

## **Proposed beam divergence estimation procedure for the ILRS**

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### **Abstract**

*Link budgets for many of the ILRS sites are estimated using divergence values that are derived from the site logs. Actual data for calculating the station divergence is often incomplete or very optimistic and based on diffraction theory from the full size of the primary mirror for monostatic systems or the full size of the Coude path and beam expander for bistatic systems.*

*A standard method to perform accurate divergence measurements is needed by the ILRS for several reasons, including GNSS retroreflector array requirements and performance prediction and reliable computation of the energy density delivered on target to deal with information requests for new missions. A new method to estimate the laser beam divergence of ILRS stations has been developed. The procedure requires scanning a single satellite and performing transmit power measurements or employing optical filters to limit the receiver efficiency. This method will be presented here with some preliminary results demonstrating its use.*

### **Introduction**

The flux delivered to a satellite retroreflector or detector array will vary as a function of the divergence of the transmitted laser beam from the SLR site. This makes it necessary to have a good estimate of the transmitter divergence. Since the real world will make the divergence measurements vary from day to day, it is necessary to try to characterize the range of divergence values that can be expected from a station transmitter. An average or mean value is necessary for general calculations of flux delivered to a satellite. The largest expected divergence is necessary for worst case link budget calculations. The tightest expected divergence is necessary for Mission Working Group assessments when data on maximum flux levels is requested. The goal of the work presented here is the development of a reasonably simple but accurate method for characterizing a station's transmitter divergence. This method is based on ideas similar to those underlying a procedure shown at a previous ILRS meeting (Burris, reference 1), presenting a number of practical and theoretical advantages.

The goal of the method is to estimate the beam divergence of a satellite laser ranging station from an expression derived from the standard link budget equation. Two quantities, obtained from ranging to a single satellite, are required to complete the calculation:

- i) Angular distance from the position at which the target is centered to the point at which no laser returns are detected
- ii) Laser power at which no laser returns are detected with the satellite in the center position.

The expression employed to estimate the beam divergence is developed below. It should be noted that the use of neutral density filters to attenuate the received energy is an alternative to measuring the power level of the laser emitter. Empirical data demonstrating both options are shown in this paper.

### **General method**

From an operational point of view, the procedure consists of a number of simple steps. A scan is performed in pointing angle on a single satellite, starting from the position at which the target is

centered. The scan is done in AZ and EL up to the points where the number of detected photoelectrons drops to approximately zero ( $N_{pe} \sim 0$ ). The scan endpoints should be noted as well as the center point, which corresponds to a pointing error of 0 degrees.

The beam is centered again on the satellite and the full transmit power is recorded. The transmit power is then reduced until  $N_{pe} \sim 0$  with 0 pointing error (satellite centered), and the power level recorded. Alternatively, neutral density filters can be employed to reduce the signal until it is no longer detected. In this case, the transmittance of the filters is the value to be recorded and used in the calculation.

The link budget equations for  $N_{pe}$  at the scan end (half angle of full scan) and at the reduced transmit power (or filter transmittance) at which the pointing error is zero are both equal to an  $N_{pe} \sim 0$ . By equating these expressions, one of which has pointing error of zero, an expression for the beam divergence can be obtained. This expression is a function of the measured maximum transmit power, the measured transmit power for  $N_{pe} \sim 0$ , and the measured endpoint of the scan for  $N_{pe} \sim 0$ . It is also a function of  $R_1$  (the range at which the scan is done) and of  $R_2$  (the range at which the transmit power is reduced to obtain  $N_{pe} \sim 0$ ). Since the satellite cross section is a function of range,  $\sigma(R_1)$  and  $\sigma(R_2)$  are also in the expression for beam divergence. If the scan and power reduction measurements are done quickly enough so that  $R_1 \approx R_2$ , then the expression for beam divergence is a function of the measured power levels (or filters transmittance) and the scan half angle only. These relationships allow the derivation of an analytical expression to calculate the divergence from the satellite scan data.

### Derivation of divergence estimation expression

#### $N_{pe}$ calculation

The expression for the expected number of photoelectrons in an SLR return is (Degnan, Reference 2):

$$N_{pe} = \eta_e * (E_r * \lambda / hc) * \eta_t * G_t * \sigma * (1/4\pi R^2)^2 * A_r * \eta_r * T_a^2 * T_c^2.$$

The terms in the  $N_{pe}$  expression are defined below with a descriptor of constant or variable which describes how the particular term is treated in this derivation. For some of the terms, the descriptor of constant is only approximately true and requires that the taking of data be accomplished reasonably quickly.

- $\eta_e$  = detector quantum efficiency      Constant
- $E_r$  = laser pulse energy      Variable
- $\eta_t$  = transmit optics efficiency      Constant
- $G_t$  = transmitter gain      Variable
- $\sigma$  = satellite optical cross section      Variable
- $R$  = slant range to target      Variable
- $A_r$  = effective receive aperture area      Constant
- $\eta_r$  = receive optics efficiency      Constant
- $T_a$  = one-way atmospheric trans.      Constant
- $T_c$  = one-way cirrus cloud trans.      Constant

#### Transmitter gain

The expression for transmitter gain (Degnan, Reference 2) is:

$$G_t = (8/\theta_t^2) * \exp[-2(\theta/\theta_t)^2]$$

where the half angle terms are defined as

$\theta_t$  = far field divergence half angle between beam center and  $1/e^2$  intensity point

$\theta$  = beam pointing error; or in this case, the half angle of the scan, so that  $\theta$  is now  $\theta_s$ , scan angle.

The expression for transmitter gain during the scan is then:

$$G_{t\text{-scan}} = (8/\theta_t^2) * \exp[-2(\theta_s/\theta_t)^2].$$

For the case where the beam is centered in the scan of the satellite, when transmit power is reduced to the point where  $N_{pe} \sim 0$ , the pointing error or scan angle is 0 resulting in the simplified expression for  $G_{t0}$ :

$$G_{t-0} = (8/\theta_t^2)$$

### Single satellite scan and assumptions

Assume that scan measurements are taken on a satellite quickly enough that all factors in the expression for  $N_{pe}$  are approximately constant with the exception of  $\sigma$ ,  $R$ , and  $G_t$ , which changes due to pointing error (i.e. scan angle) change.  $E_r$  is constant during the scan and is held at  $E_{max}$ , the typical maximum transmit energy used at the site for ranging. Let the range at which the scan was done be  $R_1$  and the cross section at this range  $\sigma_1$ . The expression for  $N_{pe}$  given above can be simplified by grouping all the constants together into one term,  $K$ . Then the expression for  $N_{pe1}$  becomes:

$$N_{pe1} = K * E_{max} * (\sigma_1/R_1^4) * G_{t\text{-scan}},$$

where  $K$  is approximately constant and

$$G_{t\text{-scan}} = (8/\theta_t^2) * \exp[-2(\theta_s/\theta_t)^2]$$

### Transmit Power Reduction and assumptions

Assume that after the satellite scan in AZ and EL is performed, the beam is centered on the satellite such that the pointing angle (or scan angle) is 0. Then transmit power is reduced in a controlled manner that does not change the beam divergence, until the  $N_{pe}$  is  $\sim 0$ .  $E_r$  is reduced to  $E_{min}$ , measured and recorded, and the pointing error is held constant at 0. Let the range at which this procedure was done be  $R_2$  and the cross section at this range  $\sigma_2$ . The expression for

$N_{pe}$  then becomes the expression for  $N_{pe2}$ :

$$N_{pe2} = K * E_{min} * (\sigma_2/R_2^4) * G_{t-0},$$

where  $K$  is approximately constant as discussed above and the transmitter gain is

$$G_{t-0} = (8/\theta_t^2),$$

since the scan angle is held at 0.

### Equate expressions where $N_{pe} \sim 0$

Since  $N_{pe1}$ , the number of photoelectrons at the end point of the scan, and  $N_{pe2}$ , the number of photoelectrons when transmit power is reduced to  $E_{min}$ , are both equal to 0, we can set the expressions to be equal such that:

$$K * E_{max} * (\sigma_1/R_1^4) * (8/\theta_t^2) * \exp[-2(\theta_s/\theta_t)^2] = K * E_{min} * (\sigma_2/R_2^4) * (8/\theta_t^2).$$

By cancelling like terms and rearranging, we can solve for the square of the beam divergence:

$$\theta_t^2 = -2\theta_s^2 / \ln[(E_{min}/E_{max}) * (\sigma_2/R_2^4) * (R_1^4/\sigma_1)].$$

### Beam divergence expression, transmit power reduction

The general expression for the half-angle of the  $1/e^2$  beam divergence is:

$$\theta_t = [-2\theta_s^2 / \ln F(E, R, \sigma)]^{1/2},$$

where  $F(E, R, \sigma) = [(E_{min}/E_{max}) * (\sigma_2/R_2^4) * (R_1^4/\sigma_1)]$ .

Since  $E_{min}$  and  $E_{max}$  are the minimum and maximum pulse energies, their ratio is equal to the ratio of the measured minimum and maximum laser transmit powers. Ideally,  $R_1$  and  $R_2$  are approximately equal, which implies that the cross sections  $\sigma_1$  and  $\sigma_2$  are also equal. In these circumstances, the expression for  $F$  will reduce to the ratio of the measured transmit powers. Otherwise,  $R_1$  and  $R_2$  could

be recorded and the cross sections calculated from knowledge of the range and the LRA construction. However, this is best avoided for there is much uncertainty in the theoretical estimations of LRA cross sections, and the computation is far from trivial. Thus, the resultant simplified expression for the laser beam divergence is:

$$\theta_t = [-2\theta_s^2/\ln(P_{\min}/P_{\max})]^{1/2} .$$

Beam divergence expression, neutral density filter insertion

As mentioned above, the step involving reduction of the transmit power until  $N_{pe} \sim 0$  can be substituted by inserting calibrated neutral density filters in front of the detector until the  $N_{pe}$  drops to zero. Using neutral density filters at the receiver input results in an equivalent final half-angle beam divergence expression:

$$\theta_t = [-2\theta_s^2/\ln(T_{nd})]^{1/2} .$$

In this expression,  $T_{nd}$  is the fractional transmission of the neutral density filter. All other assumptions are the same as above.

Beam divergence expression for difficult to measure beam divergence values

Very large beam divergence values, where the end points of the scan can be difficult to judge, may not be easily measured with this procedure as described so far. Once the scan procedure has been performed several times to characterize the divergence at a first setting  $\theta_{t,1}$ , the beam divergence can be changed and the transmit power (or ND value) adjusted to again obtain  $N_{pe} \sim 0$  while the beam is still centered on the satellite. This allows determination of  $\theta_{t,2}$  from  $\theta_{t,1}$  by the alternative equation shown below in which  $P_{\min,1}$  and  $P_{\min,2}$  are the respective transmit powers at divergence settings 1 and 2:

$$\theta_{t,2} = [\theta_{t,1}^2(P_{\min,2}/P_{\min,1})]^{1/2} .$$

In the case of using neutral density filters, the transmit power levels are substituted with the transmittances of the filters employed in each case. Again, this expression involves the assumption that the ranges and satellite cross-sections are approximately equal so that they cancel out in the derivation.

Some results from early scan data

NERC Herstmonceux Data

Data was taken on October 6 and 7 at the NERC Space Geodesy Facility at Herstmonceux (UK) on several different satellites. This data was taken using two different laser systems (2 kHz and 12 Hz systems) and with several different divergence or beam expander settings (note that the beam expander settings are not directly comparable between the two laser systems). The raw scan data is shown below in Table 1. The scan results in Table 1 are in arcseconds, and NERC uses the method of attenuating the receiver detector input with ND filters instead of transmit power reduction.

Table 1 – Satellite scans and ND measurements taken at SGF so far.

Date	Satellite	Elevation	Laser	Beam	Offsets		ND
					az.	el.	
06 Oct	Lageos-2	40°	12 Hz	b30	17"	16"	6
06 Oct	Beacon-C	35°	2 kHz	b37	38"	38"	9
06 Oct	Ajisai	33°	2 kHz	b37	36"	39"	10
07 Oct	Lageos-1	56°	2 kHz	b12	27"	18"	11
07 Oct	Lares	35°	2 kHz	b25	38"	34"	9
07 Oct	Glonass 101	41°	12 Hz	b25	18"	11"	7
07 Oct	Glonass 118	72°	12 Hz	b25	23"	19"	11
07 Oct	Glonass 118	72°	12 Hz	b35	19"	19"	9

12 Hz Laser	12 Hz Laser	2 kHz Laser	2 kHz Laser
Beam expander	$\theta_t$	Beam expander	$\theta_t$
b25	8.5"	b12	10"
b25	9.3"	b25	18"
b30	10.7"	b37	18"
b35	9.5"	b37	19"

Table 2: Beam divergence estimates for the NERC data shown in Table 1.

The divergence estimates reported in Table 2 for the NERC laser systems are half-angle values and are in units of arcseconds. The values are the average of the azimuth and elevation scans performed for Table 1.

### Stafford Data

Data was taken on LAGEOS1 from the Stafford site in October, 2013 on two different passes to test the divergence estimation procedure. The raw data is shown in Table 3 below. The data in Table 3 was taken with the NRL 10 Hz laser system and the transmit power reduction method was used. The raw data was used to estimate the divergence of the laser, assuming that the range and satellite cross section was approximately constant as discussed above. The divergence estimates for azimuth and elevation scans as well as the average and standard deviation of all estimates are shown in Table 4 below. The divergence values shown in Table 4 are the full angle divergence from  $1/e^2$  to  $1/e^2$  point and are in units of microradians.

Date	Satellite	Step size	AZ steps	EL steps	Elevation	Power ratio
10/21	Lageos1	5 $\mu$ rad	16	14	55°	0.088
10/21	Lageos1	5 $\mu$ rad	18	16	70°	0.026
10/21	Lageos1	5 $\mu$ rad	18	18	17°	0.286
10/21	Lageos1	5 $\mu$ rad	17	17	29°	0.140
10/21	Lageos1	5 $\mu$ rad	17	14	32°	0.187
10/21	Lageos1	5 $\mu$ rad	16	18	31°	0.118

Table 3: Early data taken on LAGEOS1 at Stafford on 21 October.

$\odot$ , AZ scan	$\odot$ , EL scan
72.59	63.52
66.64	59.24
113.8	113.8
85.74	85.74
92.80	76.42
77.41	87.08

Mean	StDev
82.89 $\mu$ rad	17.63 $\mu$ rad

Table 4: Divergence estimates obtained from the LAGEOS1 scan data of 21 October.

## Possible error sources and sensitivity to measurement errors

In general, since the measurements involve propagation of the laser through the atmosphere, the divergence estimates are always going to be stochastic in nature. Therefore a single measurement will be only one sample of the variable. A much better approach will be to take as many measurements as time permits and use the mean and variance (or standard deviation) for link budget calculations.

There are many factors which will affect the measurements and cause possible errors: atmospheric conditions (humidity, pressure, temperature gradients, clouds and fog), laser temperature gradients, system operators (this is a subjective measurement that needs to have time minimized) and the assumptions made in the derivation of the divergence expression to name a few. Also satellite elevation should be considered since at lower elevations, the transmit and return beam pass through a much greater length of dense atmosphere which can increase the effective divergence of the beam. All of these conditions should be considered when trying to characterize the transmit divergence of the SLR laser system.

### Sensitivity to measurement errors

A sensitivity analysis was done with the Stafford data (Lageos1 measurement number four at 20 degrees elevation) to determine how sensitive the divergence estimate is to errors in the measurement data. As is fairly obvious from the form of the divergence expression, the estimate is most sensitive to scan errors. The sensitivity plots are shown in Figure 1 below. Note that an error in the measured power ratio of 25% results in an error in the estimated divergence of only about 7%, while a scan error of 25% results in 25% divergence estimate error.

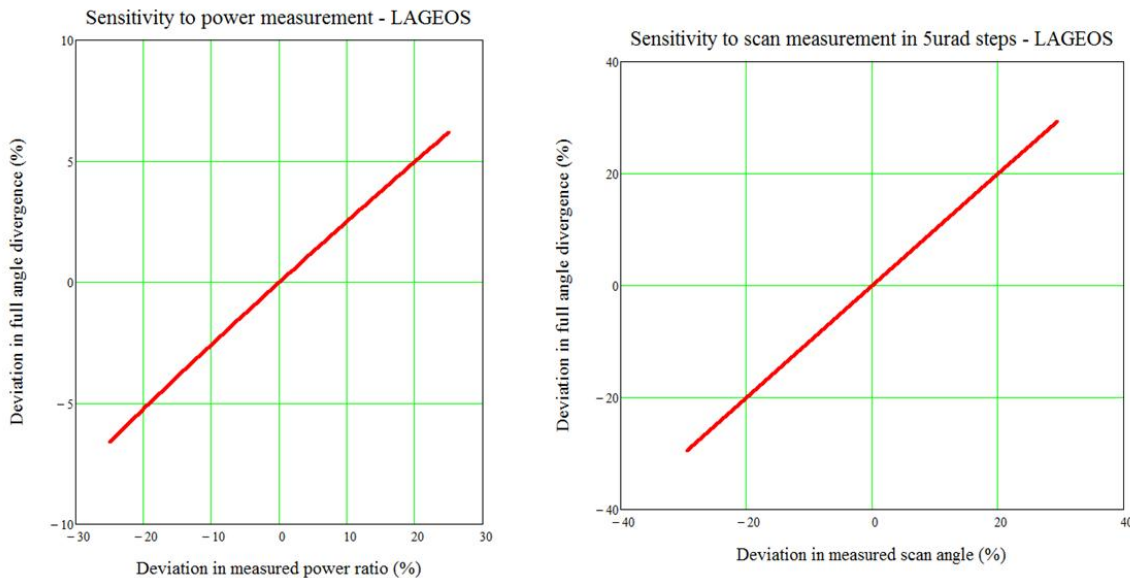


Figure 1: Sensitivity of Stafford divergence estimates to errors in scan measurements and power ratio measurements.

A sensitivity analysis of the dependence of the divergence estimate on the pulse energy ratio as well as the dependence on scanning and centering errors are shown in Figure 2 below. The top two subplots show that the beam divergence angle changes very rapidly at the extreme ends of the energy ratio (or filter transmission ratio). Therefore, divergence estimates performed with data where very high or low

ratios of these quantities have been used will be subjected to a greater potential error. The impact of errors in the determination of the center position is moderate, only of concern for situations where the scan angle is small (tight beams or low return energy from the satellite). Interestingly, deviations in the determination of the center position will always lead to overestimations of the beam divergence. An alternative view of the impact of scan errors on the divergence estimate is shown in the last plot of figure 2, where the final error in divergence is plotted for scan angle deviations of up to plus/minus 4 arcseconds. Clearly, accuracy in the estimation of the satellite scan end point is the most critical parameter of this method. As with the case of misjudgment in the center position, the severity of the errors increases where small scan angles are concerned.

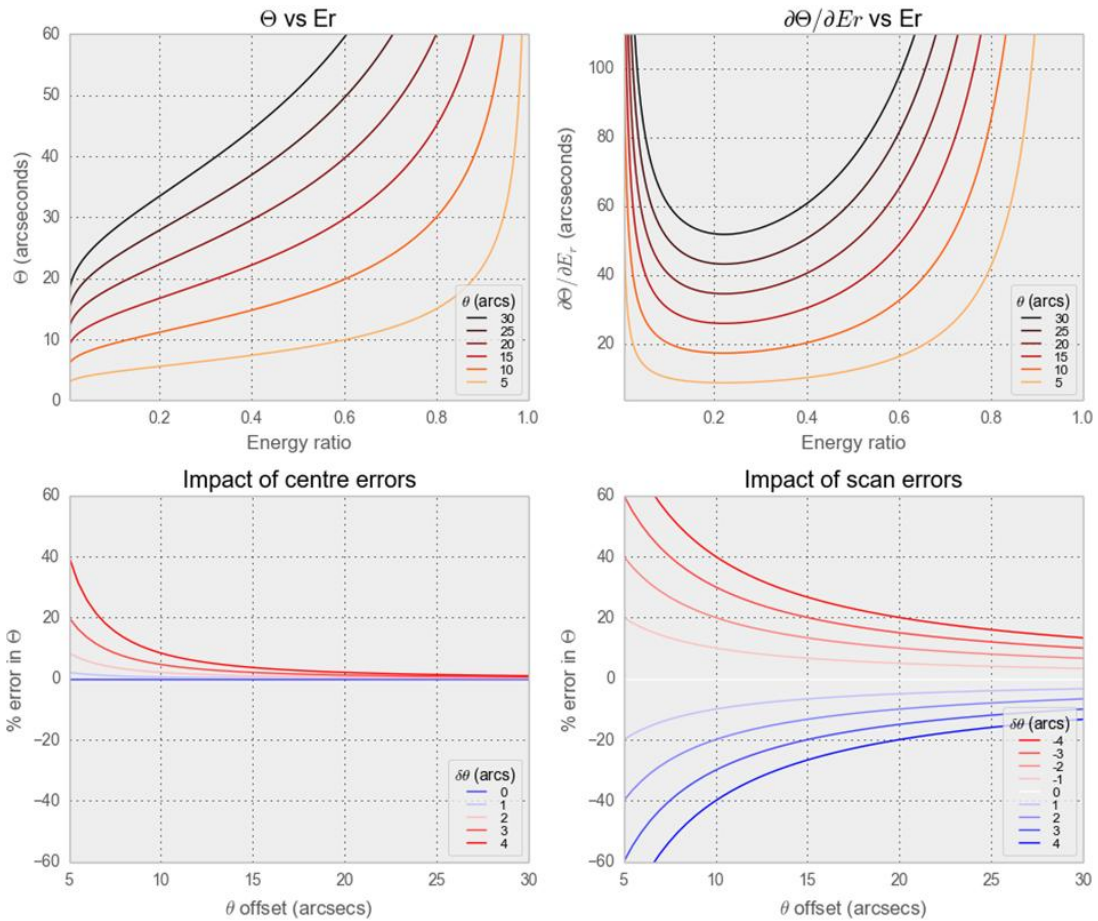


Figure 2: Dependence of divergence estimate on the energy ratio and the impact of scan errors.

**Conclusions and future efforts**

A method has been developed which allows the estimation of a station’s beam divergence with a scan of a single satellite and a power ratio measurement. Initial measurements have been taken at the U.S. Naval Research Laboratory’s Stafford SLR site and at NERC Hertmonceux’s SLR site. The results from the initial measurements seem promising and appear to offer a reasonably simple method to measure SLR station divergence.

Future efforts:

- 1) Work through the Networks and Engineering Working Group to get more stations involved in taking data sets to test the effectiveness of the method.

- 2) Take larger data sets at each station with identical settings and similar atmospheric conditions to look at average divergence estimation and statistics.

## **References**

1. Burris, Ray; Davis, Mark; Thomas, Linda; Huber, Dave; (2012) ; “*Divergence Estimation Procedure and Calculation*”; Proceedings of the International Technical Laser Workshop 2012 ITLW-12, Frascati ( Rome) Italy; November 5 – 9, 2012.
2. Degnan, John J., “*Millimeter Accuracy Satellite Laser Ranging: A Review*”, Contributions of Space Geodesy to Geodynamics: Technology Geodynamics 25, American Geophysical Union, 1993.