An additional important advantage of the two-way system over its one-way counterpart is the better robustness and reliability of the estimation results. The addition of the clock parameter estimation in the data processing of the one-way data results in clock and state signal potentially being misinterpreted in the estimation. This behaviour is hard to quantify properly without detailed numerical simulations, as shown by the sometimes erratic and unexpected results of the one-way range simulations discussed here. The behaviour of the two-way range simulations, however, is unaffected by correlations between clock and state parameters. Also, the quantitative influence of a clock noise realization on one-way system performance that occurs for a given time interval is a stochastic quantity. As a result, to meet a given positioning requirement over the entire mission, a more stringent mean positioning requirement is needed for the one-way than for the two-way system. However, the predictability of the estimation quality is still dependent on the force model error profile for both the one- and two-way system results.

4 Phobos Lander Results

We now present the results for the Phobos lander one- and two-way laser tracking simulations. The estimation error of Phobos' position, as obtained from one-way range measurements (omitted here for the sake of brevity, see Dirkx et al. (2015)) initially decreases with increasing number of clock arcs, a result of the estimation having more possibilities to remove clock

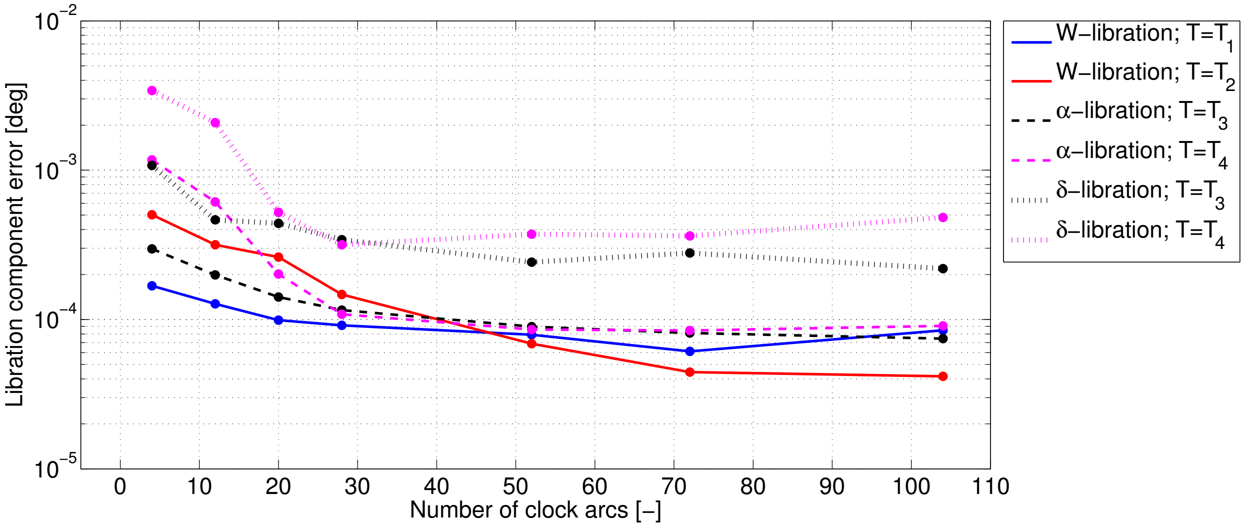


Figure 2: Plot of the estimation error of Phobos libration amplitudes as a function of number of clock estimation arcs.

noise from the measurements, similar to the results discussed in Section 3. However, for large number of clock arcs, the position error again increases, albeit slightly. Resultantly, there is a limit in how well clock errors can be removed from the range error profiles before the correlations between clock parameters and other parameters begin to degrade the position estimates, limiting the position accuracy to about 30 cm rms.

The estimation results for the simulation of the Phobos lander tracking using two-way ranging are shown in Table 1, where the mean absolute estimation error of each of the parameters is given. Also given is the mean observation residual after estimator convergence, which can be seen to be close to the single-shot Gaussian measurement error (1.5 cm), indicating that the time-correlated errors are not of strong influence in the estimation. This was expected due to the combination of small retransmission time and highly stable ground clock. It should be noted that the results shown here (as well as in the previous section) are contingent upon highly accurate gravitational models of Phobos' orbit, both classical and relativistic.

Comparing the estimation accuracy of the libration components for the two-way system (Table 1) to those obtained with the one-way system (Fig. 2), there is again a clear difference between the behaviour of the primary libration component W_{T_1} and the other, smaller, libration components. Specifically, the estimation quality improvement for the small libration amplitudes is much greater than for the primary libration when going from a one- to a two-way system. This indicates that the small libration amplitudes are much more sensitive to the presence of clock noise than the larger libration. We find that the estimation accuracy of $\bar{C}_{2,2}$ is relatively close to that which can be obtained from the one-way system (a factor two).

However, as highlighted by both Le Maistre et al. (2012) and Dirkx et al. (2014), geophysical interpretation of improved estimated of Phobos' moments of inertia at the level discussed here is currently limited by uncertainty in Phobos' volume. To compare estimated values of Phobos' moments of inertia to values that are obtained from models of Phobos interior

Table 1: List of selected estimated parameter errors for Phobos two-way range simulations; mean observation residual = 2.28 cm

Parameter	Rms estimation error
$\lvert\mathbf{x}_{P,0}\rvert$	0.0266 m
$\bar{C}^{P}_{2,2}$	$5.14{\cdot}10^{-9}$
W_{T_1}	$1.36{\cdot}10^{-5}$ deg
W_{T_2}	$1.90{\cdot}10^{-6}$ deg
α_{T_3}	$5.35{\cdot}10^{-6}$ deg
α_{T_4}	$1.04{\cdot}10^{-6}$ deg
δ_{T_3}	$2.59{\cdot}10^{-5}$ deg
δ_{T_4}	$2.47{\cdot}10^{-5}$ deg

structure with various degrees and types of heterogeneities, such as is done by Rosenblatt (2011), an accurate shape model of Phobos is needed. However, the currently best estimate for Phobos' volume (Willner et al., 2014) has an uncertainty of 35 km^3, which translates into a homogeneous moment of inertia error on the order of ≈1 %. As such, both the one- and two-way range systems are capable of providing geophysical parameter estimates, but they cannot be used to their fullest potential in analyzing Phobos' interior structure without improved Phobos shape models. Additionally, it highlights that, although the one-way range estimation is clearly inferior to the two-way range system's performance in terms of estimated parameter accuracy, both systems may result in similar science return for Phobos interior, unless strong improved estimates of additional physical parameters are made available through the use of synergistic next-generation missions and instrumentation.

5 Conclusions

We have simulated one- and two-way laser ranging measurements for both a lunar polar orbiter and a Phobos lander and have used these synthetic measurements as input for orbit determination and parameter estimation. By doing so, we have quantified the influence of clock noise on one-way laser ranging performance and compared the performance of both types of laser ranging systems.

For the lunar orbiter, we find that the one- and two-way laser ranging systems provide similar mean position estimation errors, at about 5 m for a 1 day state arc duration and a clock arc duration of less than one day (for the one-way system), due to the fact that the dynamical model errors, and not the observation errors, are the dominant source of error in the estimation. Nevertheless, the two-way system has the advantage of being more robust, since it is not influenced by the specific vagaries of the spacecraft clock.

Our simulations of the Phobos lander show large differences between one- and two-way laser ranging performance. The correlations between clock and state parameters result in an optimal clock arc length of about one week for our one-year simulation period, for which a mean Phobos position estimation error of 0.3 m is obtained. The two-way system results in a mean position estimation error of 2.5 cm (contingent upon highly accurate dynamical models of Phobos). We find that the estimation of small libration amplitudes, which are largely decoupled from Phobos' dynamics, are much more sensitive to the presence of clock noise in the one-way range measurements. However, the science return from the Phobos mission would largely stem from improved estimates on Phobos' interior and evolution.

Our geodetic parameter estimate must therefore be mapped to a space of Phobos interior compositions. In this mapping, the dominant error source at the levels of accuracy that we get for both the one- and two-way systems is Phobos' volume error, not the uncertainty in our estimated parameters.

The two missions that we have analyzed differ in a number of key aspects. Nevertheless, it is interesting to note that the general conclusion that we draw on the comparative performance of the one- and two-way systems is similar. Specifically, although the two-way system has the potential to facilitate superior mission performance, this potential cannot be fully exploited due to the uncertainties in other models used to process the tracking data.

Acknowledgements

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