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Ring retroreflector array, composed by “double-spot” CCRs

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Abstract. *The ring retroreflector array (RRA) consisting of “double-spot” corner-cube reflectors (CCRs) with a special coating of reflecting surfaces is considered. The error of distance measurement caused by both the laser pulse delay in the CCR and its spatial position (CCRs configuration) is studied. It is shown that the ring RRA, formed by the CCR with a double-spot radiation pattern, allows the distance measurement error to be essentially reduced.*

1. Introduction

At present the international network of laser stations makes use mainly of the laser range finders operating in the so-called single-electron mode, the pulse repetition rate being able to achieve 10 kHz [1 – 5]. This fact means that the system is able to receive not greater than one photon of reflected radiation from each pulse of the laser. Provided that the small interference and quantum effects are neglected, the given photon comes from a certain CCR of the array. Since in the general case the CCRs in the RRA are located at different distances from the receiver, the distances being dependent on the light incidence angle, the times of arrival of different signal photons do not coincide.

In the reception path of the laser range finder the signal photoelectrons from the collection of received pulses are ‘stored’ in the similar temporal cells with the time shift of the pulses taken into account. A collection of photoelectrons stored during the measurement time that includes more than a thousand of pulses, forms the response continuous signal with the measured parameters, namely, the position of the so-called centre of gravity (mathematical expectation) on the time axis, duration [root-mean-square (RMS) deviation], etc.

The main goal of ranging is to determine the distance to the space craft (SC) centre of gravity by analyzing and processing the obtained signal. For this purpose it is necessary to take into account the delay of the laser pulse in the CCR, the shape distortion and centre of gravity displacement of the response pulse, as well as the refraction correction. The goal of the present work is to analyze the precision characteristics of the ring retroreflector array (RRA). The specific feature of these systems is that the CCRs are placed within a ring, the central part of the RRA being free of reflectors. Provided that the specially designed reflectors are used, the RRA allows reduction of the ranging error.

2. Correction of systematic and random errors of the pulse propagation time measurements, caused by the size and configuration of the RS

Let us consider the errors caused by the joint operation of CCRs in the RA. When a symmetric probe pulse is reflected from the RA in the general case the pulse is elongated, together with the distortion of its symmetry and the shift of its centre of gravity [4, 5]. We restrict our studies to three types of plane symmetric RAs, namely, the rectangular array, the ring RA with uncoated CCRs [6], and the RRA with the so-called double-spot CCR of increased size (Fig. 1).

Let us compare the distortions of the signal by these RAs in application to the satellite-based laser rangers, in which the single-electron mode of response pulse reception is used. Let us carry out approximate analysis, assuming that a single CCR does not distort the laser pulse, but only delays it, and the pulses reflected from CCRs are incoherent, i.e., their intensities are additive without taking the phase relations into account. In this case it is necessary to account only for the individual displacements of the pulse along the temporal axis, introduced by each CCR. Assume that the zero individual displacement is attributed to the pulse reflected from the CCR located in the array centre of symmetry.

We define the coordinate system *xyz* with the origin at the symmetry centre of the RA, the axes *x* and *y* lying in the plane that passes through the vortexes S_i of all CCRs, and the axis *z* directed perpendicular to this plane (see Fig. 1). For the ring RA we will also use the polar coordinate system (r, ψ).

Each CCR in both cases is characterized by the vector ***r*** lying in the plane and directed from the origin to point S_i. The pulse delay time for the given CCR (with the sign both plus and minus) is determined by the projection of ***r*** onto the sight line, i.e., the vector ***R*** directed from the laser transducer to the centre of the RA symmetry. Let us define this vector in the system *xyz* by the angles θ_i (the angle of incidence) and φ (the azimuthal angle). Let us denote by α_i the angle between the vectors $\boldsymbol{r_i}$ (r_x, r_y, 0) and R(sinθ cosφ, sinθ sinφ, cosθ), then the projection of $\boldsymbol{r_i}$ onto ***R*** expressed as $|r_i|\cos\alpha_i$, $= r_x \sin\theta\ \cos\varphi + r_y \sin\theta\ \sin\varphi$, and the corresponding time delay is $\Delta\tau_i = |r_i|\cos\alpha_i / c$.

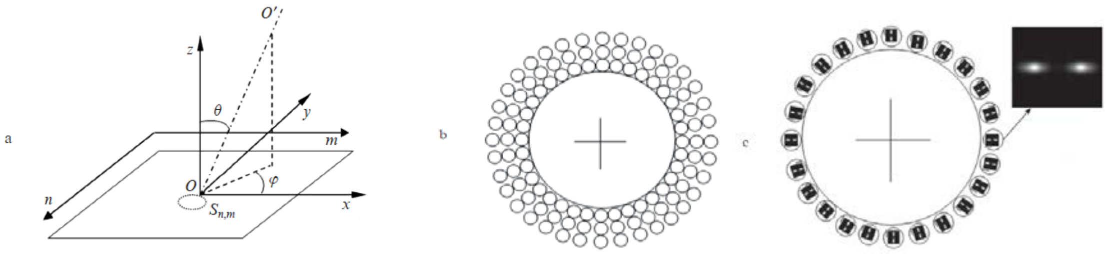

Figure 1. Types of reflector systems: (a) planar rectangular array, (b) ring retroreflector array formed by uncoated CCRs, and (c) the RA consisting of double-spot CCRs. It is shown how the radiation patterns for each CCR are oriented.

Consider first the plane array consisting of 117 CCRs, 13 rows parallel to the *x* axis and 9 rows parallel to the *y* axis. Each CCR is labeled by two subscripts, characterizing its location in the panel, *m* along the *x* axis and *n* along the *y* axis. The zero values of *x* and *y* correspond to the CCR located in the array centre. Let the distance *d* between points S_i of the CCR equal, e.g., 50 mm (similar along the axes *x* and *y*). Then for the time delay of the pulse, reflected from the CCR with the coordinates *m*, *n*, we get the following expression:

$$\Delta\tau_{mn} = \frac{d}{c}\sin\theta(m\cos\varphi + n\sin\varphi) + \Delta\tau_S\,,$$

$$\Delta\tau_S = c^{-1}h(-\cos\theta + \sqrt{n^2 - \sin^2\theta})\,. \tag{1}$$

Here h - the CCR's height; n - index of refraction; θ - incident angle.

Using Eqn (1), let us determine the intensity of a single reflected laser pulse in the form of the Gaussian function with the width 2τ at the level of 1/e of the maximal value

$$I_{mn}(t) = \frac{1}{\sqrt{\pi}\,\tau}\exp\left[\frac{-(t - \Delta\tau_{mn})^2}{\tau^2}\right]. \tag{2}$$

The total reflected pulse is expressed as the sum of all $I_{mn}(t)$:

$$I(t) = \sum_{m=-4}^{4}\sum_{n=-6}^{6} I_{mn}(t)\,. \tag{3}$$

The result of summation is shown in Fig. 2 for different values of the angles φ and θ and for different duration of the pulses 2τ. If the projection of the laser beam at the array is not parallel to the axis *x* or *y* ($\varphi \neq 0$, 90°), then for a short (e.g., 1 ps) incident pulse the reflected pulse has the shape of a trapezium. In the opposite (parallel) case the pulse is rectangular.

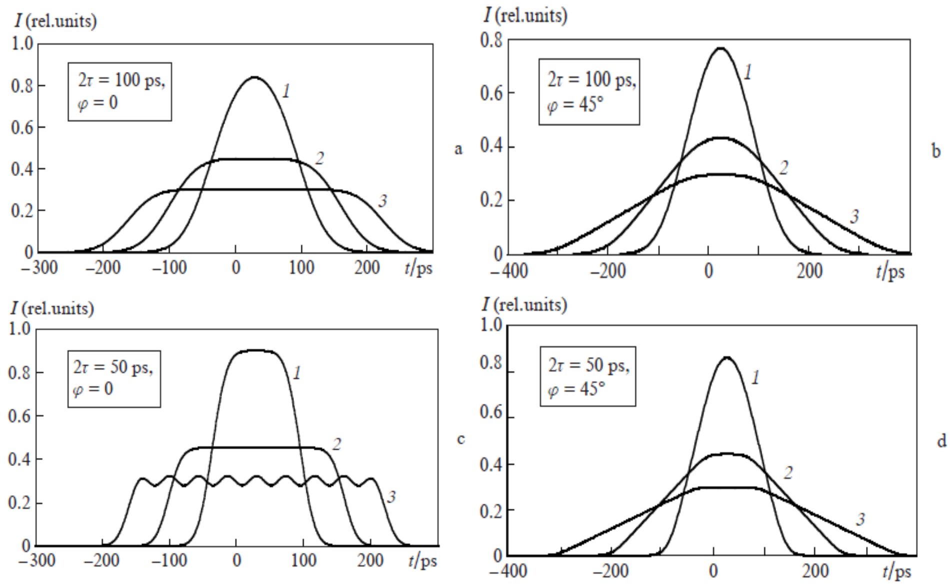

Figure 2. Broadening of Gaussian pulses at different values of 2τ and φ: θ = (*1*) 5°, (*2*) 10° and (*3*)15°.

The broadening of the incident pulse leads to smoothening of this shape. The shortening of the incident pulse duration leads to modulation of the reflected pulse, caused by the effect of individual CCRs of the array. The centers of gravity of all curves possess similar shift along the temporal axis due to the pulse delay in a single CCR. Therefore, ideally the array size and the incident ray azimuth do not give rise of ranging error. It is necessary to take only the systematic error correction into account.

Now consider the RRA consisting of three rows of CCRs with uncoated faces. As is known, in the case of total internal reflection of light from the CCR faces the diffraction pattern of the reflected radiation in the far zone consists of one central and six peripheral spots. By special turn of each CCR in the plane of the array one can obtain the intensity ring, formed by the lateral spots of each CCR. Since the diffraction pattern of the RRS is produced by all CCRs, the photons, reflected from all CCRs, arrive at the receiver. Let the RRS consist of three rows of CCRs, 36 pieces in each (see Fig. 2b), where n is the number of the CCR in the ring (n = 0, 1, . . . , 35); m is the number of the row (m = 0, 1, 2); and R is the radius of the RRS. Taking the ring symmetry of the RRS into account, one can put φ = 0 without any loss of generality.

The for the time delay of the pulse reflected from the CCR with the subscripts m and n we get the expression it is presented in Fig. 3 for different values of the incidence angle θ and durations of the initial pulses 50 and 100 ps at R = 230 mm and d = 40 mm.

$$\Delta\tau_{mn} = -\frac{R}{c}\sin\theta\cos\left(\frac{\pi}{18}n\right)(1+d\,m)+\Delta\tau_S\,. \qquad (4)$$

The total reflected pulse in this case is found from the expression

$$I(t)=\sum_{m=0}^{2}\sum_{n=0}^{35} I_{mn}(t)\,.$$

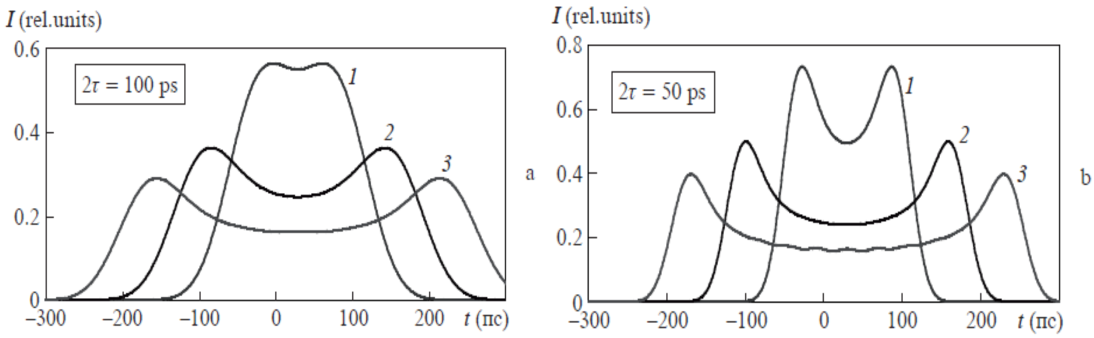

Figure 3. Broadening of Gaussian pulses at different values of 2τ; θ = (*1*) 5°, (*2*) 10° and (*3*) 15°.

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From Figs 2 and 3 it follows that the centre of gravity of the distribution of the response pulse arrival times is shifted only by the value equal to the delay of the pulse in a single CCR. If the number of responses is large enough, then the mean time of photoelectron appearance in the receiving tract of the laser ranger coincides with the centre of gravity of the response pulse. In other words, the systematic error of measurements (temporal shift) caused by the dimensions of the panel equals zero, and the random error is equal to the root mean square duration of the response pulse. Conservation of the measurement accuracy requires an appropriate increase in the repetition rate of probe pulses. The RMS obtained using the normal point appear to be $\sqrt{N}$ times smaller.

Thus, in the single-electron detection mode with the number of measurements N > 100 the systematic component of the range measurement error correction loses the dependence on the dimensions of the symmetric RA, but the required accuracy can be achieved only by averaging multiple measurements. Practically, due to the influence of external factors (e.g., weather conditions) the number of measurements may be only 10 – 20, and then the RMS of a single measurement becomes important.

3. Ring retroreflector array, consisting of double-spot CCRs of increased size

Consider the problem of reducing the single measurement random error upon an increase in the energy of the reflected signal in the direction of the receiver of the laser ranger. In order to increase the energy they make the CS larger in the appropriate direction. As is known, the phenomenon of velocity aberration [1] in the process of the space vehicle motion leads to the deviation of the reflected ray in the plane, formed by the SC velocity vector and the sight line. This means that the reflected radiation pattern, or the dependence of the CS on the angular coordinates, should be adapted to the SC with definite parameters and orbit height. The GLONASS satellites have only uniaxial orientation; therefore, the SC rotates in the plane, orthogonal to the direction towards the Earth. The angular velocity aberration for these satellites is nearly equal to ~5".

One of the promising methods for solving the problem of enlarging the CS is the use of a ring RA, consisting of double-spot CCRs with greater size. The double-spot diffraction pattern is formed due to the controlled variation in one of the dihedral angles (Fig. 4). The faces of such CCRs should be covered with a special dielectric coating, on the one hand in order to provide the appropriate radiation pattern, and on the other hand, to reduce its thermal distortions [7].

The optimal radiation pattern is provided by the choice of the CCR size (42 – 48 mm) and the deviation 2.2" – 2.5" of the angle between the reflecting CCR faces from 90°. If the line connecting the spots also lies in this plane, then for the angular distance between the spots equal to twice the angle velocity aberration one of the spots will hit the receiver of the reflected signal. This allows the energy loss reduction that arises if the diffraction pattern has the form of one or seven spots [6].

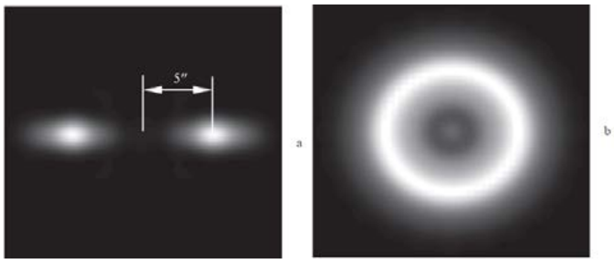

Figure 4. Radiation patterns of (a) the double-spot CCR and (b) of the entire RRA.

However, the reasonability of using the RRA is not reduced to the CS enlargement. Let us show that in this case it is possible to reduce the RMS of a single measurement. Let the orientation of the RRA be defined by the projection of the edge of the dihedral angle that differs from 90° onto the plane in which the bases of all CCRs lie. To provide reflection of light in the required direction by at least one CCR, one should rotate all CCRs in the array by a certain angle with respect to each other; e.g., for 18 CCRs this angle equals 10°. The angular size of each spot with respect to the centre of the diffraction pattern depends on the CCR size: the larger the CCR the smaller this size (for example, for the CCR aperture diameter of 48 mm this is about 30° at half-maximum intensity level). In this way the adjacent spots add to form the ring-shaped diffraction pattern.

The principle of operation for the RRA with turned double-spot CCRs is the following. Consider the hypothetic situation when the tangent component of the SC velocity is zero and the angular aberration is absent. In this case the centre of the ring diffraction pattern at the surface of the Earth coincides with the receiver, and there is no the signal. If the satellite moves with the velocity for which the angular aberration equals the angular size of the diffraction pattern, a certain part of the ring overlaps the receiver. This part is formed by one of the two spots from the CCR, definitely oriented in the RA. There can be several such CCRs.

Thus, the signal, reflected in the direction of the laser ranger located on the surface of the Earth is produced not by all CCRs, as, e.g., for the planar array of uncoated CCRs [6, 10], but by only a few of them, in the ideal case, by two identically oriented CCRs located at the opposite sides of the RRA (see Fig. 1). Using not one but two CCRs with the same orientation, separated by a certain distance, allows the solution of the important problem of reducing the RMS of the single distance measurements. If 36 CCRs are used in the RRA, each of them being rotated by 10° with respect to the next one from each side of the RRA, then the signals of three adjacent CCRs are added. Generally, the choice of the optimal CCR size and their total number allows obtaining the maximal possible value of the CS for such RRA independent of the azimuth and tilt angle of the SC observed.

Let us consider the single-measurement errors for two RRSs, consisting of uncoated CRs (Fig.3) and of double-spot CCRs (Fig. 5). In the first case the RMS σ of the single measurement is proportional to the array size. If the signal has the duration 50 ps (at the 1/e level of the maximal value) then depending on the incidence angle θ the following broadening in the temporal scale arises: for θ = 5°, 10° and 15° the RMS is σ = 54, 103, and 153 ps, respectively (see Fig. 5). With the fact that the delay of the pulse by 10 ps leads to the change in the distance by ~3 mm taken into account, for the RMS of the distance single measurement we obtain 16, 31 and 46 mm, respectively. At the signal duration 100 ps the broadening values are 62, 108 and 156 ps.

In the case of the RRA consisting of double-spot CCRs (Fig. 5), in contrast with the planar array and the RRA with uncoated CCRs, the reflected pulse does not broaden, but splits into two pulses that in the ideal case conserve the initial Gaussian shape. Every RMS in determining the centre of gravity of these two signals equals the square root of the sum of their RMS squares (provided that the signal photons from the CCRs from both sides of the RRA are present).

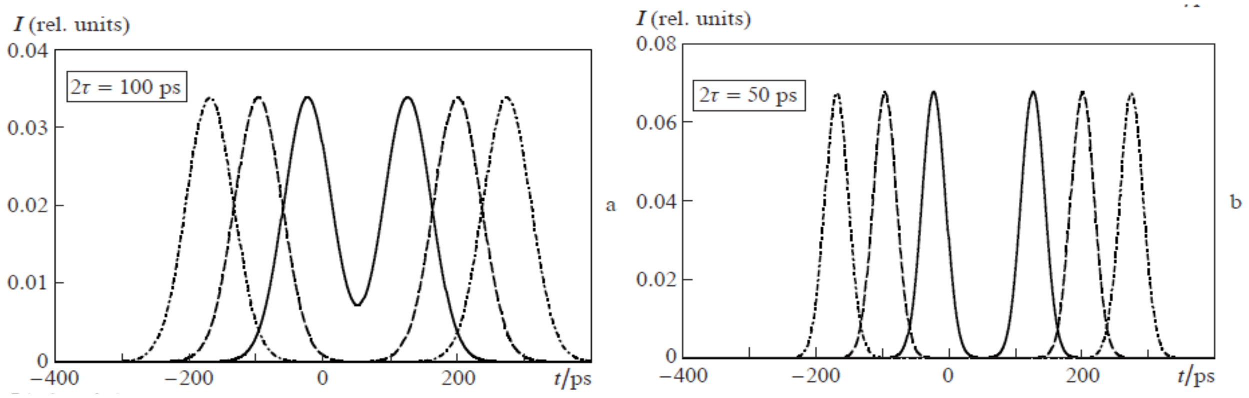

Figure 5. Broadening of the Gaussian pulse at different values of 2τ; θ = 5° (solid curve), 10° (dashed line) and 15° (dot-dashed line).

For the duration of the initial pulse 50 ps this is $18\cdot\sqrt{2}=25$ ps, and for the duration 100 ps this is $35\cdot\sqrt{2}=49$ ps, independent of the angle of incidence. For the angle of incidence θ = 15° and the pulse duration 50 ps the RMS for double-spot CCRs with the aperture 48 mm is six times smaller than the RMS for the uncoated CCRs with the aperture 28 mm.

6. Conclusions

The systematic error in measuring the distance to symmetric retroreflector systems by means of a laser ranger is determined by the delay of the laser pulse in a single cube-corner reflector. At the same time the RMS of single distance measurement is determined by the RA dimensions and CCR arrangement, as well as, generally, the CCR parameters, namely, the dimensions, the angles between the CCR faces, the type of the face coating. The single-measurement RMS is reduced if the ring retroreflector system is formed by double-spot enlarged CCRs. In this case the diffraction pattern is formed at the expense of the specified angle between the two reflecting faces and the special interference coating of the faces, providing a definite phase shift of the orthogonal components of the electric vector in the process of reflection. Theoretically the RMS can be reduced by several times using the CCRs with the aperture of 42 – 48 mm and the deviation of the dihedral angle by 2.4''. The advantage of such RRA is realized using short probe pulses (to 100 ps) and large angles of incidence onto the RRA (greater than 5°). Wherein CS can be increased by 1,5 times.

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