Lunar Laser Ranging
What is it Good for?

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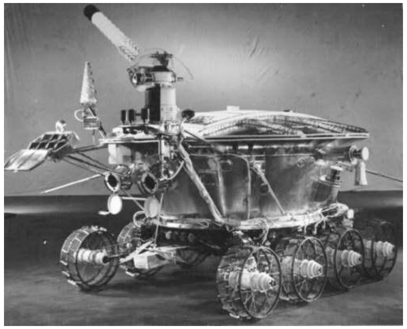
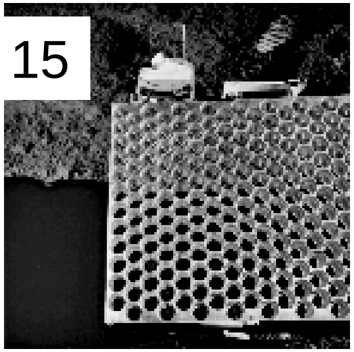
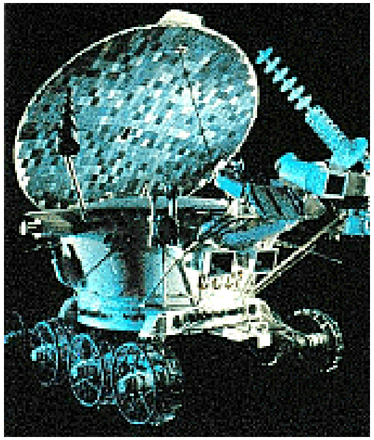
Retro-reflectors on the Moon



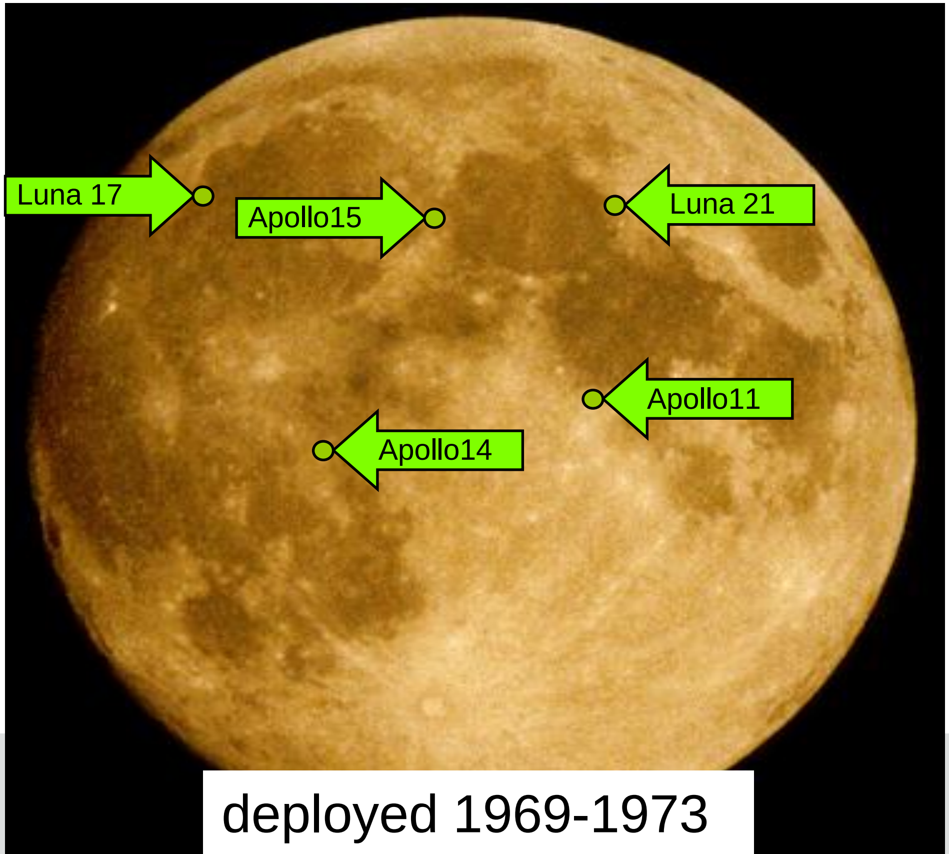
Luna 17

Apollo 15

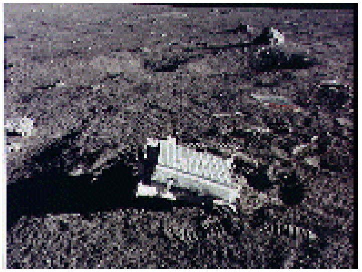




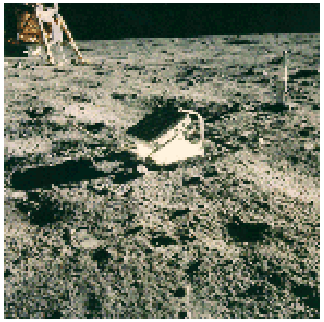
Luna 21



deployed 1969-1973

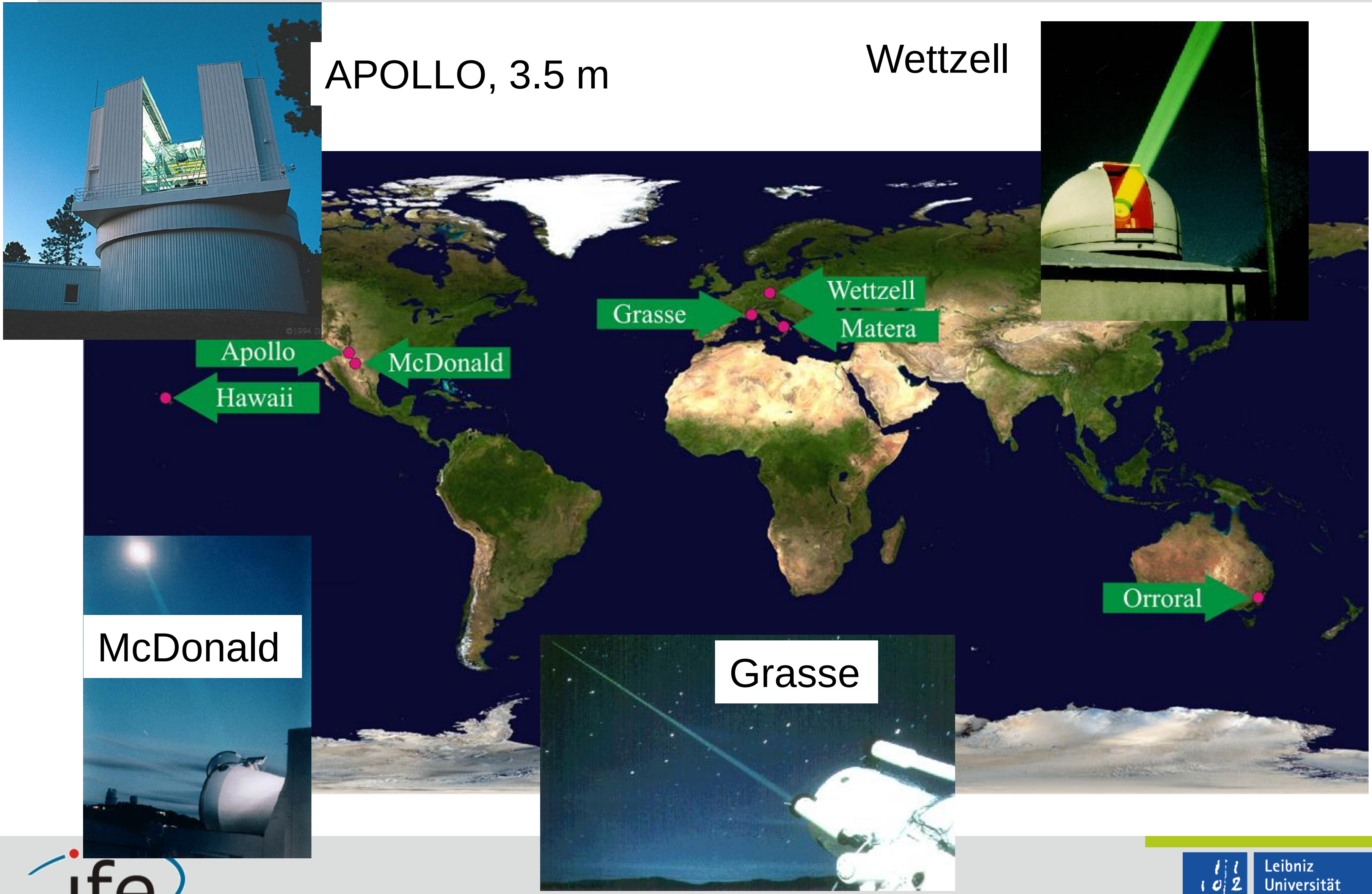


Apollo 14



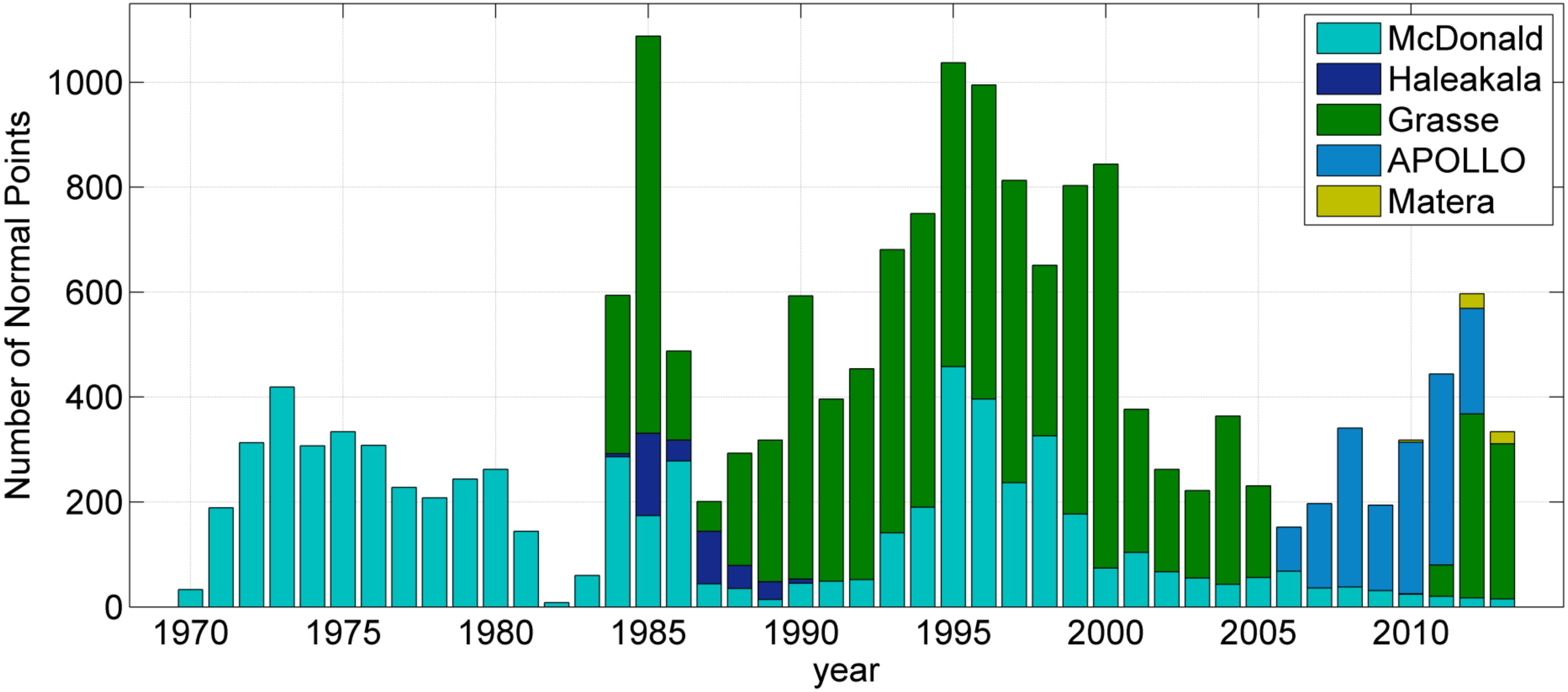
Apollo 11

Lunar Laser Ranging observatories on Earth

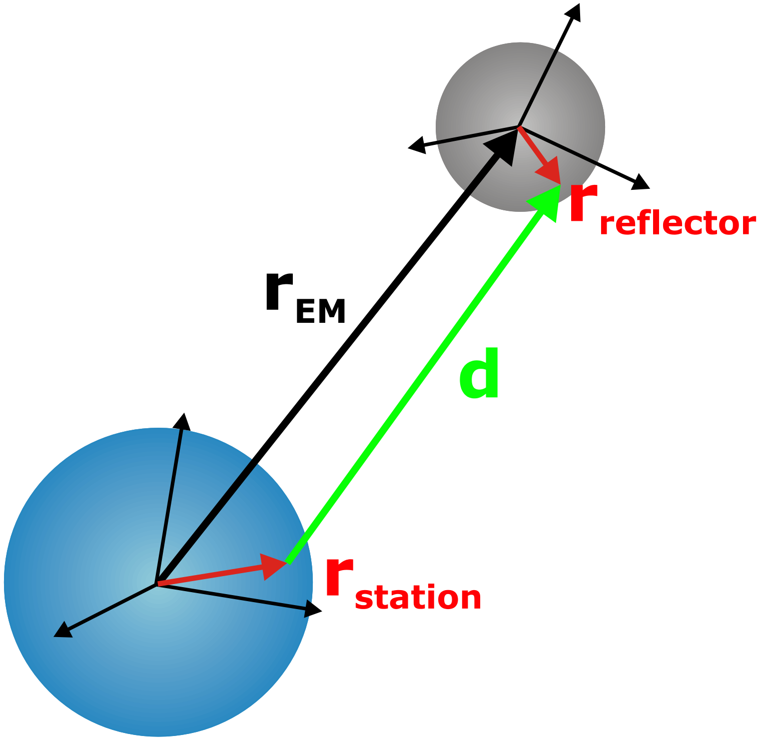


Number of normal points

1970 - 2013: ca.18,100 normal points



Basic formulas



- Basic equation

$$d = \left| \mathbf{r}_{EM} - \mathbf{r}_{station} + \mathbf{r}_{reflector} \right| + c\Delta\tau \approx c\frac{\tau}{2}$$

- Analysis in a quasi-inertial frame

$$\mathbf{r}_{reflector} = \mathbf{R}^{moon}\ \mathbf{r}_{reflector}^{SRF}$$

$$\mathbf{r}_{station} = \mathbf{R}^{earth}\ \mathbf{r}_{station}^{ITRF}$$

$$\mathbf{r}_{station} = \mathbf{S}(\mathrm{x_p}, \mathrm{y_p}, \mathrm{UT1})\,\mathbf{N}\,\mathbf{P}\,\mathbf{B}\,\mathbf{r}_{station}^{ITRF}$$

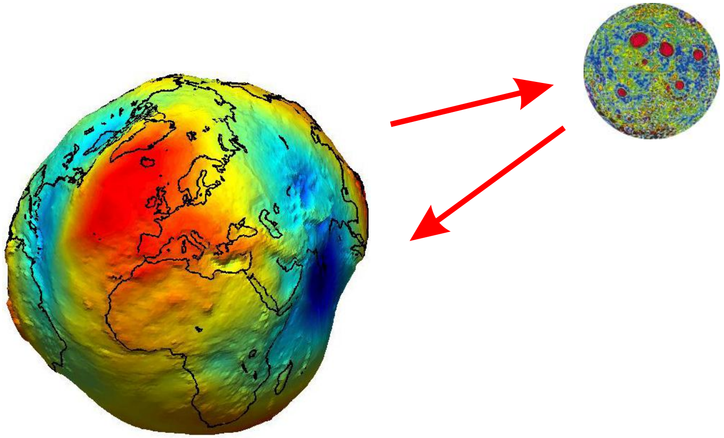
- Temporal variations of the reflector and station coordinates (tides ….)

Relativistic analysis model

- Lunar orbit, ephemeris
  - full relativistic model for motion of major solar system bodies (Einstein-Infeld-Hofmann equations of motion)
  - multipoles of Earth and Moon, lunar tidal acceleration

- Rotation of the Moon
  - elastic Moon, core, external torques
  - relativistic precessions

- Signal propagation
  - atmospheric correction
  - Shapiro time delay



Model refinement – for relativistic tests

- Optional extension of the ephemeris model
  - time variable gravitational constant $G = G_0 + \dot{G}\Delta t + \frac{1}{2}\ddot{G}\Delta t^2$
  - geodetic precession of the lunar orbit in addition to EIH
  - violation of equivalence principle (m_g / m_i)
  - acceleration due to dark matter in the galactic center (violation of equivalence principle)
  - Yukawa term for modifying Newton‘s $1/r^2$ law of gravity
  - preferred-frame effects and metric parameters (Will, 1993)
  - gravitomagnetic effects (Soffel et al., 2008)
  - optional spin-orbit coupling (Brumberg/Kopeikin)

LLR parameter fit

Analysis

- model based upon Einstein's theory
- weighted least-squares adjustment
- determination of various parameters of the Earth-Moon system (about 200 unknowns, without EOPs)

Results of major interest

- coordinates and velocities (selenocentric frame, ITRFxx)
- Earth orientation, σ = 0.5 mas (IERS)
- relativity parameters (grav. constant, equivalence principle, $1/r^2$-law, geodetic precession, metric ...)
- lunar interior, dynamic realisation of ICRS by the lunar orbit

Further LLR parameters

- Earth $k_2\delta$, lunar tidal acceleration (dr_{EM} = 3.8 cm/year)
- rotation of the Moon
- lunar gravity field coefficients up to degree and order 4
- dynamical flattening β and γ
- lunar k_2 (elasticity) and time lag (dissipation)
- *mass of Earth-Moon system* GM_{EM}
- $C_{20\text{-}Sun}$ (fixed to $2\text{x}10^{-7}$)
- …
- and various relativity parameters

Example: Yukawa-like perturbation

Test of 1/r² law (Yukawa)

$$\ddot{r}_{EM} = -\frac{GM_{E+M}}{r_{EM}^2}$$

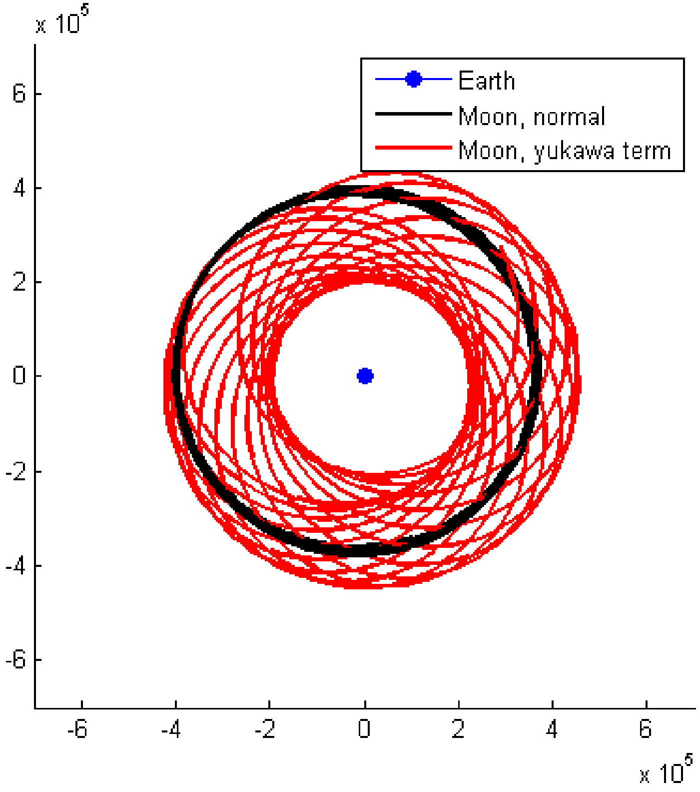
λ interaction range (400,000 km)

α coupling parameter

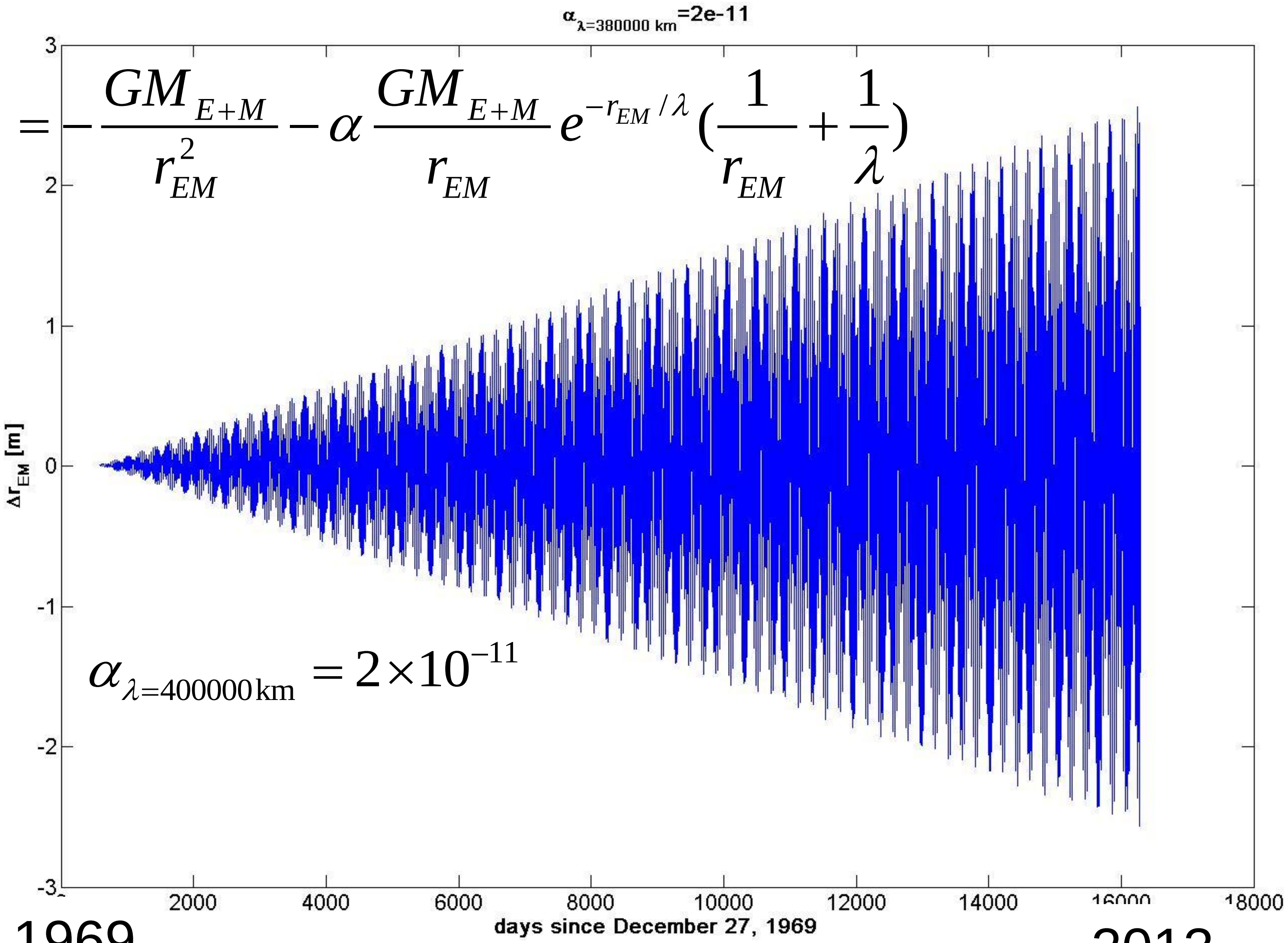
Result

$$\alpha_{\lambda=400000\,\text{km}} = (-0.6 \pm 1.8) \cdot 10^{-11}$$

strongly correlated with geodetic precession



Effect of α_{Yukawa} perturbation on r_{EM}



$$\ddot{r}_{EM} = -\frac{GM_{E+M}}{r_{EM}^2} - \alpha\frac{GM_{E+M}}{r_{EM}} e^{-r_{EM}/\lambda}\left(\frac{1}{r_{EM}}+\frac{1}{\lambda}\right)$$

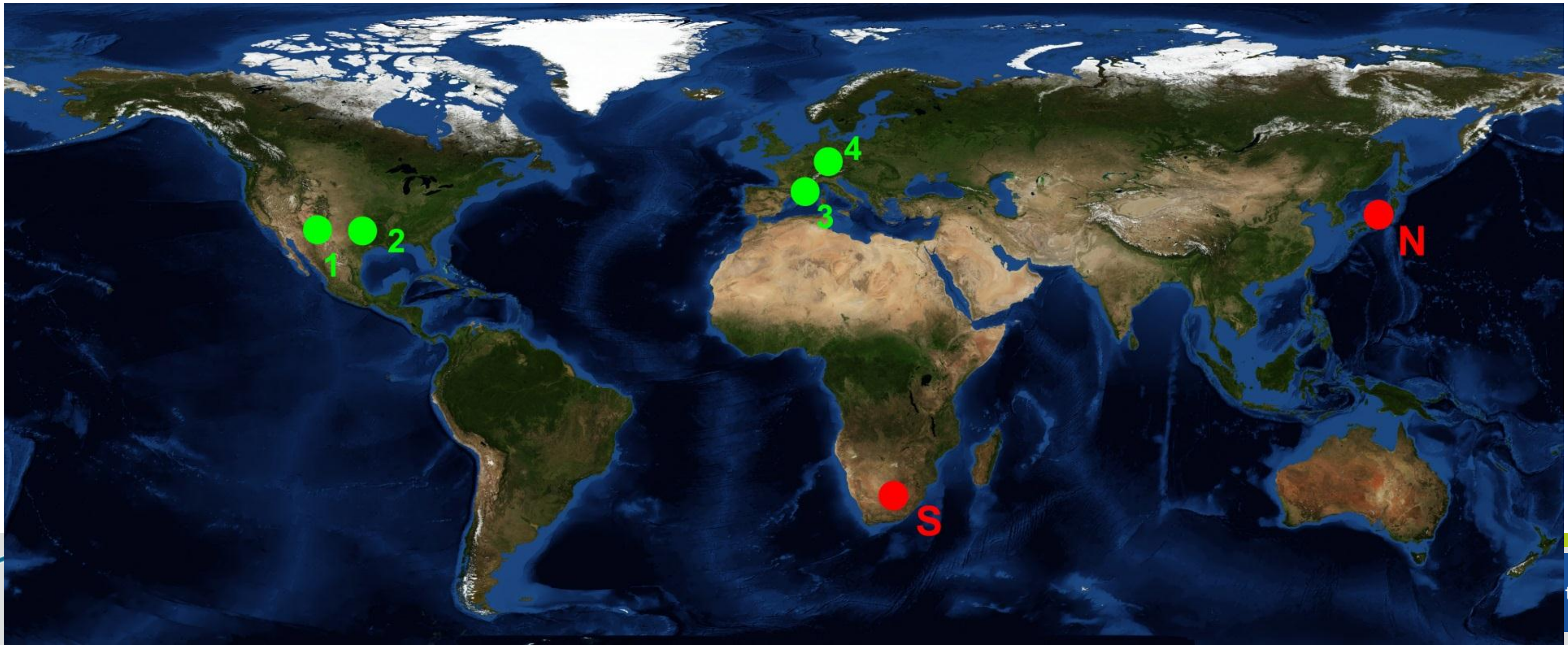
$$\alpha_{\lambda=400000\,\mathrm{km}} = 2\times10^{-11}$$

1969

2013

Benefit of more LLR observatories

- Simulation of one additional LLR observatory
  - existing constellation simulated with 4 sites (green dots) and 5 lunar reflectors
  - additional site
    - Northern hemisphere, Japan (label „N“)
    - Southern hemisphere, South Africa (label „S“)



Simulation scenario

- Simulated measurements
  - noise assumed such that annual wrms ~ 3-5 cm
  - 40 years of data homogenously distributed (is not in reality!)
  - lunar elevation > 40°
  - case 1: only reflectors which are in the dark
  - case 2: all available reflectors

- Analysis
  - estimated parameters: initial lunar orbit and rotation, reflector and station coordinates (one site fixed), lunar gravity field, tidal parameters, GM_{E+M}
  - comparison of 1σ standard deviations from different runs

Simulation results

Case 1

only reflectors in the dark

		basis solution
X_{ref}	x [mm]	242
	y [mm]	72
	z [mm]	243
Euler angles	φ [as]	1.15
	θ [as]	0.012
	ψ [as]	1.15
GM_{E+M}	[km^3s^{-2}]	6.73×10^{-4}

Case 2

all reflectors

		basis solution
X_{ref}	x [mm]	73
	y [mm]	23
	z [mm]	132
Euler angles	φ [as]	0.63
	θ [as]	0.006
	ψ [as]	0.63
GM_{E+M}	[km^3s^{-2}]	1.45×10^{-4}

Discussion of simulation results

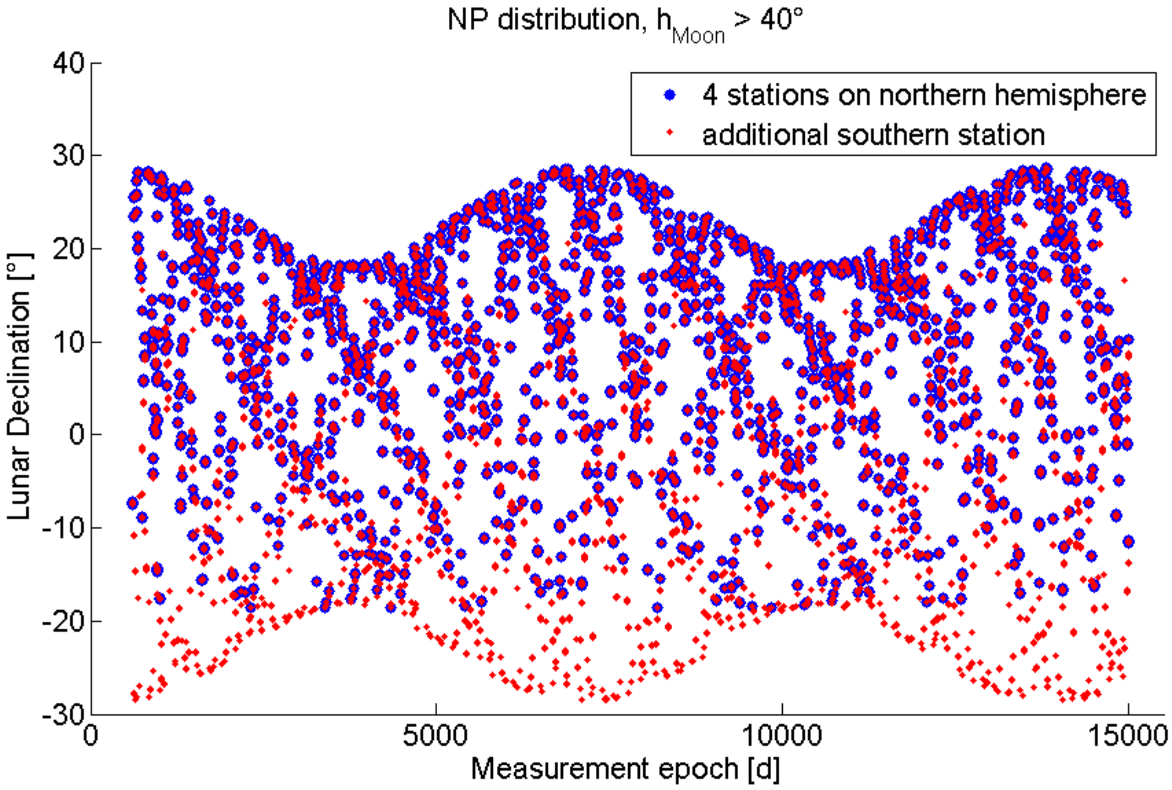
- Best results when observing as many reflectors as possible

- Higher accuracy (~ 10% - 15%) when adding a new site at „opposite“ northern hemisphere or somewhere at southern hemisphere

- Accuracy seems to be almost equal whether site in Japan or South Africa is added, but
  - simulated measurements do not account for atmospherically caused loss of accuracy
  - additional site at southern hemisphere has an advantage (see following slide)

NP distribution w.r.t. lunar declination

With condition lunar elevation > 40° the Moon is not observable from the northern stations at its southernmost declinations



With a site at southern hemisphere, the whole lunar orbit can be observed at high elevations > 50° i.e. less atmospheric effects

Results - relativity

Parameter	Results
Nordtvedt parameter η (violation of the strong equivalence principle)	$(3 \pm 3.6) \cdot 10^{-4}$
time variable gravitational constant $\dot{G}/G\,[yr^{-1}]$ $\ddot{G}/G\,[yr^{-2}]$ (→ unification of the fundamental interactions)	$(1 \pm 1.5) \cdot 10^{-13}$ $(4 \pm 5) \cdot 10^{-15}$
difference of geodetic precession $\Omega_{GP} - \Omega_{deSit}$ [″/cy] (1.92 ″/cy predicted by Einstein's theory of gravitation)	$(-3 \pm 5) \cdot 10^{-3}$
metric parameter γ - 1 (space curvature; $\gamma = 1$ in Einstein)	$(3 \pm 4) \cdot 10^{-3}$
metric parameter β - 1 (non-linearity; $\beta = 1$) or using $\eta = 4\beta - \gamma_{Cassini} - 3$ with $\gamma_{Cassini}-1$ ($\sim 10^{-5}$)	$(1.7 \pm 2) \cdot 10^{-3}$ $(0.8 \pm 1.0) \cdot 10^{-4}$

Results – relativity (2)

Parameter	Results
Yukawa coupling constant $\alpha_{\lambda=400\,000\ \mathrm{km}}$ (test of Newton's inverse square law for the Earth-Moon distance)	$(-0.6 \pm 1.8)\cdot 10^{-11}$
special relativity $\zeta_1 - \zeta_0 - 1$ (search for a preferred frame within special relativity)	$(-5 \pm 12) \cdot 10^{-5}$
influence of dark matter δ_{gc} [cm/s^2] (in the center of the galaxy; test of strong equivalence principle)	$(0 \pm 2) \cdot 10^{-14}$
preferred frame effects α_1 α_2 (coupled with velocity of the solar system)	$(3 \pm 3) \cdot 10^{-5}$ $(2 \pm 2) \cdot 10^{-5}$
preferred frame effect α_1 (coupled with dynamics within the solar system)	$(1.6 \pm 3) \cdot 10^{-3}$

Conclusions

- LLR is a unique tool for studying the Earth-Moon system and testing general relativity, e.g.

  Yukawa test $\alpha_{\lambda=400000\,\mathrm{km}} = (-0.6 \pm 1.8) \cdot 10^{-11}$

- Several parameters of the LLR model contribute to a multitude of geodetic applications (reference frames, long-term Earth rotation, lunar interior …)

- One further LLR site in Japan or South Africa would improve the results for many LLR parameters by 15% (or more)

- Good results are only possible because of fantastic long-term lunar tracking by observatories (> 43 years of data). Thanks!

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