Earth Rotation and Gravity Field Parameters from Satellite Laser Ranging

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Abstract. *We present the results from a simultaneous estimation of the gravity field, Earth rotation parameters, and station coordinates from combined SLR solutions incorporating up to nine geodetic satellites: LAGEOS-1/2, Starlette, Stella, AJISAI, Beacon-C, Lares, Blits and LARES. These solutions cover all three pillars of satellite geodesy and ensure full consistency between the Earth rotation parameters, gravity field coefficients, and geometry-related parameters. We address benefits emerging from such an approach and discuss particular aspects and limitations of the gravity field recovery using SLR data. The current accuracy of SLR-derived polar motion, by the means of WRMS w.r.t. IERS-08-C04 series, is at a level of 118-149 µas, which corresponds to 4 to 5 mm on the Earth's surface. The WRMS of SLR-derived Length-of-Day, when the gravity field parameters are simultaneously estimated, is 56 µs/day, corresponding to about 26 mm on the ground, and the mean bias of SLR-derived Length-of-Day is 6.3 µs/day, corresponding to 3 mm.*

Introduction

The main 'three pillars' of satellite geodesy can be summarized as:
- precise determination of geometrical three-dimensional positions and velocities, (geometry),
- modeling and observing of geodynamical phenomena including the Earth rotation parameters (ERP), (rotation),
- determination of the Earth's gravity field and its temporal variations, (gravity).

Although all three pillars describe geodetic and geodynamic phenomena within the system Earth, the gravity has typically been treated separately from the geometry and rotation. Many SLR solutions comprise the estimation of SLR station coordinates, pole coordinates and the Length-of-Day (LoD) from the 7-day combined LAGEOS-Etalon solutions, whereas the gravity field parameters are not provided. On the other hand, when estimating gravity field parameters from SLR data, the parameters related to geometry and rotation have typically been fixed so far and not simultaneously estimated. We present the results of a simultaneous estimation of the gravity field, ERPs, and station coordinates from combined SLR solutions incorporating up to nine geodetic satellites.

Earth Rotation Parameters (ERP) from SLR

We process 10 years of SLR observations to LAGEOS-1/2, 10 years of SLR observations to Starlette, Stella, and AJISAI (here called SLR-LEO), and a combined solutions using all 5 satellites (here called multi-SLR). A description of the solution strategy can be found in Sośnica et al., (2014a). Table 1 shows the comparisons between SLR-derived ERPs and IERS-08-C04 series (Bizouard and Gambis, 2012) for the 7-day SLR solutions with and without estimating geopotential coefficients up to degree/order 4/4. In the LAGEOS-1/2 solutions and in the multi-satellite SLR solutions without estimating the geopotential, the once-per-revolution empirical orbit parameters in

the out-of-plane direction (W_S/W_C) are additionally estimated, because they are capable to absorb the large variations of C_{20} (Sośnica et al., 2012). Omitting estimation of W_S/W_C leads to inferior SLR solutions when C_{20} is neither estimated (Sośnica et al., 2012).

Table 1. Differences in ERPs estimation and station coordinate repeatability in SLR solution with and without estimating the Earth's gravity field coefficients. Comparison with IERS-08-C04 series.

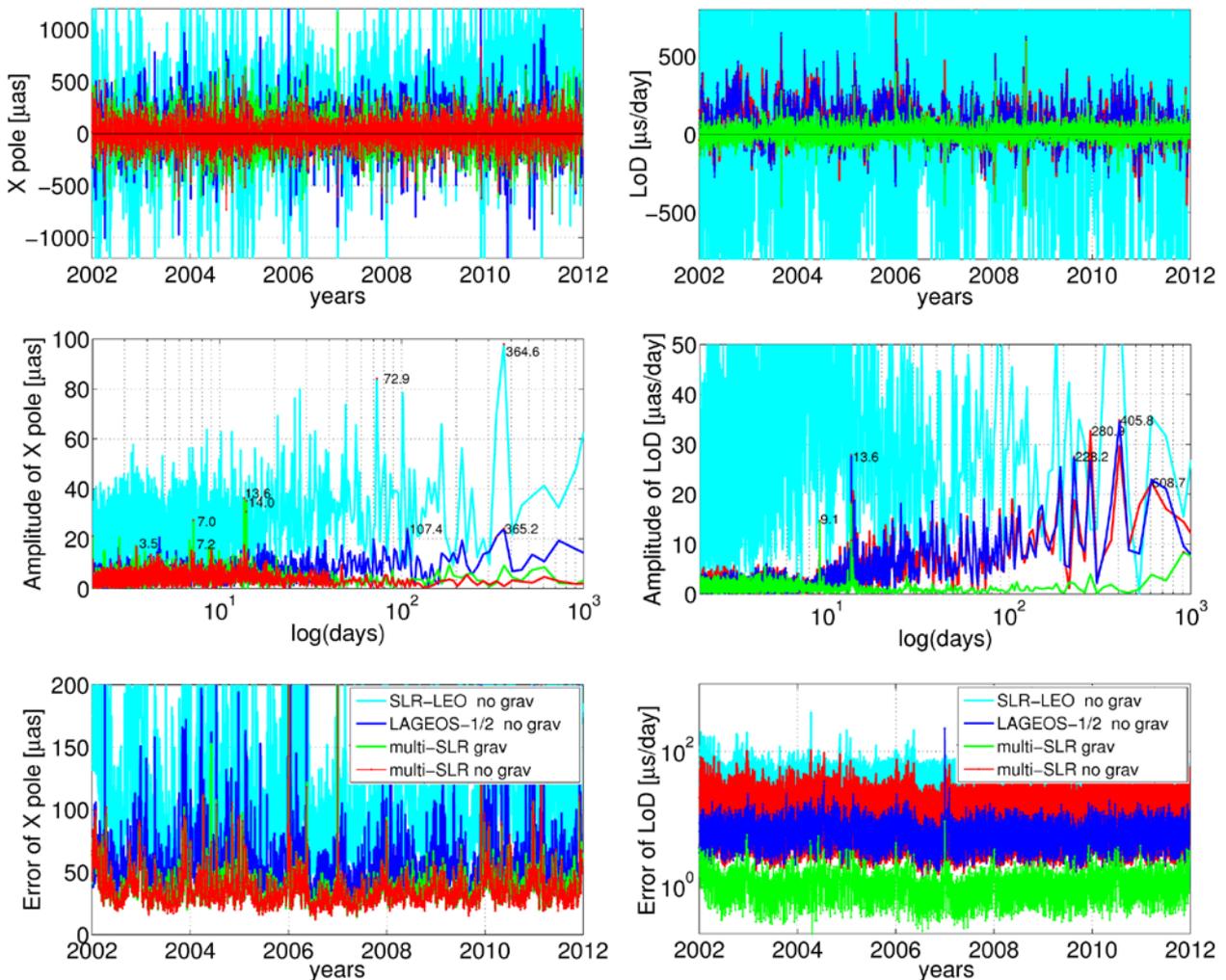| Solution type		X pole [µas]		Y pole [µas]		LOD [µs/day]	
		Bias	WRMS	Bias	WRMS	Bias	WRMS
LAGEOS-1/2	d/o 4/4	4.1	160.0	-8.0	155.2	6.1	57.0
LAGEOS-1/2	no grav	45.8	168.5	-54.1	153.5	77.3	120.5
SLR-LEO	d/o 4/4	38.3	267.9	-7.8	217.6	-38.5	105.6
SLR-LEO	no grav	190.1	437.5	-61.1	315.9	189.6	359.3
multi-SLR	d/o 4/4	6.4	148.9	8.5	140.3	6.3	56.3
multi-SLR	no grav	12.3	118.8	-3.2	120.7	73.2	120.9

Figure 1. Left: Differences of the X pole coordinate w.r.t. IERS-08-C04 series (Top), spectral analysis of the differences (Middle), a posteriori errors of the X pole coordinates (Bottom). The Y pole coordinate shows similar variations, thus, it is not shown here. **Right**: Differences of the LoD w.r.t. IERS-08-C04 series (Top), spectral analysis of the differences (Middle), a posteriori errors of LoD estimates (Bottom). Note the logarithmic scale for the y axis in the bottom figure.

In the LAGEOS solutions, the mean biases w.r.t. IERS-08-C04 for the X and Y pole coordinates become larger in the solution without geopotential parameters (see Table 1). The mean biases amount to 4.1 and -8.0 µas for the X and Y pole coordinates, respectively, in the LAGEOS solutions with estimating geopotential and 45.8 and -54.1 µas in the LAGEOS solutions without estimating geopotential parameters. A particular degradation of LoD estimates is observed for the solution without estimating geopotential parameters, namely the WRMS grows from 57.0 to 120.5 µs. LoD absorbs the part of C_{20} that is not accounted for by a priori C_{20} values in the solution without estimating geopotential, which leads to a shift in LoD series (Sośnica 2014). As a result, the C_{20} estimates and the shift of LoD from the solution without estimating the geopotential are of the same order of magnitude. Thus, the estimation of C_{20} is beneficial for the LAGEOS solutions, when estimating LoD values using the piece-wise-linear representation.

Similar problems with LoD estimates are found for the Starlette-Stella-AJISAI solutions (LEO-SLR). The degradation of pole coordinates is significant in the Starlette-Stella-AJISAI solutions, reflected in WRMS of 267.9 and 437.5 µas for the X pole coordinate in the solutions with and without estimating the geopotential, respectively. We conclude that the estimation of Earth's gravity field parameters is beneficial for low orbiting SLR satellites when a static a priori gravity field model is used. Neglecting the estimation of geopotential parameters for Starlette, Stella, and AJISAI, leads to a serious orbit and ERP degradation, whereas the station coordinates only are marginally affected (up to 1.6 mm for the height component of station repeatability).

Figure 1 (left) shows the X pole coordinates and Figure 1 (right) shows LoD estimates as differences w.r.t. IERS-08-C04 series for different SLR solutions. The LAGEOS solution without estimating gravity field parameters is closest to the official ILRS solutions. Figure 1 clearly shows that including low orbiting satellites is beneficial for ERP estimation.

Table 1 however, shows a degradation of the pole coordinates in the multi-SLR solutions when the geopotential parameters are additionally estimated. The WRMS increases from 118.8 to 148.9 µas for the X pole coordinate and from 120.7 to 140.3 µas for the Y pole coordinate in the multi-SLR solutions without and with estimating the gravity field parameters, respectively. Figure 1 (left, middle) shows peaks of about 3.5, 7.0, and 14.0 days, which are related to the lengths of the solution batches and its harmonics or to the orbit alias with tidal waves. The gravity field parameters are estimated in 7-day interval batches, which lead to an inferior quality of pole coordinates due to, e.g., the correlations between C_{21}, S_{21} and pole rates. Besides these peaks, both, the multi-SLR solutions with and without estimating the gravity field parameters, have a similar quality and a posteriori errors at a similar level, which are much smaller as compared to LAGEOS-1/2 solutions. Figure 1 (left, middle) shows that all multi-SLR solutions (regardless whether estimating geopotential or not) do reduce the peaks in the X pole coordinate, which are apparent in the LAGEOS-1/2 solutions. These peaks are related to LAGEOS orbit modeling deficiencies or to the alias with tidal waves, e.g., the annual signal and an eclipsing period of LAGEOS-2 (107.4 days on periodogram).

For LoD, the simultaneous estimation of the gravity field parameters:

- reduces the offset of LoD estimates (Fig 1, right top), which is mostly due to absorption the C_{20} variations by LoD estimates (Sośnica 2014)
- reduces peaks in the spectrum analysis (Fig 1, right middle), which correspond, e.g., to orbit modeling deficiencies (peaks of 222 days, i.e., a draconitic year of LAGEOS-2, 280 days, i.e., an eclipsing period of LAGEOS-1),
- substantially reduces the a posteriori error of estimated LoD (Figure 1, right bottom, notice a logarithmic scale for the y axis). The mean a posteriori error of LoD is 1.3, 16.9, 7.1, and 44.6 µs/day in the multi-SLR solution with gravity, multi-SLR solution without gravity, LAGEOS-1/2 solution without gravity, and SLR-LEO solution without gravity field parameters, respectively.

The a posteriori error of LoD in the multi-SLR solutions (16.9 µs/day) is thus more than factor of two higher than in the LAGEOS-1/2 solutions (7.1 µs/day) when the gravity field parameters are not estimated. This quality degradation implies that the estimation of the gravity field parameters is essential for high-quality LoD estimates when using SLR data to low orbiting geodetic satellites.

Earth's Gravity Field from SLR
We present the results from a simultaneous estimation of the gravity field up to d/o 10/10, Earth rotation parameters, and station coordinates from combined SLR solutions incorporating geodetic satellites: LAGEOS-1/2, Starlette, Stella, AJISAI, Beacon-C, Blits, Larets and LARES. These solutions cover all three pillars of satellite geodesy and ensure a full consistency between the Earth rotation parameters, gravity field coefficients, and geometry-related parameters. The monthly gravity field solutions are based on 10-day arcs of LAGEOS satellites and 1-day arc of low orbiting SLR satellites. Estimating short arcs for low orbiting satellites prevents from the accumulation of orbit errors in the estimated gravity field parameters. Details on the gravity field determination using SLR can be found in Sośnica et al. (2015), whereas the applications of SLR-derived gravity field models can be found in Weigelt et al. (2015).

Figure 2 shows the time series of exemplary tesseral S_{42} and sectorial S_{44} and C_{66} coefficients of degree 4 and 6. The figure clearly proves that not only the zonal coefficients can be well established from SLR solutions, but also tesseral and sectorial terms, even of degree 6. The spectral analysis of S_{44} for the GRACE solutions shows a peak of about 160 days, which corresponds to the alias of the GRACE orbit with the S_2 tide constituent. SLR solutions are free from this artifactual peak. SLR solutions comprise observations to low and high orbiting satellites at different inclinations which results in different S_2 alias periods with satellite orbits, and thus, minimizes the negative influence on SLR-derived gravity field parameters.

Figure 3 compares the monthly gravity field models obtained from GRACE up to d/o 60/60 (left) and SLR up to d/o 10/10 (right) w.r.t. reference field EGM2008. Figure 2 proves that the most pronounced temporal geoid deformations, e.g., in Greenland, Amazonia, North America, agree well between GRACE and SLR solutions and thus can be well recovered also by SLR solutions. On the other hand, the smaller geoid deformations can be recovered by SLR only to a limited extent, e.g., in Southern Africa and Southeast Asia. SLR-derived deformations are smoothed as compared to GRACE results and the amplitudes of geoid deformations are reduced.

Interestingly, SLR is also able to recover the differences in the ice mass loss in the Antarctica region. This result is very surprising, because none of the low orbiting SLR satellites were observed over this region. There is no single SLR station in Antarctica, and moreover, in March 2011 there were only six active SLR stations in the Southern hemisphere, from which only three were used for the network constraints, i.e., the so-called SLR core stations. Fine and small-scale geoid variations can only be recovered from low orbiting satellites. The geoid recovery is however possible, despite large gaps in spatial coverage of SLR stations in the Southern hemisphere. As opposed to continuous satellite-to-satellite tracking in the GRACE solutions (low-low) or in the CHAMP, GRACE, or GOCE solutions (high-low), the SLR observations are noncontinuous, sparse and limited by the inhomogeneous distribution of SLR stations. The intrinsic analysis of orbit dynamics allows, however, the SLR solutions to determine the geoid deformation even of the areas over which none of the SLR satellite was observed, because the orbit dynamics carries implicitly valuable information about the Earth's gravity potential as a whole. Thus, the large-scale mass redistribution can be recovered from the SLR analysis even for areas with no or sparse SLR observations.

Figure 4 shows the amplitudes of annual signals for low-degree coefficients up to d/o 6/6 in the SLR (left) and GRACE solutions (middle) and the differences of the amplitudes in both solutions (right). The amplitudes in SLR solutions are typically underestimated by about 10% as compared to the GRACE results. The agreement between SLR and GRACE is at 77% level in terms of low-

degree coefficients. Figure 5 reveals that the seasonal variations of, e.g., C_{50} in the SLR solutions are underestimated as compared to the GRACE results. The SLR-derived amplitude of annual signal is smaller by 48% than the amplitude from GRACE solutions. However, including LARES into the SLR solutions in February 2012 substantially improves the SLR solutions by reducing the correlations between estimated parameters (Sośnica et al., 2014b).
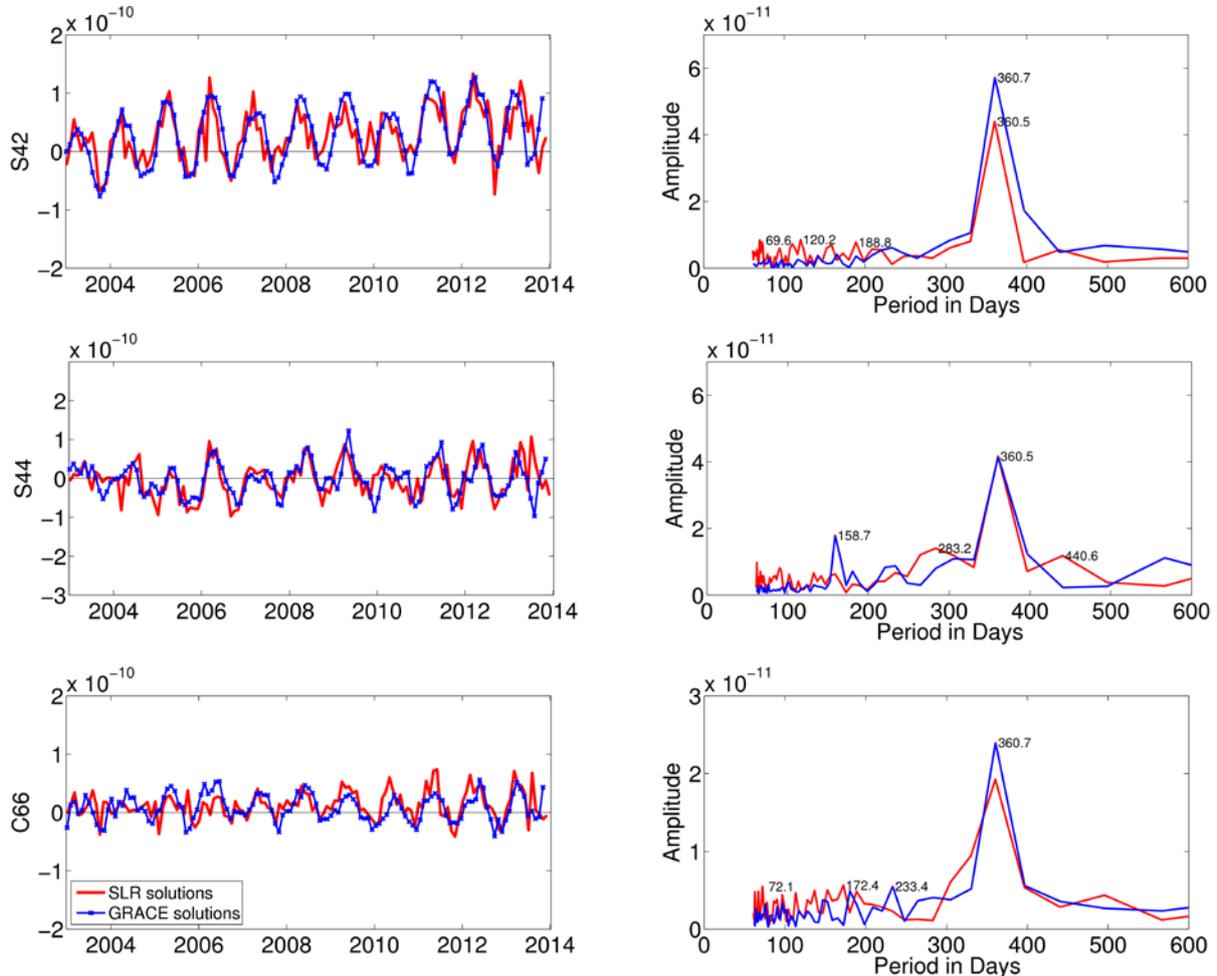

Figure 2. S_{42} (top), S_{44} (middle), C_{66} (bottom) variations w.r.t. EGM2008 derived from SLR and GRACE AIUB-RL02 solutions (left, Meyer et al., 2012) and the spectral analysis of the series (right).
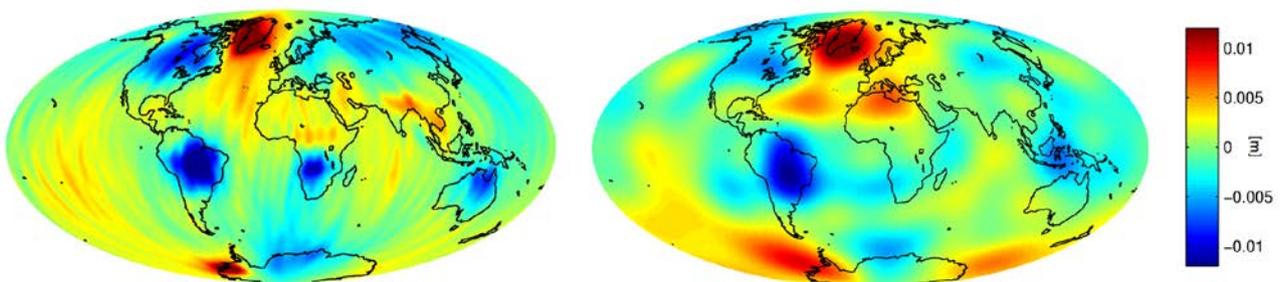

Figure 3. Monthly gravity field models w.r.t. EGM2008 for March 2011, derived from GRACE (AIUB-RL02) solutions up to d/o 60/60 with Gaussian filtering of 300 km (left), and SLR solutions up to d/o 10/10 with no filtering (right).
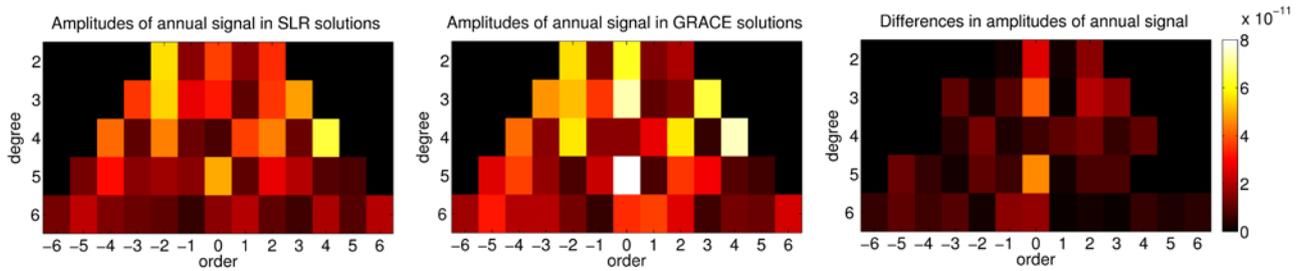
Figure 4. Amplitudes of annual signals in SLR solutions (left), GRACE solutions (middle) and the difference thereof (right).
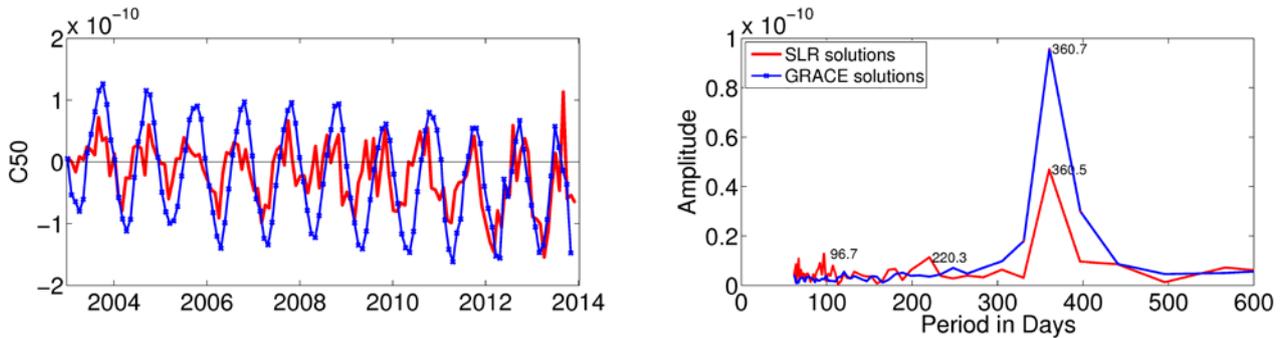

Figure 5. C_{50} variations w.r.t. EGM2008 derived from SLR and GRACE solutions (left) and the spectral analysis of the series (right).

AIUB-SLR monthly gravity field solutions are available from the International Centre for Global Earth Models (ICGEM) website: http://icgem.gfz-potsdam.de/ICGEM/shms/monthly/aiub-slr

Summary
The simultaneous estimation of the gravity field parameters, ERP, and station coordinates leads to a minor degradation of the pole coordinate quality in the multi-SLR solutions (e.g., the WRMS of the X pole is 148.9 and 118.8 µas in solutions with and without estimating gravity, respectively), but substantially improves the quality of LoD estimates. The pole coordinates benefit particularly from incorporating many geodetic satellites of different altitudes and inclinations and a better observation geometry, whereas the LoD benefits most from the simultaneous estimations of ERP and C_{20}.

In the determination of LoD the simultaneous estimation of the gravity field parameters along with other SLR-derive parameters: (1) reduces the offset of LoD estimates, which is mostly due to absorption of C_{20} variations by LoD estimates, (2) reduces peaks in the spectrum analysis, which correspond, e.g., to orbit modeling deficiencies, (3) reduces the a posteriori error of estimated LoD.

The quality of the SLR-derived pole coordinates and LoD from Starlette, Stella, and AJISAI data is by factor of two better when estimating low degree gravity field coefficients, as compared to the solution without estimating gravity coefficients. LAGEOS satellites remarkably stabilize the ERP and station coordinate estimates in multi-SLR solutions; thus, the combined solution using SLR observations to many satellites is highly preferable.

Not only degree-2 gravity field coefficients can be very well determined from SLR, but also other coefficients by using the combination of short 1-day arcs for low orbiting satellites and 10-day arcs for LAGEOS-1/2. In this way, LAGEOS-1/2 allow recovering zonal terms, which are associated with long-term satellite orbit perturbations, whereas the tesseral and sectorial terms benefit most from low orbiting satellites, whose orbit modeling deficiencies are minimized due to short 1-day arcs. The contribution of LARES, starting from February 2014, is remarkable for the quality of the estimated C_{50} series. The Antarctica and Greenland regions are essential in the geophysical studies of the mass transportations. We have shown that the SLR solutions are also able to recover the

differences in the ice mass loss in the Antarctica and Greenland; although none of the low orbiting SLR satellites were observed over Antarctica.

The amplitudes of the annual signal in the low-degree gravity field coefficients derived from SLR agree with GRACE K-band results at a level of 77%. This implies that SLR has a great potential to fill the gap between the current GRACE and the future GRACE Follow-On mission for recovering of the seasonal variations and secular trends of the longest wavelengths in gravity field, which are associated with the large-scale mass transport in the system Earth.

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