Absolute Earth Scale from SLR Measurements

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Abstract

Since the LAGEOS-I satellite was launched in 1976, the systematic instrument error of the best satellite laser ranging observatories has been steadily reduced. Advances in overall system accuracy, in conjunction with improved satellite, Earth, orbit perturbation and relativity modeling, now allows us to determine the value of the geocentric gravitational coefficient (GM) to less than a part per billion (ppb). This precision has been confirmed by observations of the LAGEOS-II satellite, and is supported by results from Starlette, albeit at a lower level of precision. When we consider observations from other geodetic satellites orbiting at a variety of altitudes and carrying more complex retroreflector arrays, we obtain consistent measures of scale, based upon empirically determined, satellite-dependent detector characteristics. The adoption of a value of GM differing by a ppb would result in a difference of a few millimeters in the height definition of a near-Earth satellite. The precision of the estimate of GM from satellite laser ranging has improved by an order of magnitude in each of the last two decades, and we will discuss projected advances which will result in further refinements of this measure of Earth scale.

Background

The technical applications of SLR data cover a variety of scientific areas. The accurate satellite position defined by a network of SLR stations enables us to improve the gravity model of the Earth and to investigate other force model effects on the orbit. The instruments can also be employed to define high resolution Earth orientation parameters from observations of geodetic satellites in stable orbits, such as the LAGEOS and Etalon constellations, and the scale of the measurements allows very accurate definition of the center of mass of the Earth, as well as the dimensions of the planet and its gravitational constant. The Global Laser Tracking Network has collected measurements from retroreflector carrying satellites for over three decades, and tracking of the first passive laser geodetic satellite, Starlette, which orbits at 950 km altitude, was initiated in 1975. The LAGEOS-I satellite, launched in 1976, provided a more stable platform at the higher altitude of about one Earth radius, and was a more sensitive monitor of the scale of the measurements. Several other passive, retroreflector-carrying satellites have since been launched, in particular, LAGEOS-II, which occupies a stable orbit at the same one-Earth-radius altitude as LAGEOS-I, but at a lower inclination. Etalon-I and -II were launched into higher, twelve-hour orbits with even better intrinsic stability and scale sensitivity than LAGEOS. Ajisai occupies a near-Earth orbit of 1400 km, a little higher than Starlette at 950 km., and Stella, the most recent addition to the constellation, is in the lowest orbit at 800 km altitude.

SLR Scale Measurement

The ability of the network to define Earth scale is directly proportionally to the ranging accuracy, and by the time of LAGEOS-I launch, this had reached the decimeter level in most of the systems. During the 1980's, advances in technology further improved instrument accuracy to centimeters and recent developments indicate millimeter capability [*Degnan*, 1993] in the most advanced systems. The distribution of the network also has an effect on its ability to determine global parameters such as the geopotential, the motion of the geocenter and GM.

The MERIT campaign of 1983 initiated an era of improved data coverage from a worldwide distribution of stations. Regular upgrades in the instruments have taken place during the 1980's (see, for example, *Bosworth et al.*, 1993). Significant campaign-related changes in the accuracy of several important stations also occurred in the early 1990's, in preparation for the support for the TOPEX/Poseidon mission, for which SLR was chosen as the prime tracking system.

Scientific Relevance of Earth Scale Determination

Satellite geodesy depends on the integrity of the geocentric gravitational coefficient for its definition of scale. An accurate value of the universal gravitational constant times the mass of the Earth (GM) enables us to monitor the behavior of the Earth, as sensed by satellite observations, in an absolute reference frame. Improved orbit scale definition provides an important component to the definition of the geopotential model, and has among its most practical applications a contribution to the refinement of the positioning of altimeter satellites, which carry stringent radial accuracy requirements. This work will help to define the long-term stability of the SLR reference frame and improve the positioning capability of the network. The rate of sea level rise caused by global warming is currently measured over decadal time scales with tide gauges which provide observations relative to the Earth's surface. The scale inherent in laser ranging observations to stable high-altitude satellites enables us to determine accurate geocentric height at the observatories. The work will also contribute to the refinement of the positioning of altimeter satellites, which carry stringent radial accuracy requirements in order to define sea level relative to the Earth's center. The precision of the estimate of GM has improved by an order of magnitude in each of the last two decades, and refinements by another order of magnitude will allow us to test contemporary limits on Gdot/G. Furthermore, we anticipate that routine monitoring of Earth processes to which ranging scale is sensitive, such as seasonal and secular variability in Earth albedo, can also provide important indicators of the effects of global change.

The Evolution of Scale Definition

During the 1970's, the Earth's geocentric gravitational coefficient was independently determined from observations of several interplanetary spacecraft, including the Ranger, Surveyor, Lunar Orbiter, Pioneer, Mariner and Viking flights. *Lerch et al* [1978] give a summary of these results, as well as those from laser ranging to lunar retroreflectors, in the introduction to their analysis of the first six months of LAGEOS-I data. Perhaps it should be noted that the vertical scale which was used for the plot of those estimates had a full range of 1 km^3/sec^2, corresponding to the sixth decimal digit of GM, and equivalent to about one part per million. The early, decimeter-accuracy LAGEOS observations were included in the development of the GEM-L2 gravity model, most of whose data was collected in the 1970's, when the prevailing scale knowledge was based on the speed of light 299729.5 km/sec. An uncertainty of 0.02 km^3/sec^2 (50 parts per billion) was assigned to the new estimate for GM with a value of 398600.44 km^3/sec^2 after appropriate scaling for the speed of light currently adopted (299729.458 km/sec).

Lerch et al. noted a variation of 0.05 km^3/sec^2 in estimates for individual satellites, which nonetheless established a significant improvement on earlier GM determinations during that decade and moved the uncertainty another decimal place to the right. *Smith et al.* [1985] list a variety of error sources which affect LAGEOS-I determinations of GM at the level of a few parts per billion, which include the relativity model, instrument bias, refraction, and several Earth model parameters. Their published annually determined estimates of GM varied within a range of 0.005 km^3/sec^2, and this variability provided a measure of the effect of unmodelled error on the prevailing resolution of orbit scale. *Tapley et al.* [1985] conducted an analysis of a full 7.7-year span of LAGEOS data to determine a value of 3986400.440 km^3/sec^2, which was subsequently assigned an uncertainty of 0.002 km^3/sec^2 (5 ppb). *Tapley et al.* note the important influence of the adopted value of GM on the scale of laser station coordinates, particularly when comparing positioning results from other techniques, in that case from Very Long Baseline Interferometry measurements.

Table 1 summarizes some of the historical values of GM based on analyses of observations predominantly from LAGEOS-I, as well as some more recent determinations. The values listed in Table 1 between 1976 and 1990 were determined with the satellite, Earth and force models prevailing in 1990. These early results have been compensated with a correction for the satellite center-of-mass offset which was too small (by 11 mm) in the original work of *Lerch et al.*, *Smith et al.*, and *Tapley et al.* At the level of data and modeling accuracy of 1990, this correction was less than the quoted 0.002 km^3/sec^2 uncertainty of these later estimates. At this SLR accuracy level, the shorter satellite ranges now shared the sensitivity of the lunar ranging results to the relativity model.

Table 1. Estimates of GM from LAGEOS-1.

Data period	GM-398600 km^3/sec^2	Error in km^3/sec^2	Source
1976	0.44	0.02	Lerch et al.1978
1976-1984	0.441	0.002	Tapley et al. 1985(1)
1976-1982	same	same	Smith et al. 1985(1)(2)
1986-1992	0.4415	0.0008	Ries et al. 1992
1992-1996	0.4419	0.0002	Dunn et al. 1999 (with LAGEOS-II)

(1) + 0.001 km^3/sec^2 (for a center-of-mass offset of 251 mm)
(2) + 0.006 km^3/sec^2 (for coordinate time scale factor in relativity model)

The Effect of General Relativity

The analysis of the SLR observations is conducted in a geocentric reference frame, which is more suitable for the treatment of Earth-orbiting satellites than the solar system barycentric coordinate system in which the relativistic equations of motion are formulated (see, for example, *Ries et al.*, 1988). The measurement computation of the software used in our analysis (GEODYN, [*Putney*, 1977]) allows us to operate in a geocentric frame for which the secular component of Earth's rotation is defined by Very Long Baseline Interferometric quasar observations. General relativistic corrections to the satellite accelerations, the light time measurement, station clock times and station coordinates are made according to the treatment of *Martin et al.*, [1985]. *Ries et al.*, [1989], showed that the determination of GM at the level of accuracy of 0.001 km^3/sec^2 (2.5 ppb) requires careful consideration of all relativistic effects. Their comparison of the barycentric and geocentric formulations for the effect of general relativity on near-Earth satellites established that relativity could be fully modeled in the geocentric reference frame if the light-time correction is applied to the ranging data.

Current Capability for Scale Definition

The analysis of *Ries et al.*, [1992] included all significant effects of general relativity, as well as improved knowledge of the LAGEOS satellite center-of-mass correction, which had been indicated in pre-launch tests of the LAGEOS-II satellite [*Minott et al.*, 1993]. The GM thus determined, employing data collected between November 1986 and November 1991, was 398600.44150 +/- 0.00080 km^3/sec^2 (2 ppb). This value was well within the uncertainty assigned to the lunar laser ranging (LLR) estimate of *Newhall et al.* [1987], after the required scale difference of 1.4808*10-8 between barycentric dynamical time and terrestrial dynamic time had been applied for a comparison in compatible reference frames. *Dickey et al.* [1994] combined an accurate value of the mass ratio of the Sun/(Earth+Moon) from LLR with the solar GM and the lunar GM from lunar-orbiting spacecraft [*Ferrari et al.*, 1980]. They arrive at a value of GM in an Earth centered reference frame of 398600.44300 +/- 0.00400, an accuracy of ten parts per billion. The estimate of *Ries et al.* [1992] was the basis of the IERS standard (IERS92), but they employed measurements from systems which underwent regular upgrade after 1986, during the period of their analysis. *Dunn et al.* [1999] show that annual estimates starting in 1990, after improvements to many systems had been made in anticipation of the TOPEX/Poseidon mission, exhibit more stable behavior. We have extended the work to consider GM determinations which can take full advantage of the observations of LAGEOS-II, which was launched in 1992, for the confirmation of any revised GM estimate from LAGEOS-I.

Technical Approach

The goal of this analysis is to use the scale properties of modern SLR observations tracking LAGEOS-I and -II to simultaneously determine GM. The solution parameters were based on a speed of light of 299792.458 km/sec, and the force model included geopotential perturbations to degree and order 20 from a recently determined gravity model [*Lemoine et al.*, 1997] derived from observations of many different satellites. Full Earth and ocean tidal models were employed, and third body perturbations from the Sun, Moon, and Mercury through Neptune included. The effects of general and special relativity were modeled, together with Earth albedo, and ocean loading and solid Earth tides are modeled at each station. Tidally coherent diurnal and semi-diurnal geocenter and Earth Orientation Parameters are applied for fourteen tidal frequencies.

Non-conservative pertubations contribute a significant component to the orbit error budget of the LAGEOS satellites. Solar radiation pressure and Earth albedo are applied to both satellites using solar reflectivity coefficients which remain constant for each satellite. Thermal thrusting effects due to solar and Earth-reflected radiation [*Rubincam*, 1988, *Ries et al.*, 1991, *Scharroo et al.*, 1991] are accommodated by empirical force model parameterization which is satellite-dependent. Direct and once-per revolution along-track accelerations are estimated at 5-day intervals and the solar radiation pressure coefficient held at 1.13. Monthly estimates of GM are made simultaneously with orbit and force model parameters for each satellite with fixed station positions and Earth orientation parameters.

Error Analysis

The determination of parameters using the Bayesian least-squares process [*Putney et al.*, 1990] which we have adopted in this analysis depends on a priori assumptions for the Earth and satellite model, the ability of the adjusted parameters to accommodate errors which are not correlated to the sought–for variables, and the quality of the data. In order to test the sensitivity of the proposed analysis to errors in our assumed Earth, force model and instrument models, a series of experiments was conducted [*Smith et al.*, 1999]. The strong dependence of the GM estimates on instrument accuracy is shown in Table 2, demonstrating their sensitivity to instrument range bias and atmospheric refraction error. The correction of a ranging measurement for atmospheric refraction is approximately 2 meters at zenith, and as large as 7 meters at 20 degrees elevation, which is the usually observed minimum for most systems. The average effect on the measurement is about 4 meters and so a 0.25% refraction error is equivalent on average to a one centimeter range difference.

The similar influence of refraction error and range bias is clear from Table 2, which shows an empirically determined effect on the estimated value of GM in the data reduction process using observations collected between 1992 and 1996. The global estimate of GM from each satellite changes by amounts which depend on the satellite altitude. The sensitivity is compared in Table 2 to a theoretical estimate (labeled in the table as a one centimeter height change) based on the assumption that the satellite's period is independently determined by the epoch timing properties in the data and GM is directly related to a change in satellite height according to Kepler's third law.

Table 2. Effect of orbit or data errors on GM determinations from several satellites.

		Effect in km^3/sec^2 of		
Satellite	**Altitude in km**	**one cm height**	**one cm range**	**0.25 % refraction**
Etalon-I	19,000	0.00047	0.00047	0.00040
Etalon-II	19,000	0.00047	0.00047	0.00040
LAGEOS-I	5,900	0.00092	0.00104	0.00092
LAGEOS-II	5,900	0.00092	0.00105	0.00087
Ajisai	1,400	0.00146	0.00190	0.00175
Starlette	950	0.00155	0.00215	0.00201
Stella	800	0.00157	0.00230	0.00221

The possibility of errors in the adopted station position and velocity tracking complement must also be considered, although it is expected that the estimates of GM are insensitive to these parameters. Improvement of the Earth and force model (gravity, tidal and non-conservative) will, however, probably improve the formal error estimate for GM. There is one perturbation to the satellite which has a direct effect on the scale of the solution: the effect of Earth-reflected radiation. The albedo will exhibit seasonal and long-term variations, which provide a measure of global 'health', as it monitors the cloud cover which results from global temperature changes. *Martin et al.* [1988] have investigated the albedo radiation and find that the effect is significant as it affects orbital evolution through interaction with spacecraft properties and that albedo should be considered if the full accuracy of the observations is to be utilized. The total effect of the Earth albedo amounts to about one-tenth ppb.

Satellite Signature

The range measured by an SLR instrument to a spherical satellite is always longer than the distance to its exterior surface. Retroreflector cubes are recessed into the body of the target, and the refractive properties of the visible array cause the laser light to effectively penetrate the target. Reflections from LAGEOS, which has an outside radius of 298 mm, would theoretically return from points between 200 and 258 mm from the satellite center [*Neubert*, 1990]. The signal which arrives at the station's receiving telescope will be a convolution of the optical transfer function with the laser pulse shape. The finally measured return distribution is skewed towards longer ranges by an amount depending on the laser pulse width and the response characteristics of the photo-detector.

Systems operating at the multi-photon return level generally use a detector, such as a micro channel plate, which is sensitive to return pulse shape. The measured time-of-flight is typically defined by the triggering of a discriminator at the pulse's leading edge. A system, which is calibrated with a similarly detected measurement from a terrestrial target at a known distance, will produce accurate observations with a noise level of a few millimeters [*Degnan*, 1985]. The evolution of future systems such as SLR2000 [*Degnan et al.*, 1997] will take a different emphasis and will rely on low light-level, eye-safe instruments, which must detect a much weaker return signal.

Single photon systems detect returns with a probability proportional to the density profile of the reflected pulse, and so individual range observations will be influenced by the skewness of the satellite signature. The noise level of the resulting measurements is higher than those from the high-energy instruments, but consistent performance can be maintained by calibrating with terrestrial measurements collected at the same energy level as the satellite returns. Accuracy can be achieved in these systems if any difference in the satellite and ground target data distribution is accommodated in the computation of the final, 'normal' measurements. When avalanche photo-diode detectors are used for increased light sensitivity, another skew tail is added to the satellite signature which will increase the noise level of the data and impose a further requirement for accurate calibration.

The instruments of the Global Laser Tracking Network undergo continuous improvements in performance. Early transmitters with longer pulses and early detection systems with lower return rates measured longer than the improved systems which now provide measurements corresponding to the expected satellite signature model. In particular, extensive upgrades of NASA-developed SLR systems were implemented in the early 1990's in preparation for the TOPEX/Poseidon altimeter mission. An elaborate TOPEX laser retroreflector array model allowed the improved systems to successfully produce TOPEX orbit height definition of unprecedented centimeter accuracy and helped to focus attention on satellite signatures which can affect results at the millimeter level.

Improvement in Scale Definition

Dunn et al. [1999] describe a series of independent annual determinations of GM from LAGEOS-I and -II data over ten years. The data between 1986 and 1992 were those used to determine the IERS92 standard, and the annual values were seen to be consistent with the results of *Ries et al.* [1992]. The scatter of values during the first five years was also compatible with the quoted uncertainty of 0.0008 km^3/sec^2. The data collected since 1990 suggested higher estimates of GM with generally better consistency from each LAGEOS satellite. The average value for LAGEOS-II was 498600.44187 km^3/sec^2, which was chosen as the most appropriate value from this analysis. The scatter of annual determinations of GM from several other geodetic satellites over a four-year period was found to contribute no absolute scale information, but their scatter provided an indication of overall network consistency for the tracking of these satellites

The scatter (standard deviations) about the mean of monthly values gives a realistic error measure, and the formal errors of single estimates give optimistic assessments. The formal errors would hold if the ranging observations were randomly distributed about the orbits at the level of the final residual fit for each satellite. The orbit fits are about 3 cm for LAGEOS-I and -II and the orbital residuals are, of course, far from random. The high range residual level is caused by unmodeled Earth, satellite and orbit errors, and as these models improve in the future, the associated uncertainty in the scale parameters which we seek will be reduced. The systematic effect of refraction errors on the determination of GM was noted in the latest analysis, whose results are shown in Figure 1. The process adopted in earlier analyses included a step in which the atmospheric pressure at the site was truncated to the nearest millibar. The inclusion of the more precise pressure measurements yielded a GM value which is about one half ppb lower than the earlier analysis, and which remains one half ppb higher than the adopted standard with an uncertainty of about the same magnitude.

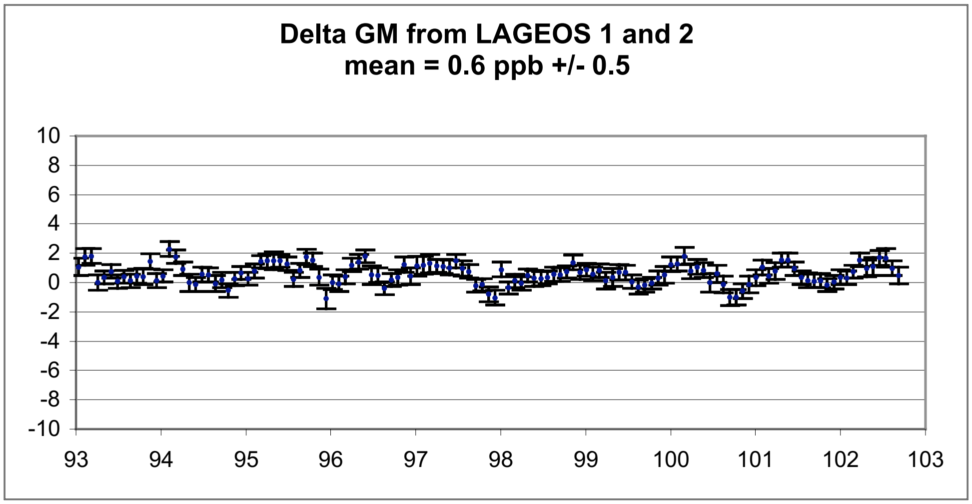

Figure 1. Scale difference from the IERS standard from ten years of data.

Conclusions

Orbit fits at the centimeter level suggest that satellite and station uncertainties currently restrict our ability to determine scale. Improved fits at the millimeter intrinsic data accuracy would yield formal errors of multi-year estimates of an order of magnitude of a few parts per trillion (ppt). At this accuracy level, improved bounds on any change in the gravitational constant (G) in time could be determined. Lunar range data has provided such a test through the lunar orbit sensitivity to solar longitude [*Dickey et al.*, 1994] who cite the suggestion of *La and Steinhardt* [1989], that the early history of the universe has seen quite large changes in G. Recent estimates of limits on Gdot/G range from 20 ppt per year from binary pulsar data [*Damour et al.*, 1988] to 4 ppt per year from solar system data [*Heilings et al.*, 1983]. An important part of the advances which would allow us to test these limits will entail full consideration of the properties of individual receiver characteristics within the network. Together with anticipated improvements in Earth positioning and orbit modeling, we can then expect ever decreasing uncertainty in the determination of Earth scale from the Global Laser Tracking Network.

References

Appleby, G., Center of Mass Corrections for LAGEOS and Etalon for Single-Photon Ranging Systems, Proc. Eurolas Meeting, Munich, 1995.

Bosworth, J.M., R.J. Coates, and T.L. Fischetti, The Development of NASA's Crustal Dynamics Project, Contributions of Space Geodesy in Geodynamics: Crustal Dynamics, AGU Geodynamics Series, 25, 1-20, 1993.

Damour, T., G.W. Gibbons, and J.H. Taylor, *Phys. Rev. Lett.*, 61,1151, 1988.

Degnan, J.J., Satellite Laser Ranging: Current Status and Future Prospects, *IEEE Transactions on Geoscience and Remote Sensing*, GE- 23, 4, 398-413, 1985.

Degnan, J.J., Millimeter Accuracy Satellite Laser Ranging, Contributions of Space Geodesy to Geodynamics: Technology, AGU Geodynamics Series, 25, 133-162, 1993.

Degnan, J.J., and J. McGarry, SLR2000: Eyesafe and autonomous satellite laser ranging at kilohertz rates, Proc. Conf. on Laser Radar Techniques, European Symposium on Aerospace Remote Sensing, 1997.

Dickey, J.O., P.L. Bender, J.E. Faller, X.X. Newhall, R.L. Ricklefs, J.G. Ries, P.J. Shelus, C. Veillet, A.L. Whipple, J.R. Wiant, J.G. Williams and C.F. Yoder, Lunar Laser Ranging: A Continuing Legacy of the Apollo Program, *Science*, 265, 22 July, 1994.

Dunn, P., M. Torrence, R. Kolenkiewicz, and D. Smith, Earth Scale defined by Modern Satellite Ranging Observations, *Geophys. Res. Lett.*, 26, 10, 1489-1492, 1999.

Ferrari et al, JGR 85, 1980.

Heilings, R.W. et al., *Phys. Rev. Lett.*, 51, 1609, 1983.

La, D., and P. J.Steinhardt, *Phys. Rev. Lett*., 62,376, 1989.

Lemoine, F.G., D.E. Smith, L. Kunz, R. Smith, E.C. Pavlis, N.K. Pavlis, S.M. Klosko, D.S. Chinn, M.H. Torrence, R.G. Williamson, C.M. Cox, K.E. Rachlin, Y.M Wang, S.C. Kenyon, R. Salman, R. Trimmer, R.H. Rapp, and R.S. Nerem, "The Development of the NASA GSFC and NIMA Joint Geopotential Model", Gravity, Geoid and Marine Geodesy, Vol. 117, International Association of Geodesy Symposia, J. Segawa, H. Fujimoto, and S. Okubo (editors), pp 461-469, 1997.

Lerch, J.F., R.E. Laubscher, S.M. Klosko, D.E. Smith, R. Kolenkiewicz, B.H. Putney, J.G. Marsh and J.E. Brownd, Determination of the geocentric gravitational constant from laser ranging on near-earth satellites, *Geophys. Res. Lett.*, 5(12), December, 1978.

Martin, C.F., M.H. Torrence, and C.W. Meisner, Relativistic Effects on an Earth-Orbiting Satellite in the Barycenter Coordinate System, *JGR* 90, 9403-9410, 1985.

Martin, C.F., and D.P. Rubincam, Earth albedo affects on LAGEOS-I satellite based on Earth radiation Budget (ERBE) satellite measurements, *EOS*, paper G32A-3, Spring Meeting of the AGU, Baltimore, MD, 1993.

Minott, P.O, T.W. Zagwodzki, T. Varghese, and M. Selden, "Prelaunch Optical Characterization of the Laser Geodynamics Satellite (LAGEOS 2)", NASA TP-3400, Sept, 1993.

Neubert, R., Satellite signature model: Application to LAGEOS and TOPEX, Proc. Eurolas Meeting, Munich, 1995.

Otsubo, T., J. Amagi and H. Kunimori, The Center-of-mass Correction of the Geodetic Satellite AJISAI for Single-photon Laser Ranging, *JGR* 1999, in press.

Putney, B.H., R. Kolenkiewicz, D.E. Smith, P.J. Dunn and M.H. Torrence, "Precision Orbit Determination at the NASA Goddard Space Flight Center", *Adv. Space Res.*, V.10, No.3, pp.197-203, 1990.

Ries, J.C., R.J. Eanes, C.K. Shum, and M.M. Watkins, Progress in the determination of the gravitational coefficient of the Earth, *Geophys. Res. Lett.*, 19(6), 529-531, 1992.

Ries, J.C., R.J. Eanes, C. Huang, B.E. Schutz, C.K. Shum, B.D. Tapley, M.M. Watkins and D.N. Yuan, Determination of the gravitational coefficient of the Earth from near-Earth satellites. *Geophys. Res. Lett.*,16(4),271-274, April 1989.

Ries, J.C., C. Huang and M.M. Watkins, Effect of General Relativity on a Near-earth Satellite in the Geocentric and Barycentric Reference Frames, *Phys. Rev. Ltrs.*, 61,8, 1988.

Scharro, R., K.F. Wakker, B.A.C. Ambrosius, and R. Noomen, On the Along-Track Acceleration of the LAGEOS Satellite, *J. Geophys. Res.*, Vol. 96, No. B1, pp. 729-740, January 10, 1991.

Smith, D.E., D.C. Christodoulidis, R. Kolenkiewicz, P.J. Dunn, S.M. Klosko, M.H. Torrence, S. Fricke and S. Blackwell, A Global Geodetic Reference Frame from LAGEOS Ranging (SL5.1AP), *JGR*, 90 (B11) 9221-9234,1985.

Smith, D.E., R. Kolenkiewicz, P.J. Dunn and M.H. Torrence, "Earth Scale Below a Part per Billion from Satellite Laser Ranging", Proc. IUGG99, G6/E03-B1, 1999.

Tapley, B.D., B. E. Schutz, and R. J. Eanes, Station Coordinates, Baselines, and Earth Rotation from LAGEOS Laser Ranging: 1976-1984, *JGR* - V.90, pp.9235-9248, 1985.