



# **Automated Transmitter Beam Size and Divergence Control in the SLR2000 System**

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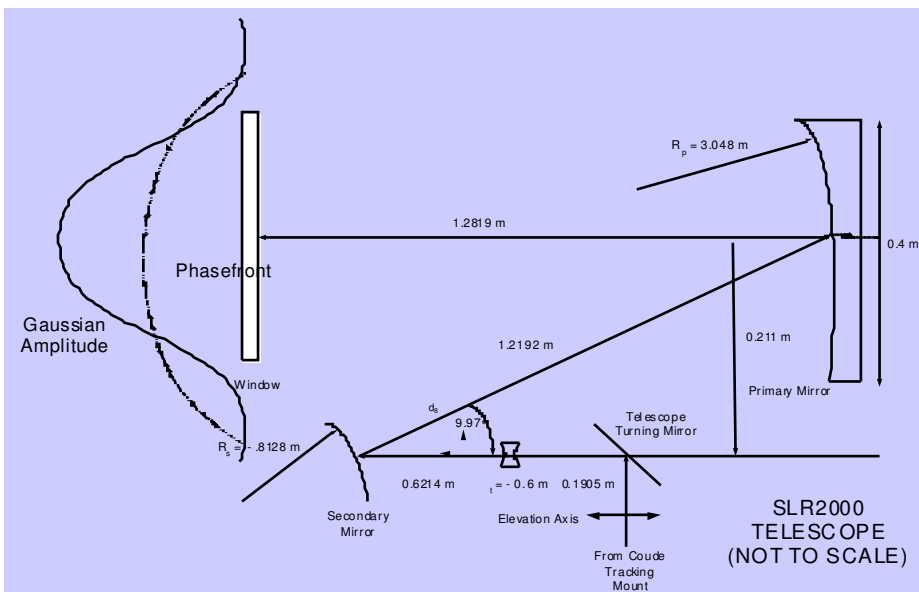
**October 16-20, 2006**

# Goals

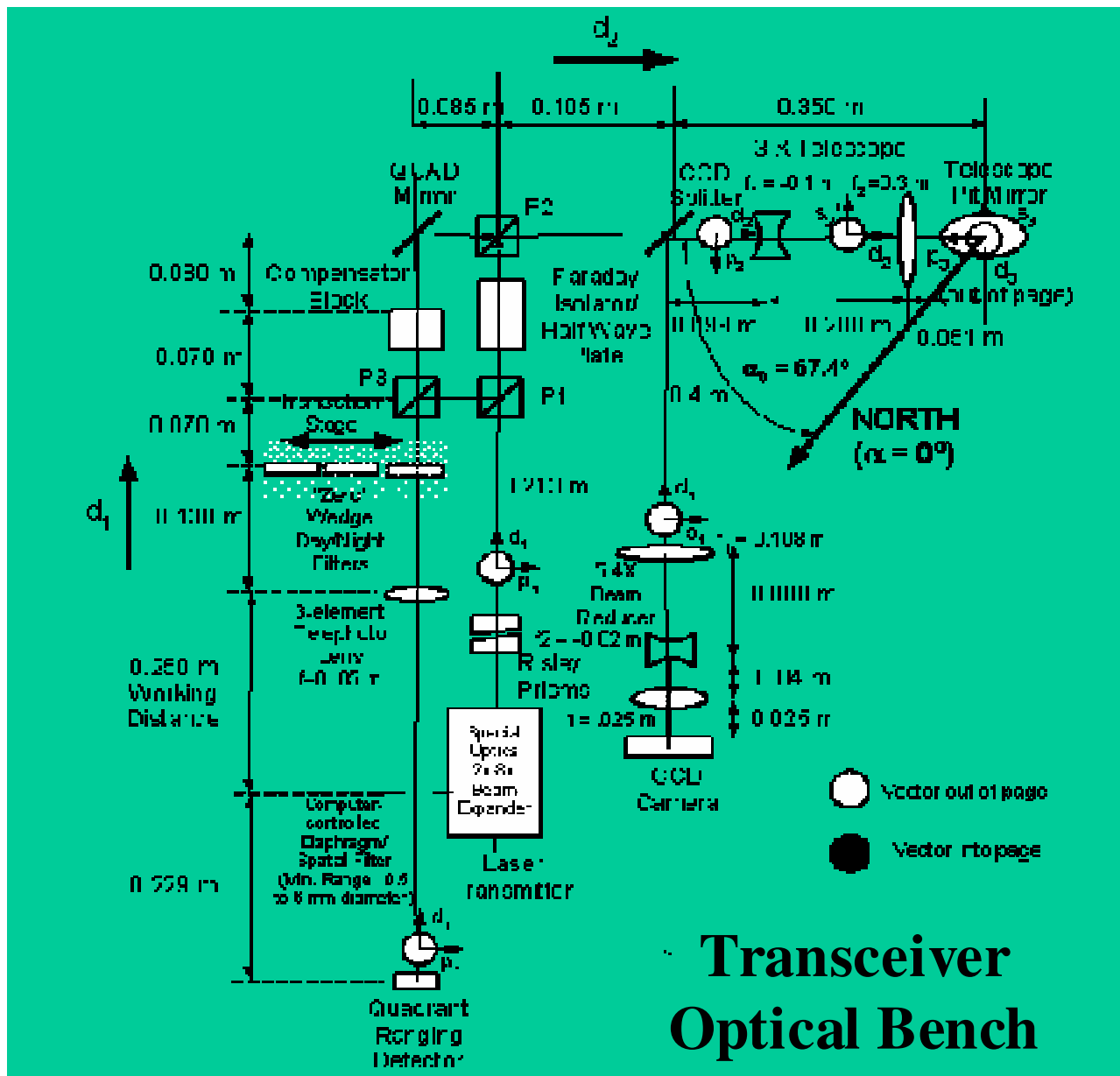
- SLR2000 adjusts transmitter beam divergence based on satellite altitude and orbital knowledge
  - Narrow for high satellites ( $\pm 4$  arcsec min)
  - Wider for low satellites ( $\pm 13$  arcsec max)
- Gaussian beam diameter must remain fixed at optimum value\* of 35.7 cm (40 cm/1.12) at the telescope exit aperture to achieve maximum far field gain at lowest divergence setting and to maintain eye safety for all other divergence settings. Final divergence is set by adjusting phasefront curvature at the telescope exit window.
- The beam expander is located in the transmitter path and the beam must then pass through two telescopes with a total magnification of 30.48 before exiting the system.

\*Klein, B. J. and J. J. Degnan, "Optical Antenna Gain: 1. Transmitting Antennas," *Applied Optics*, Vol. 13, pp. 2134 – 2141, 1974.

# SLR2000 Optical System



**Telescope**



**Transceiver Optical Bench**

# Ray Matrices and Gaussian Beams

Paraxial ray matrix theory can be applied to gaussian beam propagation if we define the following complex parameter:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi \omega^2(z)}$$

If propagation from a point  $z_0$  to  $z$  can be described by the ray matrix

$$M = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

then the gaussian beam properties at  $z$  are given by

$$\frac{1}{q(z)} = \frac{C + D \frac{1}{q(z_0)}}{A + B \frac{1}{q(z_0)}}$$

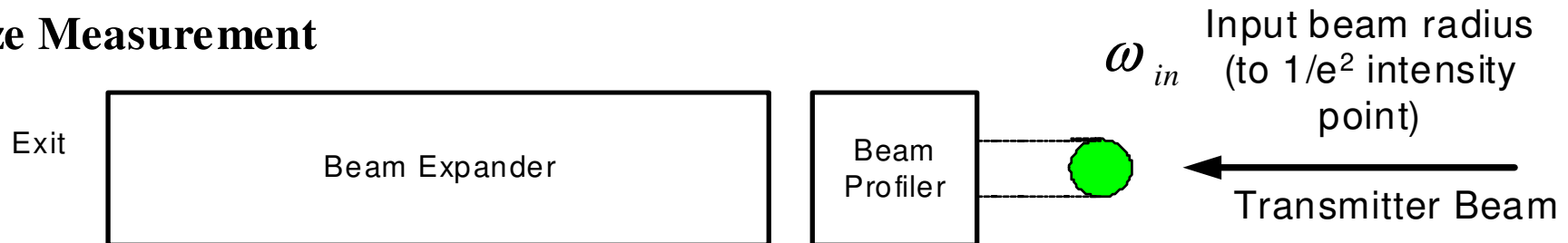


# Technical Approach

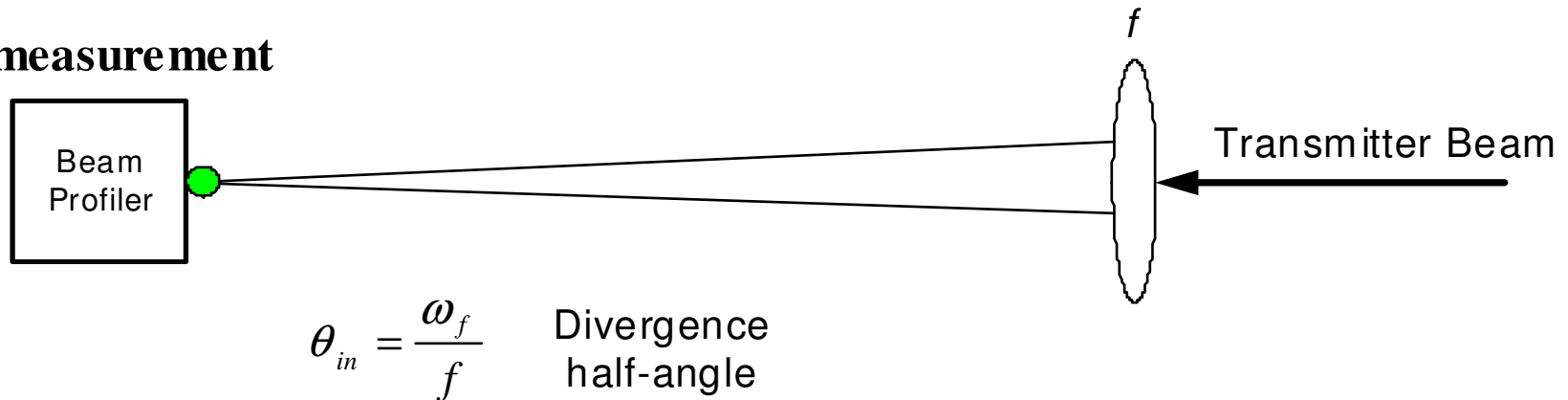
1. Measure transmitter gaussian beam radius (.969 mm) at entrance plane to beam expander, raw beam half divergence, and compute gaussian complex q-parameter for the input beam
2. Choose COTS beam expander with adequate
  - exit aperture ( $>40\text{cm}/30.48 = 13.1\text{ mm}$ )
  - magnification range ( $\sim 13.1\text{ mm}/2\text{mm} = 6.5$ ) and
  - at least two control elements for independently adjusting beam size and phasefront curvature at the outputand replace vendor-provided controller and software with more flexible control system from National Aperture.
3. Develop dynamic ray model for unit including variable lens spacings
4. Test dynamic ray model against sophisticated ray tracing program such as ZEMAX
5. Calibrate beam expander servo controllers at various magnifications
6. For each divergence value, use the gaussian beam propagation law to compute the complex q-parameter of the expander output beam and the lens spacings which produce that parameter.
7. Compute lookup table specific to laser transmitter

# 1. Characterize Transmitter Beam

## a. Spot Size Measurement



## b. Divergence measurement



Compute complex gaussian beam parameter at expander input

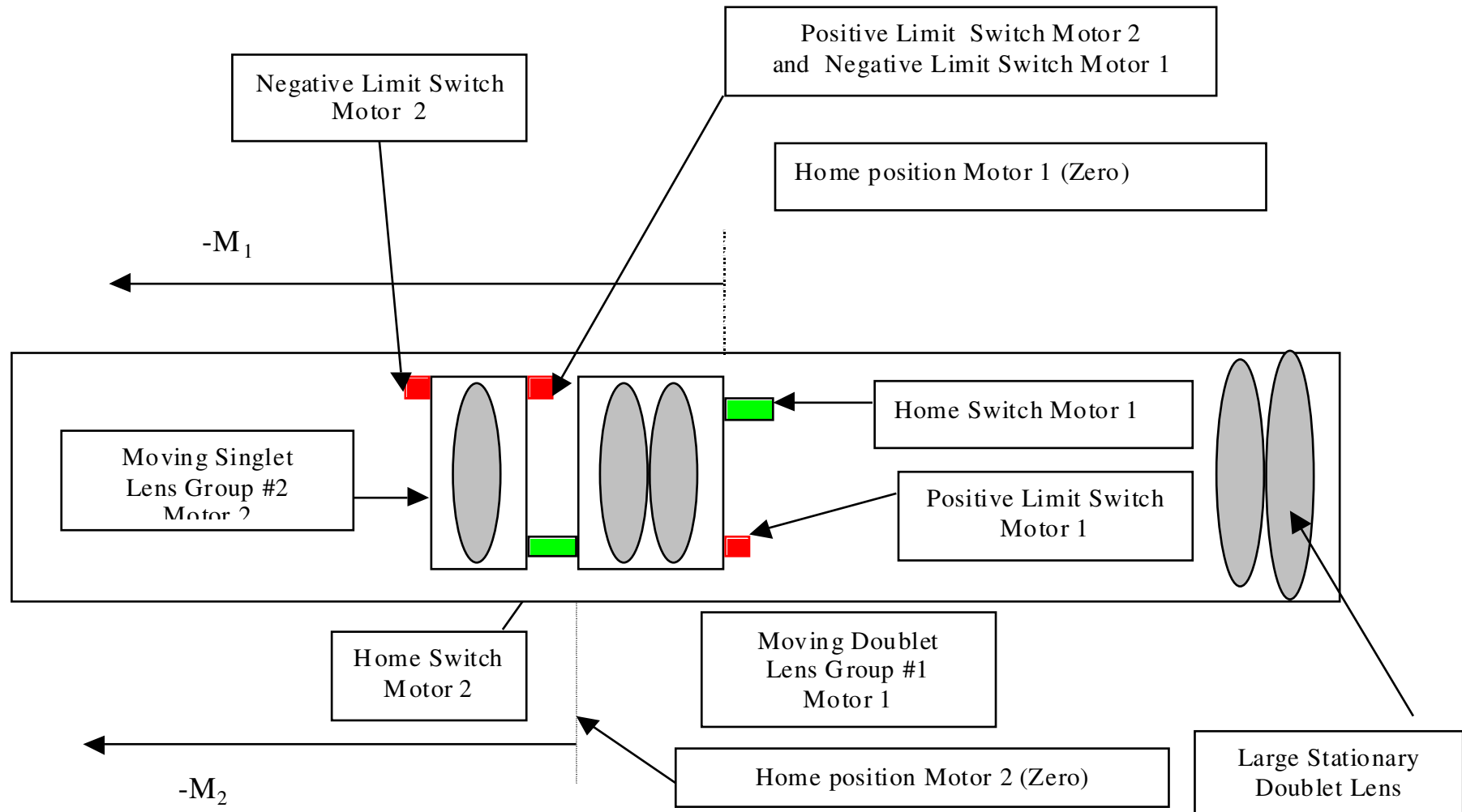
$$\frac{1}{q_{in}} \equiv \frac{1}{R_{in}} - j \frac{\lambda}{\pi \omega_{in}^2} = \frac{\lambda}{\pi \omega_{in}^2} \left( \sqrt{\left( \frac{\pi \omega_{in} \omega_f}{\lambda f} \right)^2} - 1 - j \right)$$

GSFC currently uses the long focal length lens and CCD camera to monitor the divergence of the beam expander output

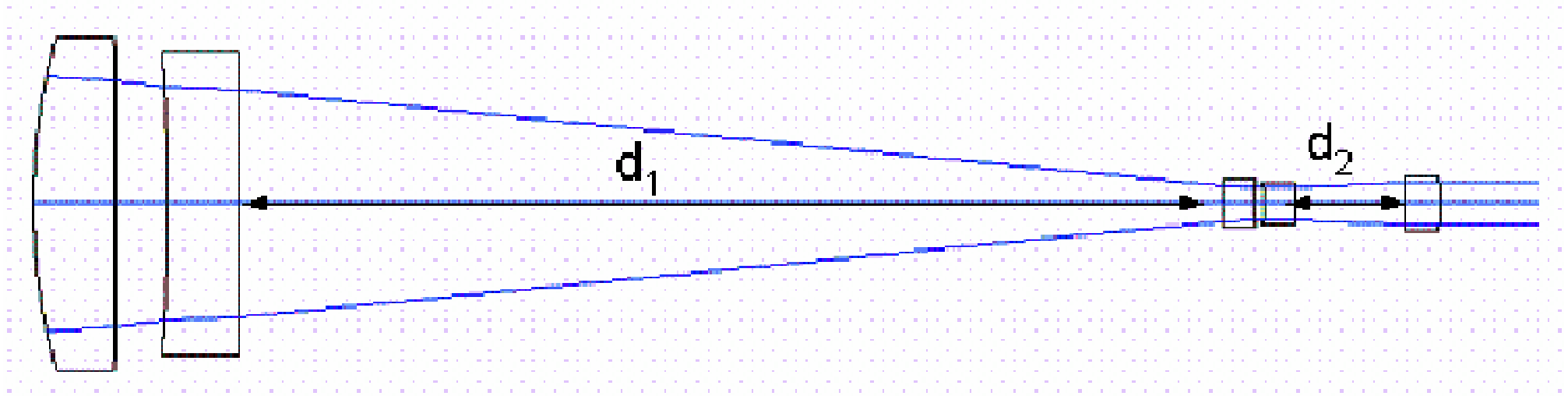
# 2. Choose Beam Expander

**Special Optics Beam Expander Model 56C-30-2-8X@450-700**

**Magnification: 2 to 8; exit aperture: 30 mm; input: 10 mm**



# 3a. Develop “Dynamic” Ray Model ( $d_1$ and $d_2$ are variable)



Stationary  
Doublet

Moving Doublet    Moving  
Singlet

Description	Lens #	$R_1$	$R_2$	$n$	$f$	Eff. Focal length, $f(m)$
Moving Singlet	1	0.02714	-35816	1.46070634	0.003591	0.005
Variable Spacing					$d_2$	
Moving Doublet	2	-0.03318	0.01577	1.46070634	0.002997	-0.011
	Spacer				0.001184	
	3	-0.01193	0.2879	1.46070634	0.003	
Variable Spacing					$d_1$	
Stationary Doublet	4	1.57946	0.23559	1.81565672	0.006995	0.135
	Spacer				0.00526	
	5	1.25341	-0.08621	1.81565672	0.008323	



# 3b. Ray Model Coefficients

$$M_{SO} = \begin{vmatrix} A_{SO} & B_{SO} \\ C_{SO} & D_{SO} \end{vmatrix}$$

$$A_{SO}(d_1, d_2) = A_0 + A_1 d_1 + A_2 d_2 + A_{12} d_1 d_2$$

$$B_{SO}(d_1, d_2) = B_0 + B_1 d_1 + B_2 d_2 + B_{12} d_1 d_2$$

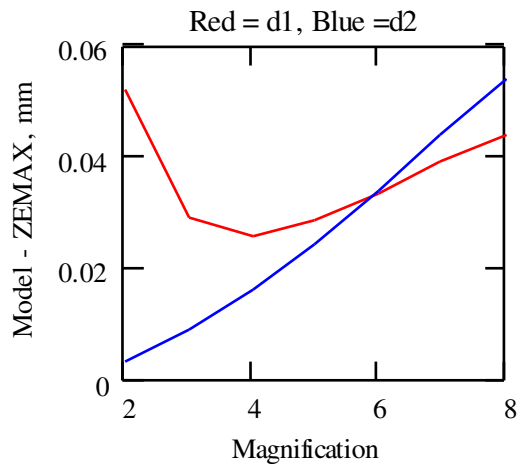
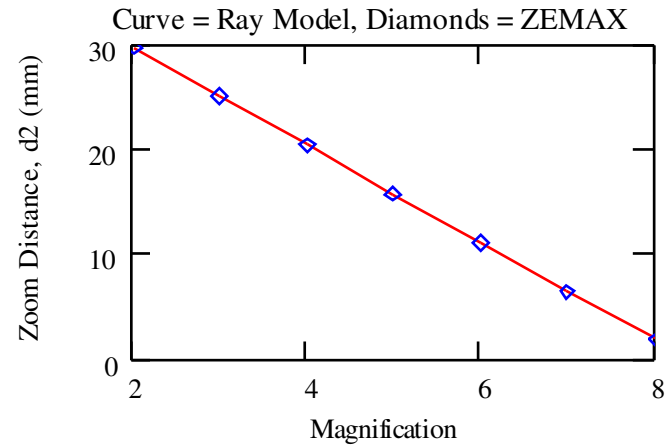
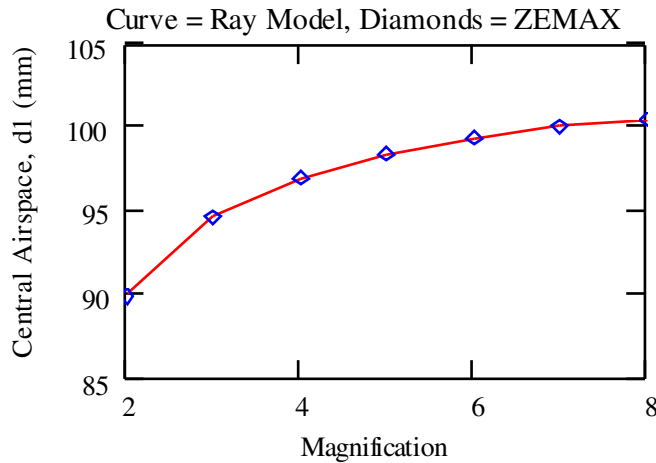
$$C_{SO}(d_1, d_2) = C_0 + C_1 d_1 + C_2 d_2 + C_{12} d_1 d_2$$

$$D_{SO}(d_1, d_2) = D_0 + D_1 d_1 + D_2 d_2 + D_{12} d_1 d_2$$

Suffix	A	B	C	D
<b>0</b>	1.995665379968	0.028752151456	46.402866180544	1.170293617808
<b>1</b>	63.683551232	1.442710470656	-460.710690944	-10.437108561152
<b>2</b>	-45.765735168	2.505955512	-1230.503704704	67.377646836
<b>12</b>	-1634.569158656	89.502715904	11825.086257152	-647.496210368



# 4. Test Ray Model vs ZEMAX



Perfectly focused magnifier implies:

$$A_{SO}(d_1, d_2) = m_{SO}$$

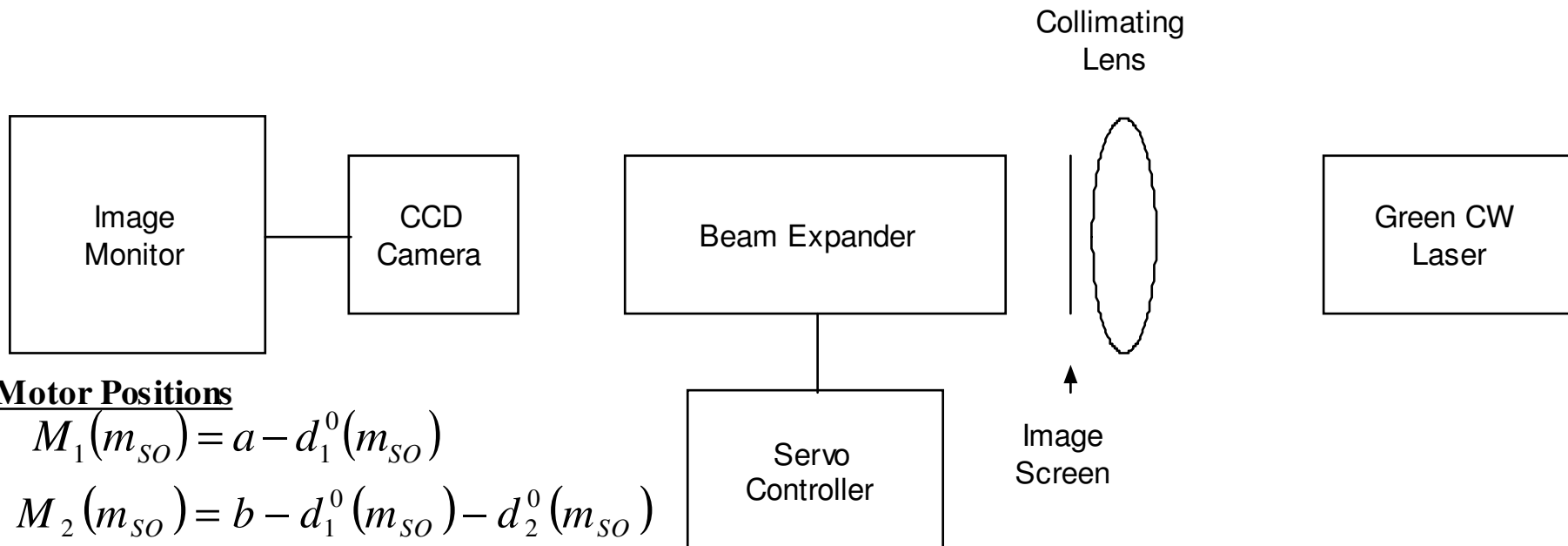
$$C_{SO}(d_1, d_2) = 0$$

or 
$$d_1^0(m_{SO}) = -\frac{0.02807695}{m_{SO}} + 0.104059$$

$$d_2^0(m_{SO}) = \frac{-637121.954 + 6325663.046 d_1^0(m_{SO})}{-16895097 + 162361136 d_1^0(m_{SO})}$$

which is compared to ZEMAX in above plots.

# 5. Calibrate Servo Controllers



Vendor values:  $a = 88$  mm,  $b = 90$  mm

## Calibration Procedure

1. Using vendor values for  $a$  and  $b$ , set lens positions for integer magnifications ( $m_{SO} = 2, 3, \dots, 7$ )
2. Using servo controller, adjust doublet and singlet lens positions for sharpest image and record motor encoder counts.
3. Apply least squares analysis to set of 4 equations on previous slide to obtain updated values for  $a$ ,  $b$ , and  $m_{SO}$
4. Repeat experiment and least squares analysis with updated constants until clear images are obtained at all magnification settings without significant correction.
5. Our final values:  $a = 88.7142$  mm,  $b = 92.9858$  mm

# 6. Use Gaussian Propagation Law



Spot size at primary: 
$$\omega(\omega_0, R_0) = m_t \omega_0 \sqrt{1 + 2 \frac{d_t}{m_t R_0} + \left( \frac{d_t \lambda}{\pi m_t \omega_0^2} \right)^2 \left[ 1 + \left( \frac{\pi \omega_0^2}{\lambda R_0} \right)^2 \right]} \cong 0.179 m$$

Far Field Half-Divergence: 
$$\theta_t(\omega_0, R_0) = \frac{\lambda}{\pi m_t \omega_0} \sqrt{1 + \left( \frac{\pi \omega_0^2}{\lambda R_0} \right)^2}$$

Solving for the beam waist radius and phasefront curvature out of the beam expander,  $\omega_0$  and  $R_0$ , we obtain for SLR2000:

$$\omega_0(\theta_t) \cong \frac{\omega}{m_t} - d_t \theta_t = 0.00585 - 30.84 \theta_t \text{ (rad)}$$

$$\frac{1}{R_0(\theta_t)} \cong \frac{m_t^2 \theta_t}{\omega - m_t d_t \theta_t} = \frac{929.0 \theta_t \text{ (rad)}}{0.1785 - 940.0 \theta_t \text{ (rad)}} m^{-1}$$

For each divergence,  $\theta_t$ , we then use the beam expander ray matrix to compute the expander lens positions,  $d_1$  and  $d_2$ , which yield the above values of  $\omega_0$  and  $R_0$ .



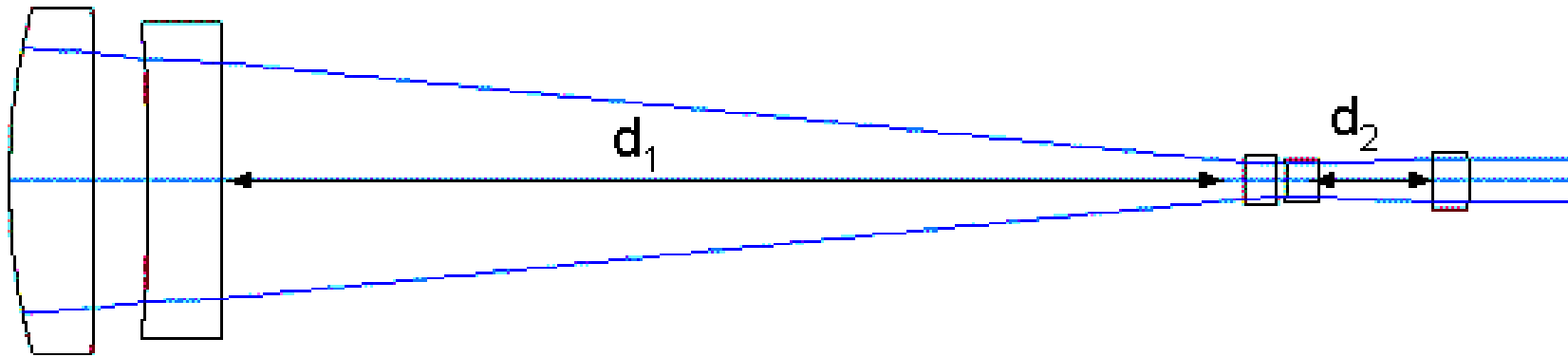
# 7. Generate Lookup Table Source: Phase II SLR2000 Transmitter

Divergence Half-Angle, arcsec	Interlens Distance $d_1$ (mm)	Interlens Distance $d_2$ (mm)	Motor 1 Position $M_1$ (mm)	Motor 2 Position $M_2$ (mm)	Motor 1 Encoder Counts $C_1$	Motor 2 Encoder Counts $C_2$
“0”	99.7472	12.9948	-11.0330	-19.7562	-36293	-64988
1	99.1793	13.6596	-10.4651	-19.8531	-34425	-65306
2	98.582	14.3243	-9.8710	-19.9238	-32470	-65539
3	97.9631	14.9891	-9.2489	-19.9664	-30424	-65679
4	97.3110	15.6539	-8.5968	-19.9790	-28279	-65720
5	96.6265	16.3186	-7.9123	-19.9593	-26027	-65656
6	95.9073	16.9834	-7.1931	-19.9049	-23661	-65477
7	95.1506	17.6482	-6.4364	-19.8130	-21172	-65174
8	94.3534	18.3130	-5.6392	-19.6806	-18550	-64739
9	93.5125	18.9778	-4.7983	-19.5045	-15874	-64159
10	92.6240	19.6426	-3.9098	-19.2808	-12861	-63424
11	91.6838	20.3075	-2.9696	-19.0055	-9768	-62518
12	90.6873	20.9723	-1.9731	-18.6738	-6490	-61427
13	89.6293	21.6372	-0.9151	-18.2807	-3010	-60134

“0” = 0.25 arcsec

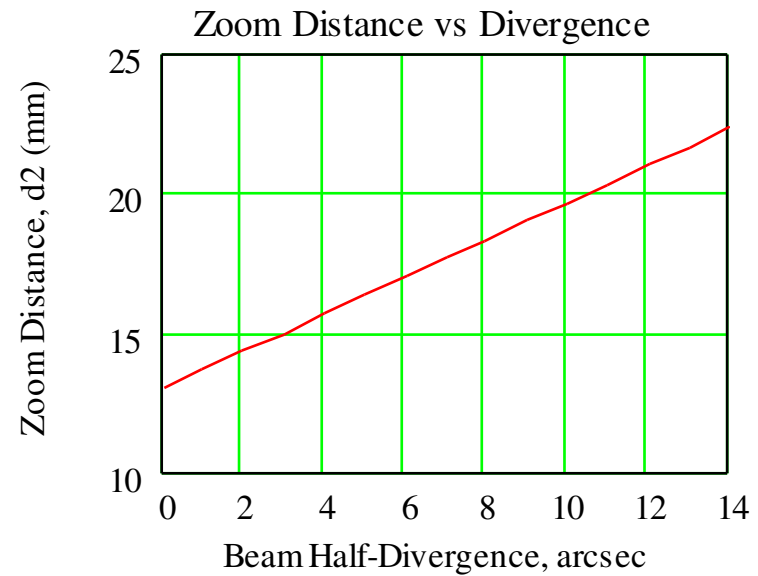
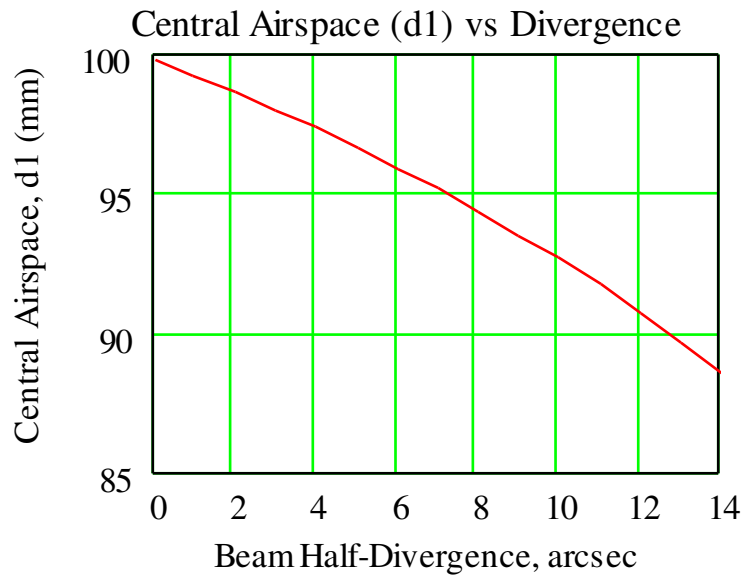
One encoder count = 0.304 microns of lens movement

# Lookup Table Plots



Stationary  
Doublet

Moving Moving  
Doublet Singlet





# Summary

- Using a computer lookup table, the SLR2000 computer can set two lens spacings in the transmit beam expander to provide a fixed beam diameter (35.8 cm) at the telescope exit aperture for eye safety while adjusting the phasefront curvature to give the desired final divergence.
- Lookup table must be adjusted for different transmitters but is an automated process.
- Optical half-divergence range of the final transmit beam is theoretically 0.25 arcseconds to 13 arcseconds (1.3 to 65 microradians) but atmosphere will set the actual lower limit.
- For verification, GSFC monitors the divergence of the beam expander output via a long focal length lens and CCD camera. Final beam divergence is reduced relative to the measured value by the transmitter magnification ( $3 \times 10.16 = 30.48$ ).
- q-parameter technique was successfully used to couple transmitter to the 1.2 meter telescope in GSFC to Mars laser link (Sept. 2005)
- For detailed analysis, see J. J. Degnan, "Ray Matrix Analysis for the Realtime Control of Automated SLR2000 Optical Subsystems", Chapter 8, Sigma Space Corporation Report, October 2005.