Earth Orientation and Relativity Parameters Determined from LLR Data

Jürgen Müller[1,2], Liliane Biskupek[1], Franz Hofmann[1]

[1]Institut für Erdmessung (IfE), Leibniz Universität Hannover, Schneiderberg 50, 30167 Hannover, Germany
[2] QUEST – Centre for Quantum Engineering and Space-Time Research, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany
Contact: mueller@ife.uni-hannover.de

Abstract

Lunar Laser Ranging (LLR) has provided distance measurements to the Moon for 45 years with steadily increasing accuracy. Using a highly accurate model based on Einstein's theory of gravity for the analysis of these measurements, LLR is able to provide, among others, (long-periodic) Earth orientation parameters as well as quantities testing General Relativity (e.g. strong equivalence principle, Yukawa-like perturbations or time-variability of the gravitational constant). Here, we briefly show the recent status of analysis model developed at IfE. We present results for selected nutation coefficients as well as for polar motion and UT0 purely derived from LLR data. We also give a few examples for relativistic quantities (including realistic errors).

1. Introduction

LLR is now carried out for more than 45 years. In this time span, more than 20,000 normal points (NP) were obtained, i.e. a collection of roundtrip travel times of photons received over 2 to 15 minutes. These data represent the longest series of space geodetic observations. Under optimal conditions, the measurements reach accuracies at the millimeter level. Currently, four active LLR sites track the Moon routinely: the McDonald Observatory in Texas (USA), the Observatoire de la Côte d' Azur (France), the APOLLO site in New Mexico (USA) and the Matera Laser Ranging station in Italy.

The analysis of the data is based on Einstein's theory of relativity and complete up to the first post-Newtonian order. The accuracy of the model is at the centimeter level. Various parameters of the Earth-Moon system can be determined in a least-squares adjustment. These include parameters of physical libration and orbit of the Moon, coordinates of LLR stations and retroreflectors as well as coefficients of the nutation series and Earth rotation parameters. With special modifications, tests of Einstein's theory of relativity are possible.

2. Earth orientation parameters

2.1 Earth rotation parameters

The determination of Earth rotation parameters (ERP), which are polar motion and Earth rotation phase (UT0), from LLR data can be performed for different configurations: specific nights, selected time spans, all observatories together or only one, trends over certain periods or

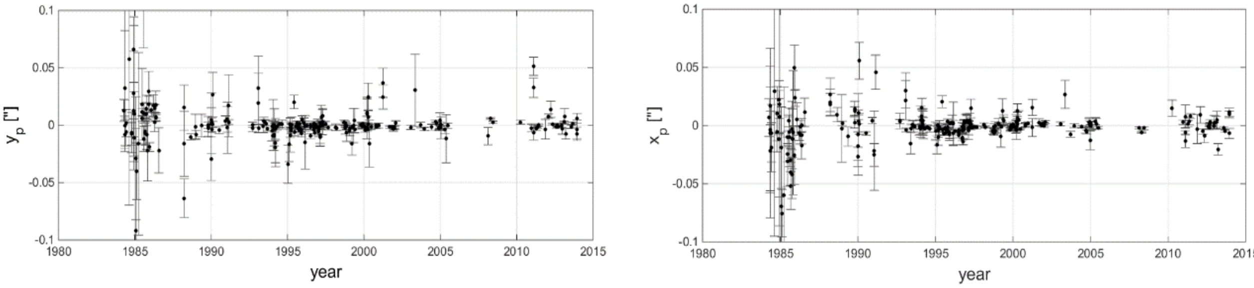

Fig. 1: *Terrestrial pole coordinates provided that 15 NP are available in the analyzed night. The dots give the corrections to the initial values from IERS EOP C04 series with their respective 1σ error bars.*

corrections to initial values for pre-selected epochs. In the next sections, results from recent fits for these different configurations are presented.

2.1.1 Pole coordinates from all LLR sites

Investigations of ERP estimates from LLR alone for selected time spans show, that the datum definition in the LLR analysis is challenging and must be done carefully to get realistic results. So for the results presented here, corrections to a-priori pole coordinates of the IERS C04 series are determined using data from all LLR observatories. The characteristics of the fit are:

- Only nights with at least 15 NPs per night for the time span 4/1984 – 12/2013 (247 nights),
- Simultaneous determination of only one component of the pole coordinates and all coordinates of the observatories,
- Velocities of the observatories fixed to the ITRF values,
- A-priori EOP values from IERS C04 series, fixed for the nights that were not considered.

The results are shown in figure 1. From 1995 on, the values vary by about ± 15 mas with an accuracy of 1 – 12 mas for the x_p component and by ± 10 mas with an accuracy of 1 – 15 mas for y_p. Correlations up to 10 % exist with the station coordinates. Occasional correlations in the same order are detected among the pole coordinates themselves.

2.1.2 Pole coordinates or Earth rotation phase from APOLLO data

In this study, only data of APOLLO are used. The characteristics of the analysis are:

- Only nights with at least 5 NPs per night for the time span 6/2006 – 9/2013 (182 nights),
- Simultaneous determination of either pole coordinates or Earth rotation phase and all coordinates of the observatories,
- Velocities of the observatories fixed to the ITRF values,
- A-priori EOP values from IERS C04 series, fixed for the nights that were not considered.

The results for pole coordinates are shown in figure 2. In comparison to the results in figure 1, the parameter corrections are scattered much more as now also nights with only 5 NP have been

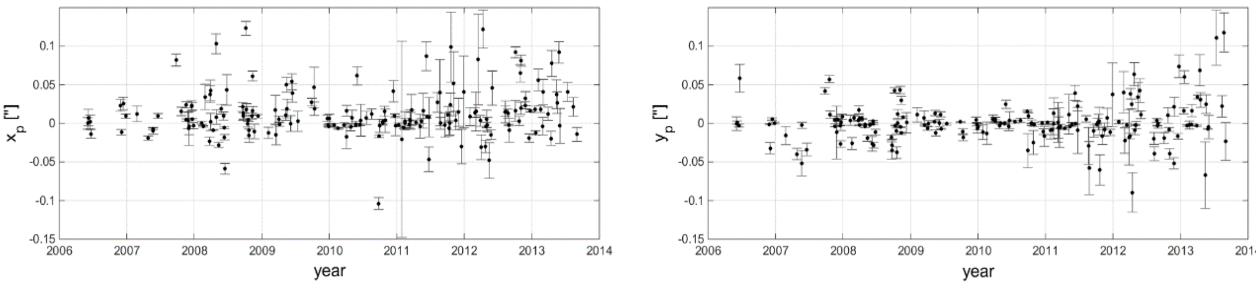

Fig. 3: *Terrestrial pole coordinates provided that 5 NP from the station APOLLO are available in the analysed night. The dots give the corrections to the initial values from IERS EOP C04 series with their respective 1σ error bars.*

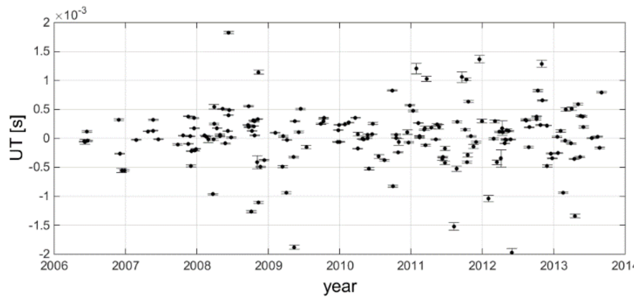

Fig. 3: *Earth rotation phase provided that 5 NP from the station APOLLO are available in the analysed night. The dots give the corrections to the initial values from IERS EOP C04 series with their respective 1σ error bars.*

included, whereas previously the constraint was 15 NP. The accuracy is between 1 and 40 mas. From end of 2010 on, the results get worse due to reduced observational accuracy as consequence of non-solved problems with the calibration of the raw data. The correlations in that determination are higher with other parameters, up to 30 % with reflector and station coordinates, 20 % with the mass of the Earth-Moon system and 40 % among the pole coordinates themselves.

The result for Earth rotation phase in figure 3 looks better than the one for the pole coordinates. Although the increase of the corrections because of the reduced observational accuracy after 2010 is also present, the results reach an accuracy of 0.003 – 0.05 ms. Correlation with the coordinates of the observatories are up to 50 %, with other ΔUT values of 20 to 40 % are obtained. This is the best ERP result from LLR and can be used to independently validate UT estimates from VLBI.

2.1.3 Discussion of ERP results

The investigations show that Earth rotation parameter determination from LLR is possible, but not easy because of the unequally distributed data, i.e. due to nights with no measurements as well as the low number of observatories and their geographical distribution. These conditions contribute to the uncertainty of the pole coordinates of 1 – 10 mas that is worse than those of other space geodetic techniques. For the determination of the Earth rotation phase, stations in east-west direction are needed. That condition is fulfilled by the LLR network. The realistic accuracy for that component is 0.03 – 0.2 ms, which is closer to results from other space geodetic techniques. Because of the difficult datum definition in the LLR analysis, the Earth rotation parameters should be determined from observations of all LLR stations together, to minimize sensitivities of one station to an individual component. Those results could then be used for a combined solution of the Earth rotation parameters that is computed by the International Earth Rotation and Reference System Service IERS.

Tab. 1: *Nutation coefficients for four periods from the analysis of LLR data applying different methods for the transformation between ITRS and GCRS. The equinox-based transformation is used in LLR 1 with precession according to Fukushima [2003] and Williams [1994]. In LLR 2, precession is modeled according to Capitaine und Wallace [2006]. MHB2000 are the values from the official nutation model from Mathews et al. [2002]. All values are given in [mas].*

Period		MHB2000 [mas]	LLR 1 [mas]	LLR 2 [mas]
18,6 years	A	-17206,42	2,70 ± 0,20	5,21 ± 0,25
	B	9205,23	-0,48 ± 0,10	-1,32 ± 0,11
	A"	3,34	-4,62 ± 0,12	-3,46 ± 0,21
	B"	1,54	-2,29 ± 0,09	-2,19 ± 0,10
182,6 days	A	-1317,09	-2,38 ± 0,08	-1,69 ± 0,11
	B	573,03	0,25 ± 0,05	0,15 ± 0,05
	A"	-1,37	1,80 ± 0,07	1,85 ± 0,09
	B"	-0,46	0,23 ± 0,05	0,22 ± 0,05
9,3 years	A	207,46	0,45 ± 0,11	0,85 ± 0,18
	B	-89,75	-0,15 ± 0,07	-0,13 ± 0,08
	A"	-0,07	-1,50 ± 0,12	-0,97 ± 0,20
	B"	-0,03	-0,87 ± 0,08	-1,35 ± 0,09
365,3 days	A	147,59	-2,91 ± 0,10	-0,51 ± 0,16
	B	7,39	0,55 ± 0,06	0,01 ± 0,07
	A"	1,12	-2,30 ± 0,09	-0,06 ± 0,11
	B"	-0,19	-0,29 ± 0,05	-0,02 ± 0,05

2.2 Nutation

In our LLR analysis, precession and nutation are modeled according to IAU Resolution 2006 and IERS Conventions 2010. Because of the different realizations of precession/nutation for GCRS-ITRS transformation, it is also possible to fit luni-solar nutation coefficients for these different scenarios. From LLR data, the estimation of the non-time dependent coefficients A', A'', B', B'' in the equations

$$\Delta\psi = \sum_{i=1}^{N} (A_i + A'_i t)\sin(ARG) + (A''_i + A'''_i t)\cos(ARG)$$

$$\Delta\varepsilon = \sum_{i=1}^{N} (B_i + B'_i t)\cos(ARG) + (B''_i + B'''_i t)\sin(ARG)$$

$$ARG = \sum_j N_j F_j$$

is possible for nutation periods of 18.6 years, 9.3 years, 1 year, 182.6 days. The determination of the 13.6-day period is difficult, because of the unequal distribution of the LLR NP over one synodic month. Corresponding results are not shown here, but are given in Biskupek [2015].

Extensive investigations and calculations were done under different conditions to determine the accuracy for nutation coefficients from LLR. Realistic errors are in the order of 0.1 – 0.3 mas in obliquity and 0.2 – 0.5 mas in longitude. As one example for four periods of nutation, LLR results are calculated under the following conditions for an equinox-based transformation:

- **LLR1:** precession according to Fukushima [2003] and Williams [1994],
- **LLR2:** precession according to Capitaine and Wallace [2006].

The values of the two transformation matrices themselves agree to each other on the level of 10^{-12} rad. The results for the fit of the nutation coefficients are given in table 1 as differences to the official nutation model MHB2000 [Mathews et al., 2002]. The results show large differences for the 18.6-year period for both transformation methods, the largest are obtained in case LLR1. Here, also large differences are seen in the coefficients for the 182.6-day and 365.3-day period. In case LLR2, the differences of the 365.3-day period are very small. In both cases larger differences show up in the longitude component.

Nevertheless, the results are interesting, as they show the changing correlation related to the taken transformation matrix. Earlier investigations in Biskupek and Müller [2009a,b] and Biskupek et al. [2012] also showed differences in the LLR determined nutation coefficients. All the results may point to still existing model inconsistencies in the LLR analysis and the problem of unequally distributed LLR data (e.g. gaps in time series, orbit coverage, only few sites, weather, less accuracy in early years). But also systematic errors in the MHB2000 model, which is based on VBLI data, cannot be excluded.

The APOLLO station is capable to measure the distance to the Moon also under difficult conditions and reaches a better distribution over the synodic month. Concerning the 13.6-day period, tests with only APOLLO data showed promising results. With more data from APOLLO in the next years, the determination of the discussed nutation coefficients should be able in future.

A planned joint analysis of LLR and VLBI data will also lead to more stable results for the nutation coefficients.

3. Variation of the gravitational constant

The last results presented here address relativity tests, where the variation of the gravitational constant G is taken as an example. The recent results of the determination from LLR [Biskupek, 2015] are:

$$\frac{\dot{G}}{G} = (1.2 \pm 1.5) \times 10^{-13}\, yr^{-1},$$

$$\frac{\ddot{G}}{G} = (1.4 \pm 3.0) \times 10^{-15}\, yr^{-2}.$$

The accuracy improved by a factor of 4 for $\dot{G}$ and by a factor of 1.5 for $\ddot{G}$ in comparison to the previous results published in Müller and Biskupek [2007]. Results for further relativity parameters are given in Müller et al. [2014]. The longer time span of data and the updated processing are responsible for the improvement. The correlations of the gravitational constant components $\dot{G}$ and $\ddot{G}$ are almost 100 % with the lag angle $k_2\delta$ of the lunar tidal acceleration, which should improve in future.

4. Conclusions

The recent investigations showed that nutation coefficients from LLR are partly well determined (e.g. annual), but the results are affected by the uneven data distribution. The pole coordinates and ΔUT can be obtained with high accuracy for those nights where good data are available. Our investigations also confirmed that LLR is a unique tool for studying the Earth-Moon system and testing general relativity, e.g. by constraining variations of the gravitational constant.

Acknowledgements

Current LLR data are collected, archived and distributed under the auspices of the International Laser Ranging Service (ILRS) [Pearlman et al., 2002]. We acknowledge with thanks, that the more than 45 years of used LLR data has been obtained under the efforts of the personnel at different observatories. This research was funded by the Centre for Quantum Engineering and Space-Time Research QUEST and the DFG, the German Research Foundation, within the research unit FOR584 "Earth rotation and global dynamic processes".

References

BISKUPEK, L. [2015]: *Bestimmung der Erdorientierung mit Lunar Laser Ranging*, Phd Thesis, Leibniz Universität Hannover. Deutsche Geodätische Kommission bei der Bayerischen Akademie der Wissenschaften, Reihe C, Nr. 742 (identical with: Wissenschaftliche Arbeiten der Fachrichtung Geodäsie und Geoinformatik der Leibniz Universität Hannover, Nr. 316).

BISKUPEK L., MÜLLER J. [2009a]: *Lunar Laser Ranging and Earth Orientation*. In: M. Soffel, N. Capitaine (Eds.) „Astrometry, Geodynamics and Astronomical Reference Systems", p. 182–185. Proceedings of the Journées 2008 «Systèmes de Référence Spatio-Temporels», Lohrmann-Observatorium and Observatoire de Paris.

BISKUPEK, L. UND MÜLLER, J. [2009b]: *Relativity and Earth Orientation Parameters from Lunar Laser Ranging*. In S. Schillak, (Ed.), Proceedings of the 16th International Workshop on Laser Ranging, vol. 1, p. 270–276.

BISKUPEK L., MÜLLER J., HOFMANN F. [2012]: *Determination of Nutation Coefficients from Lunar Laser Ranging*. In: S. Kenyon, M.C. Pacino, U. Marti, M.G. Sideris (Eds.) „Geodesy for Planet Earth", vol. 136, IAG Symposia, p. 521–525. Springer, Berlin, Heidelberg.

CAPITAINE N., WALLACE P.T. [2006]: *High precision methods for locating the celestial intermediate pole and origin*. Astronomy and Astrophysics, vol. 450, p. 855–872.

FUKUSHIMA T. [2003]. *A New Precession Formula*. The Astronomical Journal, vo. 126, p. 494–534.

MATHEWS P.M., HERRING T.A., BUFFETT B.A. [2002]: *Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth's interior*. Journal of Geophysical Research, vol. 107, 10.1029/2001JB000390.

MÜLLER J., BISKUPEK L. [2007]: *Variations of the gravitational constant from lunar laser ranging data*. Classical and Quantum Gravity, vol. 24, p. 4533–4538.

MÜLLER J., BISKUPEK L., HOFMANN F., MAI E. [2014a]: *Lunar laser ranging and relativity*. In: S.M. Kopeikin (Ed.) „Frontiers in relativistic celestial Mechanics", Vol. 2: Applications and Experiments, 103–156. Walter de Gruyter, Berlin.

WILLIAMS J.G. [1994]: *Contributions to the Earth's obliquity rate, precession, and nutation*. The Astronomical Journal, vol. 108, p. 711–724.