ASYMMETRIC RADIATION PRESSURE ON LAGEOS

David A. Arnold (1), Graham Appleby (2)
(1) 94 Pierce Rd, Watertown, MA 02472, 617-924-3811. (2) NERC Space Geodesy Facility, Herstmonceux, Hailsham, E. Sussex, UK

Introduction

This is a abbreviated version of the original paper. For the complete paper see the SPWG website 'http://nercslr.nmt.ac.uk/sig/signature.html'.

Analysis of the orbits of the LAGEOS satellites indicates that the radiation pressure on the satellites is not perfectly symmetrical. There is an unexplained force component along the spin axis. I was asked to review paper entitled "LAGEOS Satellites Germanium Cube-Corner-Retroreflectors and the Asymmetric Reflectivity Effect", David M. Lucchesi, Celestial Mechanics and Dynamical Astronomy 88: 269-291, 2004. This paper attempts to explain the asymmetry as due to the germanium cube corners. I feel that the physical model used in the Lucchesi paper is incorrect. I recommended rejecting the paper. This paper discusses my objections to the physical model and outlines a different modeling approach.

The physical problem.

The LAGEOS satellite is subjected to perturbing forces from incident radiation and thermal radiation emitted by the satellite. The incident radiation is primarily solar radiation (1412.5 w/m^2), but there are also forces due to solar radiation reflected from the earth (earth albedo 38.1 w/m^2), and thermal radiation from the earth (66.5 w/m^2). The incident radiation is partially reflected from the satellite and partially absorbed.

The satellite has an aluminum surface (bare, machined 6061-T6 aluminum) and brass core. The absorptivity of the aluminum is .15 and the emissivity is (.05). The average core temperature is 55 deg C. For a non-spinning satellite the hot side and cold side temperatures are 60 deg C and 52 deg C. The corresponding maximum and minimum retroreflector face temperatures are 16 and -8 deg C.

Thermal vacuum tests run on two of the infrared cube corners gave temperatures of 104.4 and 106.7 deg C with full solar illumination. The cube corners would be opaque at this temperature. With the solar illumination reduced by the factor $1/\pi$, the temperatures fell to 55.5 and 52.8 deg C for the two cubes. This illumination corresponds to an incidence angle of 71.4 deg. With no solar illumination, the temperatures were 12.2 and 26.6 deg C.

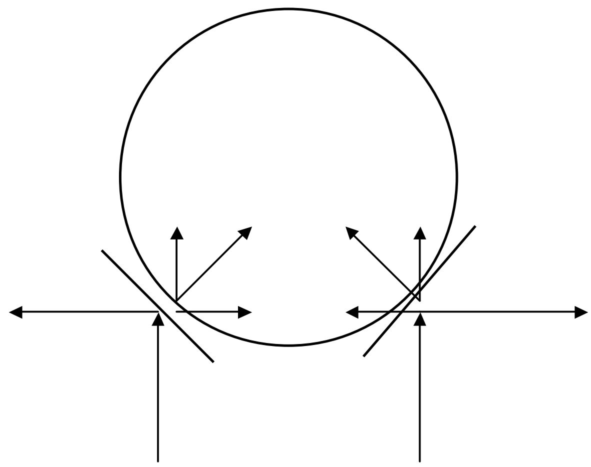
The optical cube corners are colder than the core because they have a high emissivity (.90) and low volumetric solar absorptivity (.05). The mounting cavity is designed to minimize both radiative and conductive heat transfer between the core and the cube corners to minimize thermal gradients which would distort the diffraction pattern of the cube corners.

The energy absorbed by the satellite must be emitted as thermal radiation. The force due to the incident radiation is instantaneous. As long as the reflecting properties of the satellite are uniform there is no asymmetry. However, because of the time constants involved in the thermal behavior, the thermal radiation will be in a different direction from the incident radiation. Since the spin rates of LAGEOS-1 and LAGEOS-2 are different, the thermal behavior will be different for the two satellites. Since the optical and thermal parameters of the germanium cubes are different from the parameters of the optical cubes, they run at different temperatures and the pressure of the thermal radiation is different. This results in an asymmetric thermal radiation pressure.

Asymmetry due to the germanium cube corners.

The author assumes that the optical cubes cover the LAGEOS satellite uniformly and that there would be no asymmetry in the radiation pressure on the satellite with only optical cubes. The premise of the paper is that replacing 4 of the optical cubes with 4 germanium cubes results in an asymmetry in the radiation pressure on the satellite because the reflectivity of the germanium cubes with respect to solar radiation is different from the reflectivity of the optical cubes. The asymmetry is illustrated by a simple example below.

In the diagram below, there are two mirrors on a spherical satellite which are at a 45 degree angle with respect to the incident solar radiation which is in the vertical direction. The mirrors have a reflectivity of 100 percent. The light hitting the left mirror is reflected to the left and the light hitting the right mirror is reflected to the right. The momentum transfer is perpendicular to the mirror in the direction of the center of the satellite for both mirrors. The force on each mirror can be resolved into horizontal and vertical components. The horizontal components cancel and the net force on the satellite is in the vertical direction, parallel to the incident solar radiation. There are no torques or unbalanced horizontal forces. This is the definition of a symmetrical radiation pressure on the satellite.



If the mirror on the right is replaced by a black body, the radiation is totally absorbed and the pressure on the black body is in the vertical direction. This creates an asymmetry . There is a net force to the right because the pressure on the black body has no component to the left to balance the horizontal component of the force on the left mirror.

Suppose the reflectivity of the right mirror is 50 percent. The force due to the absorbed component is in the vertical direction. The force due to the reflected component is as shown in the diagram except that the magnitudes of the components are half as great. The force on the satellite is asymmetrical since the horizontal components do not balance.

Suppose the reflectivity of the right mirror is 99 percent. The radiation pressure on the satellite would be nearly symmetrical but there would be a small asymmetrical component.

If one replaces an optical cube corner on LAGEOS with a germanium cube corner, the effect is to subtract the force on the optical cube and add the force on the germanium cube.

Perturbation to the orbit.

A. First approach

One approach to determining the orbital perturbation caused by the germanium cube corners which I think gives a good physical understanding of the problem is the following:

1. Do an orbital simulation with only optical cubes on the satellite computing all the forces on the satellite due to incident radiation on the cube corners and the core, and the force due to thermal radiation from the core and the cube corners.

2. Replace 4 optical cubes with germanium cubes. Do a second orbital simulation computing all the forces on the germanium cubes, the optical cubes, and the core from incident radiation, and the forces due to thermal radiation by the germanium cubes, optical cubes, and the core.

3. Compute the difference between the state vectors or orbital elements from the two simulations. This is the perturbation due to the germanium cube corners. Step 2 will give a different answer from step 1 because the force on the germanium cubes is different from the force on the optical cubes they replace.

B. Second approach

Since the only thing that matters in computing the perturbation is the difference between the force on a germanium cube and the force on the optical cube it replaces there is a simpler approach to computing the perturbation as follows:

1. Do an orbital simulation with only a central force and no perturbations. The orbital elements are constant for this case.

2. Compute the difference between the force on a germanium cube and the force on an optical cube. Do a second simulation using only the central force and the difference in force on the two types of cube corners.

3. Compute the difference between the state vectors or orbital elements from the two simulations. This is the perturbation due to the germanium cube corners.

The Lucchesi model

In the paper by Dr. Lucchesi, the perturbing force is computed as the difference between the force on a germanium cube and the force on a black body. This implies that the optical cube is a black body. A black body has zero reflectivity. It absorbed all the incident light.

In principle, one could use a black body as a reference in the following way. Suppose we assume that there is no asymmetry with only quartz cubes on the satellite. Let us call the force on a quartz cube $\vec{Q}$, the force on a black body with the same area $\vec{B}$, and the force on a germanium cube $\vec{G}$. Suppose one removes a quartz cube and replaces it with a black body. The imbalance created is ($\vec{B}$ - $\vec{Q}$). Suppose the black body is then replaced by a germanium cube. This introduces an additional imbalance given by ($\vec{G}$ - $\vec{B}$). The total imbalance due to the two substitutions is [($\vec{B}$ - $\vec{Q}$) + ($\vec{G}$ - $\vec{B}$)] = ($\vec{G}$ - $\vec{Q}$). Introducing the intermediate step of a black body is unnecessary and using only ($\vec{G}$ - $\vec{B}$) is incorrect.

Numerical comparison of the two models.

There is no available model for computing the radiation pressure on an optical cube corner as a function of incidence angle. However, the case of normal incidence is simple to compute. Let us assume that momentum p is incident on a cube corner at normal incidence and compute the momentum transfer for various different cases.

Case 1. Normal incidence on an optical cube corner.

In this case, the reflectivity of an uncoated cube corner is nearly 100 percent due to total internal reflection. The momentum of the radiation before reflection is $+p$ and the momentum after reflection is $-p$. The difference $2p$ is the momentum transferred to the cube corner.

Case 2. Normal incidence on a germanium cube corner with a reflectivity of 100 percent.

The answer in this case is $2p$ the same as Case 1. This was the assumption in the author's original calculation and the computations were never revised.

Case 3. Normal incidence on a black body.

The momentum of the radiation before absorption is $+p$. The momentum of the radiation after absorption is zero. The difference $+p$ is the momentum transferred to the black body.

Case 4. Normal incidence on a germanium cube with reflectivity 50 percent.

The momentum of the radiation before striking the cube is $+p$. The momentum of the 50 percent that is reflected is $-p/2$. The difference is $3p/2$ which is the momentum transferred to the cube corner. Half of the energy is absorbed by the cube and must be emitted as heat.

Case 5. Asymmetry using the Lucchesi model

In the model used by the author the perturbation is computed as the difference between the momentum transferred to a germanium cube and the momentum transferred to a black body. This is given by the difference between Case 2 and Case 3. We have

$2p - p = p.$

Case 6. Asymmetry using my model

In the model I have proposed, the perturbation is computed as the difference between the momentum transfer to a germanium cube with reflectivity 50 percent and the momentum transfer to an optical cube. This is given by the difference between Case 4 and Case 1. We have

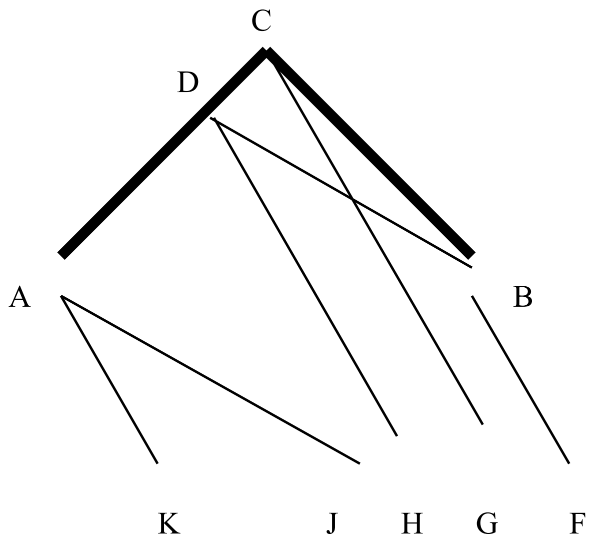
$3p/2 - 2p = -p/2.$

Discussion of the differences between the two models.

The method used by the author over-estimates the magnitude of the perturbation due to the difference in reflectivity between a germanium cube and an optical cube as shown by comparing Case 5 and Case 6. However, the author's model neglects thermal radiation. The absorbed radiation has to be emitted as heat. If all the absorbed radiation were emitted instantly perpendicular to the front face, this would add another force proportional to $p/2$ so that the total momentum transfer in Case 4 would be $2p$ the same as for an optical cube. Case 6 would then give zero instead of a negative number.

Asymmetry of the radiation pressure on an optical retroreflector.

The figure below shows a two-dimension hollow cube corner. Light entering the cube corner at F is reflected from the right side at B, the left side at D, and exits in the direction of point H. Light entering at G is reflected near the vertex at C and exits in the direction G. Light entering at H is reflected at points D and B and exits in the direction of point F. The light that enters between points H and F is retroreflected and the radiation pressure is in the direction of the incident radiation.



Light that enters the cube corner at K is reflected from the left side, misses the right side, and exits in the direction J. All light that enters between K and H is reflected only from the left side. The radiation pressure for this part of the radiation is perpendicular to the surface AC. This part of the radiation pressure is asymmetric. Appendix B gives a numerical calculation of the radiation pressure as a function of incidence angle. At normal incidence all radiation is retroreflected. Past about 27 degrees, the asymmetric component of the radiation pressure is larger than the retroreflected component. Beyond 45 degrees, which is the cutoff angle for retroreflection, all the radiation pressure is asymmetric.

A real cube corner has three reflecting faces. In addition to radiation that is reflected from only one face there may be radiation reflected from only two faces. The direction of these beams can be easily calculated. The more difficult part is calculating the amount of energy in each beam which depends on complicated geometric calculations of the reflecting area of each beam. This radiation pressure from this component is also asymmetric.

Each back face reflects one component of the incident momentum. Except at normal incidence on the cube corner, the momentum transferred to the cube corner is not normal to the front face. It is the sum of momentum transfers normal to the back reflecting faces.

The direction of the normal to each of the three faces depends on how the cube corner is installed in its holder. The cube corner can be rotated at any angle about its symmetry axis. In order to reduce systematic effects due to the angle at which the cubes are installed, the orientations of the cube corners have been randomized. As a result the direction of the asymmetric component of the radiation pressure on the optical cubes is random. The net radiation pressure on all the optical cube corners depends on the method used to randomize the orientations.

If the cube corner is solid, some of the radiation is reflected from the front face by dielectric reflection. The radiation pressure of this component is perpendicular to the front face. Since the optical cube corners cover the satellite in a reasonably uniform manner, the radiation pressure of this component is probably reasonably symmetric except for the presence of the germanium cubes.

Since there can be loss of total internal reflection in an uncoated cube, some radiation may be transmitted into the cavity and undergo further reflections or absorption by the core. Also, the light my strike unpolished surfaces of the cube where it has been cut into a spherical shape. Both these components are asymmetric.

There is no way at present to calculate the asymmetry due to the optical cubes because there is no existing model for the radiation pressure on the optical cubes. The only part of the reflection from an optical cube that has been modeled is the retroreflected part.

The radiation pressure on a germanium cube is easy to calculate. However, the equations used in the Lucchesi paper are incorrect. The correct equations are given in Appendix A.

Summary.

1. The asymmetry is the force on a germanium cube minus the force on the optical cube it replaces. Calculating the deviation from a black body is unphysical and over-estimates the asymmetry due to the germanium cubes.

2. The author's model neglects the force on the cube corners due to thermal radiation. There is an asymmetry because the germanium cubes are warmer than the optical cubes.

3. The equations used to compute the radiation pressure on the germanium cubes are incorrect by a factor of 2 even if one accepts the validity of the black body model.

4. The author neglects the asymmetry in the radiation pressure on an optical cube corner.

Conclusions and recommendations.

The apparent partial agreement between the calculations and the observed orbital perturbations should be considered accidental because the physical model and equations are not correct. The satellite can be considered to consist of 4 parts:

A. The optical cubes
B. The germanium cubes
C. The retaining rings
D. The surface of the core

The information needed to do a proper calculation of the asymmetry is the solar radiation pressure on each of the parts and the temperature of each part. The temperature is then used to calculate the pressure of the thermal radiation.

Appendix A. Radiation pressure on a germanium cube (see SPWG website)
Appendix B. Radiation pressure on a two-dimensional cube corner (see SPWG website)