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# LAGEOS-1 spin determination, using comparisons between Graz kHz SLR data and simulations

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## Abstract

*kHz SLR data contains unique information about the measured targets; this information allows e.g. determination of spin parameters (spin period, spin direction, spin axis orientation) of various satellites, using various methods for different spin periods / satellites: Spectral analysis for spin periods of 2 s (AJISAI (Kirchner et al, 2007)), simulations for spin periods of 77.5 s (GP-B), and comparing simulation results with kHz data for very long spin periods like LAGEOS-1 (about 5000 s).*

*For the long LAGEOS-1 spin periods, we developed a method to calculate spin axis orientation and spin period from Graz kHz SLR data. This method is based on simulation of returns from each retro reflector, with spin period and spin axis orientation as input parameters. Varying these parameters, the simulation generates retro tracks similar to those seen in the kHz SLR data; comparing simulated and measured tracks, allows determination of spin period, and spin axis orientation. Applying this method to a set of LAGEOS-1 passes - covering a period of 178 days - shows also the slow change of the LAGEOS-1 spin axis direction with time.*

**Keywords:** *satellite laser ranging, LAGEOS-1, satellite spin*

## Introduction

LAGEOS-1 and LAGEOS-2 are identical satellites in circular orbits, about 5,900 km above Earth's surface. Both satellites are spheres with 60 cm diameter, covered with 426 cube corner reflectors (CCRs) arranged in 20 rings symmetrically with respect to the satellite equator (Fitzmaurice et al., 1977). Because the satellites are totally passive, their orbital motions are affected only by the natural perturbations. In this paper, we analyse only kHz SLR data of LAGEOS-1, due to its very low spin rate.

Perturbations can be of gravitational, non-gravitational (for example: Yarkovsky effect, Yarkovsky-Schach effect) or magnetic nature. SLR distance measurements to the satellites allow precise determination of these orbital perturbations and consequently identification of their origin. The more accurately we can determine the effect of perturbations, the more reliably we can obtain the geodynamical parameters of the Earth, and the relativistic effects in the near space (Ciufolini and Pavlis, 2004). It is expected that a detailed knowledge of LAGEOS-1 spin behaviour should improve the accuracy of such analysis, and will help to identify and confirm the source and magnitude of the (unknown) perturbations, which are introduced presently as empirical accelerations in actual models.

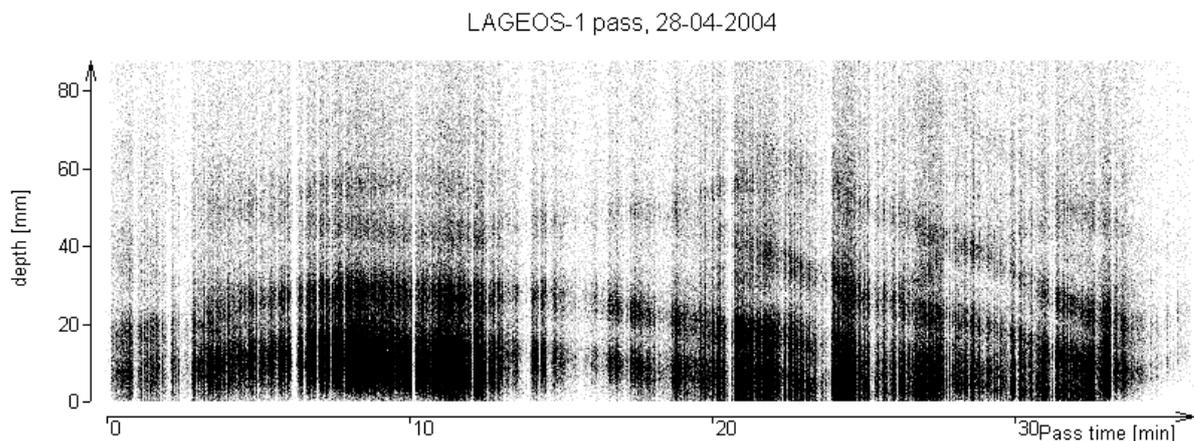
Up to now two methods were used to calculate spin parameters of LAGEOS satellites: frequency analysis of full rate SLR data (Bianco et al, 2001) and analysis of

photometric observations. The frequency analysis works well if the spin rate is not too low (e.g. 23.5 s for LAGEOS-2 in May 2000 gives good results in Bianco et al, (2001), but is not applicable anymore for larger spin periods, like the expected 5000 s for LAGEOS-1 in 2004 (Andres et al., 2004). Photometric measurements of LAGEOS-1 spin parameters were performed until 1997, when they were ceased because of a too low spin rate. In total, 57 photometric observations were carried out for this satellite (Andres et al., 2004), which allowed verification and improvement of the models of its spin motion. The most accurate model describing changes in the parameters of LAGEOS-1 spin is LOSSAM (Andres et al., 2004). According to this model, LAGEOS-1 started the third phase of its life in 1999, where the influences on spin parameters by magnetic, gravitational and non-gravitational torques are of the same order of magnitude. Bertotti and Iess (1991) have predicted that at this phase LAGEOS-1, having reached an extremely low spin rate, will start tumbling more and more, rapidly changing orientation of the spin axis, with chaotic dynamics.

### SLR Graz kHz laser measurements

Usually, SLR stations measure distances to satellites with laser repetition rates of 5 or 10 Hz. The Graz SLR station was the first station to measure with a laser repetition rate of 2 kHz (Kirchner and Koidl, 2004). Because of the very short 10 ps laser pulses, and the single photon detection system, the measurements are not only very precise (2–3 mm single shot RMS), but also allow identification of retro – reflector tracks in the data, easily seen due to their slightly different distances.

After a successfully measured satellite pass, the differences between measured and predicted distances are calculated. From these residuals the systematic trends are eliminated, e.g. by using polynomials; plotting these residuals (Fig. 1), different tracks from various retro-reflectors (or groups of them) can be identified easily. Residuals of nearer satellite prisms are on the bottom (satellite front), and residuals originating from more distant prisms are more towards the top in this figure.



**Fig. 1.** Range residuals of a LAGEOS-1 pass, measured by Graz kHz SLR system, 28-04-2004, 2 a.m. (P1)

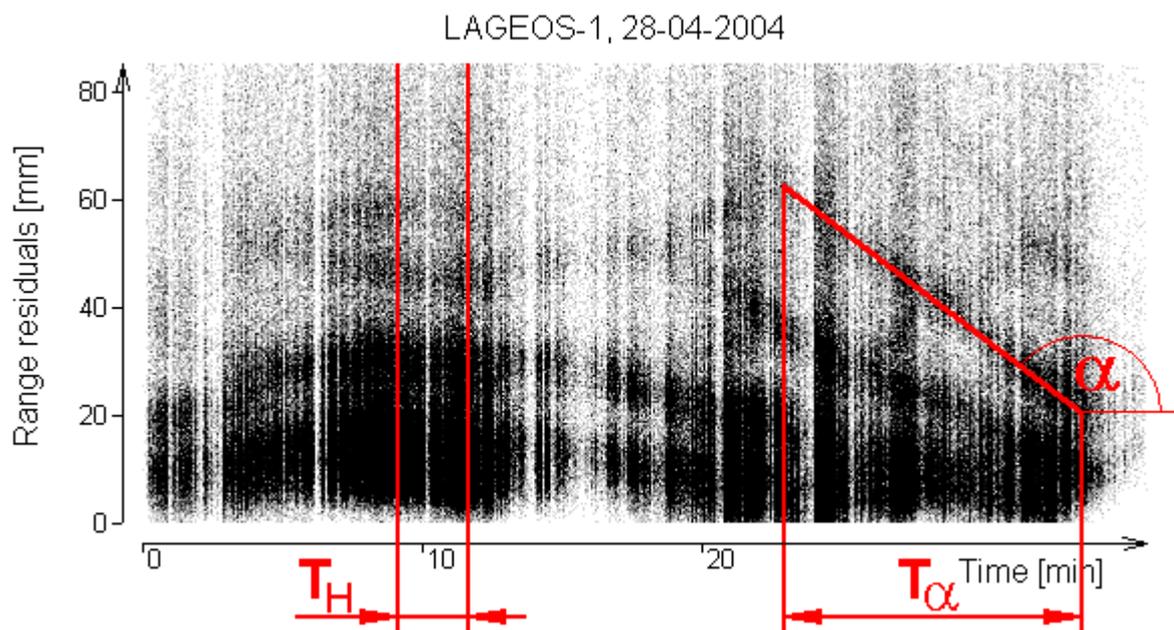
The residuals plotted in Fig. 1 refer to a LAGEOS-1 pass of April 28<sup>th</sup>, 2004 (P1). During the 35 minutes of the pass, more than 500,000 returns were measured. The majority of the returns come from the nearest retro-reflectors; the detection probability for returns from more distant retro – reflectors on the satellite’s sphere is decreasing. The reason for this effect is the geometry between the incident laser beam and the CCR. Total internal reflection of LAGEOS-1 optical retro - reflectors depends

on the angle between the incident laser beam and optical axis of the CCR as well as on the azimuth angle giving the direction of the incident beam about the normal to the front face of the CCR (Arnold, 1979; Otsubo and Appleby, 2003).

### Identification of the single prism tracks – the method

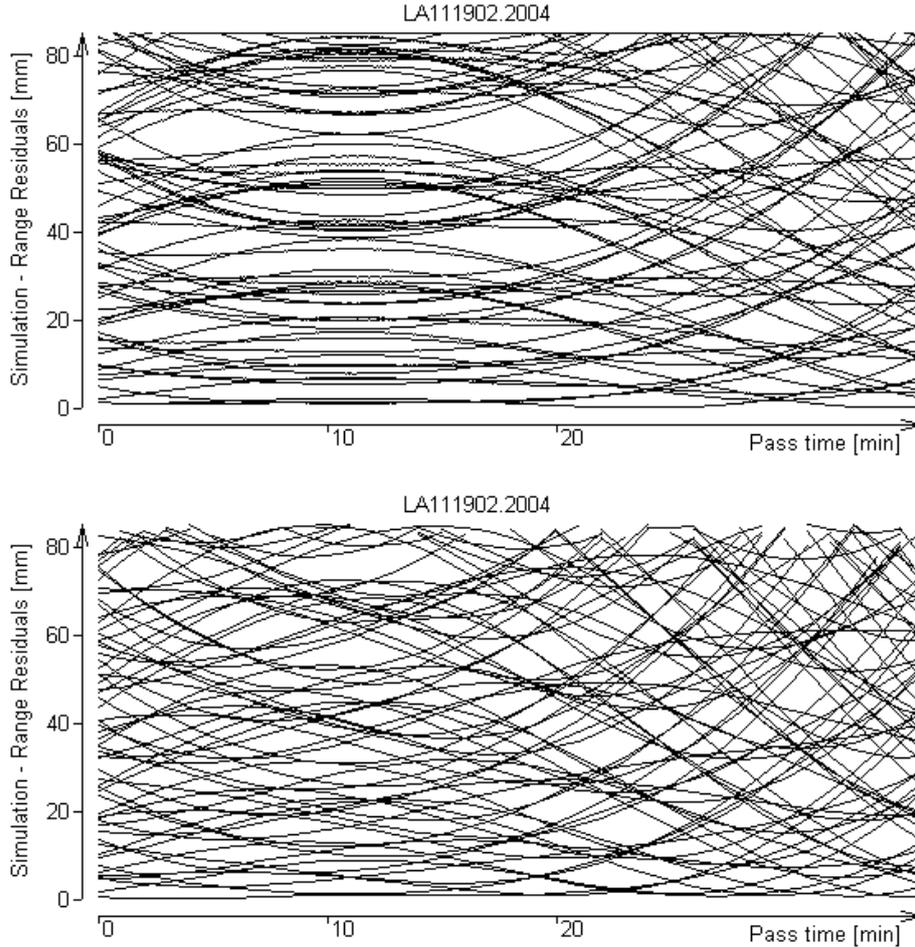
The tracks in Fig. 1 are due to the passage of retro – reflectors through the field of view of the telescope; thus they contain information on the satellite spin (Arnold et al., 2004). To recognize spin parameters out of the geometry of these spin tracks we developed a new method based on simulations of SLR measurements. The model used in these simulations is divided into two parts. The first part (macro-model) contains the Earth's rotation, the site position in ITRF2000 (Altamimi et al., 2002) and the orbital motion of the satellite. The second part (micro-model) contains the retroreflector-array arrangement and the range correction function (Fitzmaurice et al., 1977). In present study the model does not contain CCR transfer function (Arnold, 1979). The range correction function describes the photon's time of flight delay when the photon is going through the glass of the CCR. This correction depends on refractive index of the glass and the angle of incidence.

The geometry of range residuals distribution depends on spin parameters of the satellite: spin axis orientation and spin period. To calculate spin parameters it is necessary to determine epochs of the spin tracks and their tilt angles. The pass shown on Fig. 1 contains horizontal and tilted CCR tracks.



**Fig. 2.** Range residuals distribution:  $T_H$  - epoch range of horizontal tracks,  $T_\alpha$  - epoch range of  $\alpha$ -tilted track, pass start 28-04-2004, 2 a.m.

By using simulations it is possible to generate range residuals for every CCR distributed over the visible satellite's surface. Figure 3 presents examples of simulated CCR's trajectories for different spin parameters of the pass presented on Fig 2. For both charts spin period remains the same, but the second case was generated for different spin axis orientation: both angles (longitude and co-latitude) were increased by  $10^\circ$ . The geometry of the CCRs trajectories is very sensitive for spin parameters.

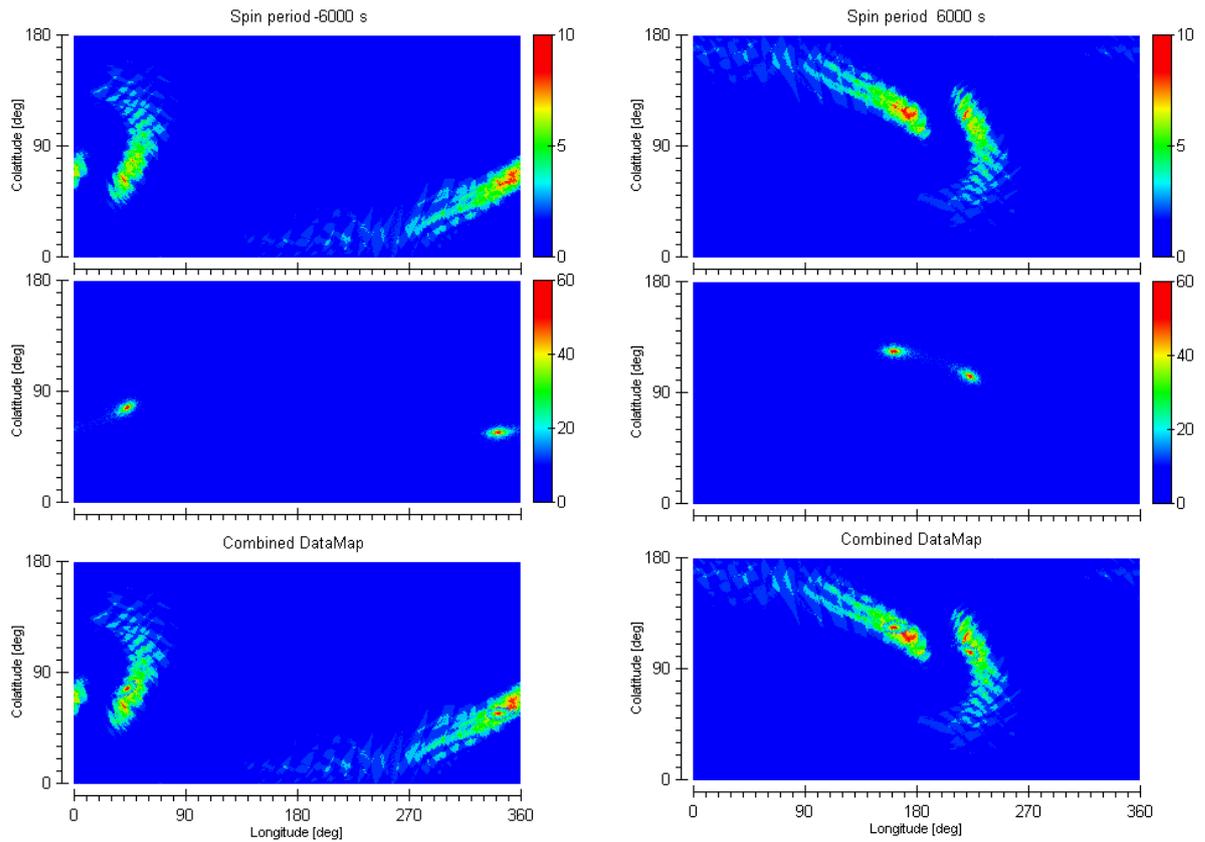


**Fig. 3.** Simulations of the CCR's trajectories for the pass presented on fig 1, for both cases spin period stays constant, but spin axis orientation for the bottom situation is shifted by  $10^\circ$  in longitude and colatitude.

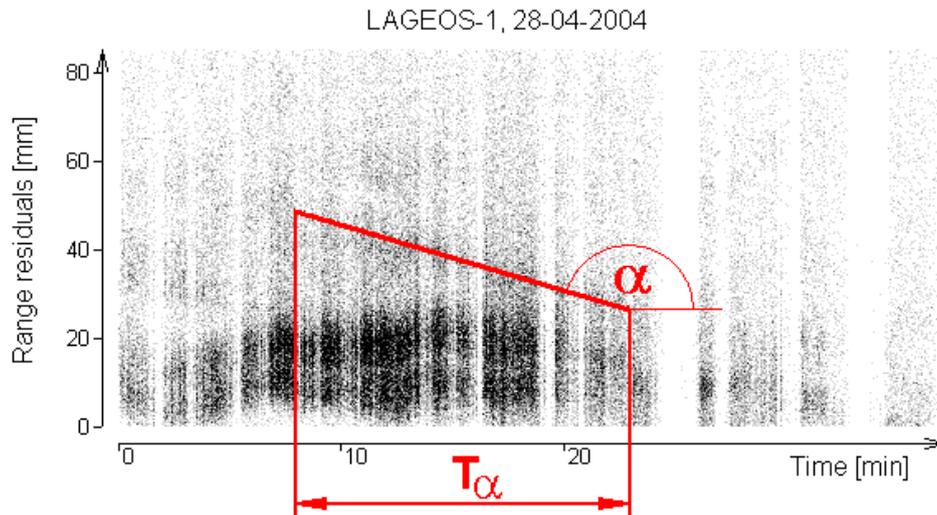
### Spin parameters determination

The LAGEOS-1 pass shown in Fig. 2 (P1) shows two significant kinds of range residuals distribution - horizontal and  $\alpha$ -tilted - which allows determination of the satellite's spin rate. LOSSAM predicts a spin period of about 5,000 s for LAGEOS-1 for the first half of 2004. Therefore we simulated range residuals for the pass P1 for spin periods  $T_S$  from  $-8,000$  s to  $-3,000$  s and from  $3,000$  s to  $8,000$  s with 50 s steps, and for all spin axis orientations with  $1^\circ$  steps.

Figure 4 shows results of simulations for all possible spin axis orientations (longitude and colatitude), for a spin periods of  $T_S = -6,000$  s and  $T_S = 6,000$  s. The top chart presents amounts (right scale - color bar) of  $\alpha$ -tilted spin tracks in  $T_\alpha$  epoch range for all spin axis orientations, the middle chart presents amounts of flat spin tracks in  $T_H$  ( $T_\alpha$  and  $T_H$  are given for the pass presented on Fig. 2). During all simulations the algorithm was searching for simulated  $\alpha$ -tilted CCR tracks within  $\alpha \pm 5$  deg. The bottom charts (Fig 4) show the sum of the top and the middle charts, evaluated pixel by pixel; as can be seen, for some spin axis orientations both kinds of spin tracks can exist. Such common spin axis orientation areas are the biggest for  $-6,000$  s (counter-clockwise rotation) and  $6,000$  s (clockwise rotation), therefore those spin periods were chosen for further investigation.



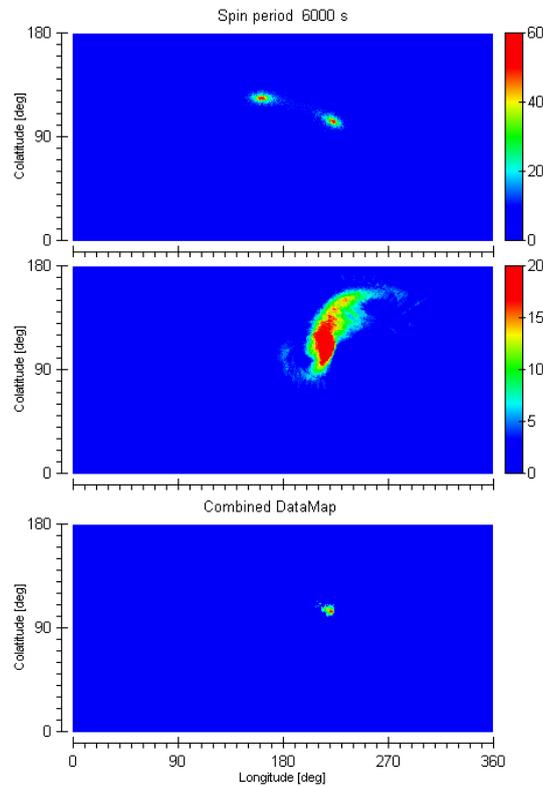
**Fig. 4.** Simulation results for spin period of  $T_S = -6000$  s (left) and  $T_S = 6000$  s (right); Top: Amounts of  $\alpha$ -tilted spin tracks; Middle: Amounts of horizontal spin tracks; Bottom: Sum of Top and Middle, pixel-by-pixel



**Fig. 5.** Range residuals of LAGEOS-1 pass tracked 12 hours later (P2), pass start 28-04-2004, 2 p.m.

For both spin periods it is possible to detect two different solution areas (Fig. 4, the bottom charts), due to the symmetrical arrangement of the CCRs over the surface of the satellite. After processing four solutions were obtained, two for CW and two for CCW spinning. To identify which is the real one we used a LAGEOS-1 pass (P2) tracked 12 hours after the main pass (P1) – Fig. 5.

Supposing that spin parameters of the satellite will not change significantly during 12 hours (from pass P1 to pass P2), one of the solutions determined for P1 should be the solution also for pass P2. Figure 6 presents three charts; the top one shows spin axis orientation solution for P1 and the middle chart for P2. The bottom chart shows common area of solutions for these two passes (pixel by pixel comparison); the appropriate spin axis orientation for both P1 and P2 was calculated as a mean value of this area.

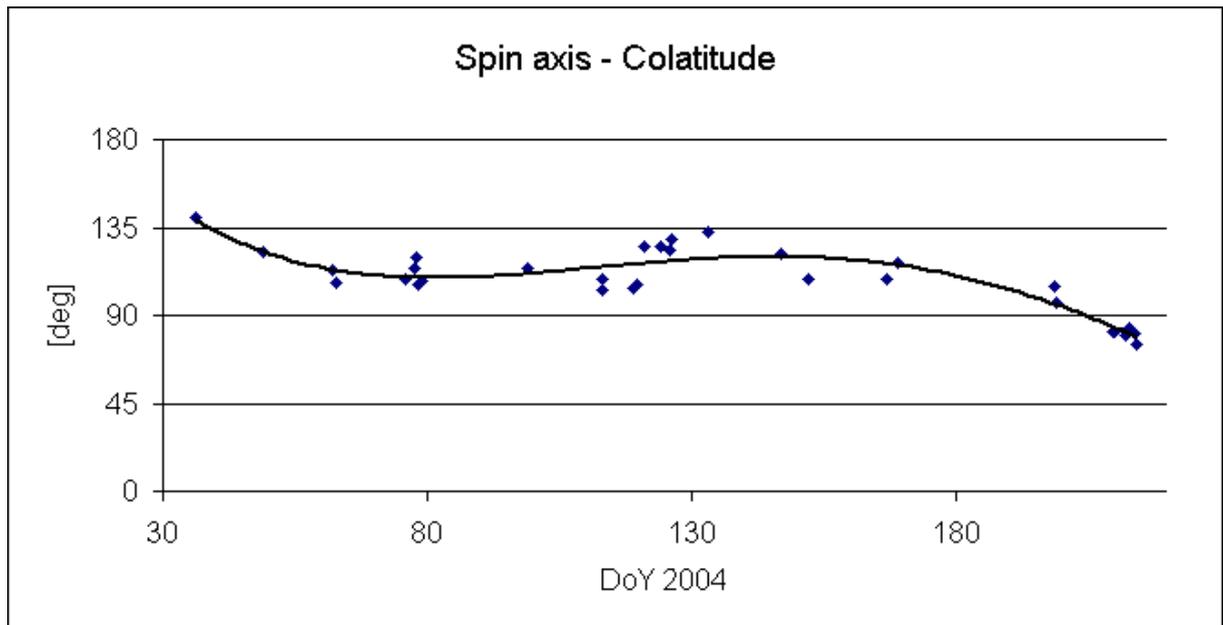


**Fig. 6.** Simulation - results; Top and middle: solutions for passes P1 and P2; Bottom: common area of the solutions

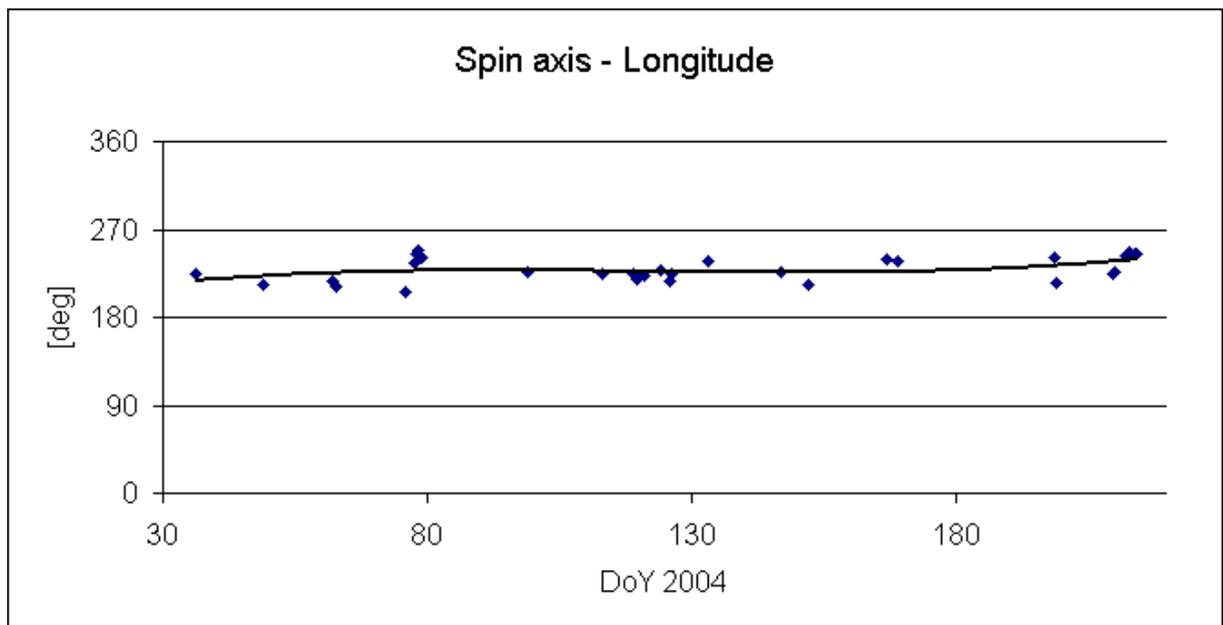
Using this pass-to-pass method reduces the amount of possible solutions from four to one; the spin parameters of LAGEOS-1 calculated from these two passes are: spin period (CW)  $T_S=6,000$  s, spin axis orientation: colatitude=103.8 deg, RMS=3.66 deg, longitude=224.2 deg, RMS=3.76 deg. All parameters are expressed in the J2000 inertial reference frame.

This pass-to-pass method was used to process 33 passes during 178 days of year 2004. Figures 7 and 8 present results for colatitude and longitude of spin axis orientation. The results were obtained for spin period  $T_S=6,000$  s, mean value of RMS for all colatitude results is  $RMS_{COL\_mean}=5.87$  deg, and for longitude  $RMS_{LON\_mean}=7.19$  deg.

For both angles the scatter around the fitted trend function is visible and has similar magnitude. That may be caused by inaccuracy of the used method or even by chaotic changes of the spin axis precession. The trend function of colatitude values shows sinusoidal decreasing during the investigated time period, while the longitude angle is more stable.



**Fig. 7.** Time-series of colatitude angle observations of the spin axis of LAGEOS-1, and trend function



**Fig. 8.** Time series of longitude angle observations of the spin axis of LAGEOS-1, and trend function

## Conclusions

The analysis presented in this paper identifies spin tracks in kHz SLR measurements to LAGEOS-1, and uses them to fully determine the spin parameters of this very slowly spinning satellite. This was possible by identifying the geometry of the observed tracks and looking for similar geometries in simulations generated for various spin parameters. This process allows to find several possible solutions, but with the pass-to-pass method it is possible to find a single common solution for two consecutive passes. This method can be applied only when spin parameters do not change significantly between the two analysed passes. Only one out of 33 investigated

passes contains both horizontally and  $\alpha$ -tilted CCR tracks, which are both necessary to determine the spin period of the satellite. The simulation model used for presented investigation is missing CCR energy transfer function, thus obtained results contain additional error. The transfer function will be taken into account with next version of the model and then analysis process will be repeated.

The accuracy of our method is a few times worse than that of photometric measurements. However, for long spin periods kHz SLR measurements and this simulation-based method is the only source of information about spin parameters of LAGEOS-1.

kHz SLR measurements, as started for the first time at the Graz SLR station, have opened new possibilities, allowing determination of the satellite spin parameters when all other methods fail. Additionally, the expected increase of the number of kHz SLR stations in the near future will improve the accuracy of spin parameter determination by a few orders of magnitude.

### **Acknowledgments**

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