Validation of Mapping Functions

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Abstract

The present paper focuses on techniques, both numerical and experimental, which are currently used to validate mapping functions relating the zenith path delay correction to other angles of elevation. The paper concentrates on the application of two colour laser ranging data, taken with the TIGO SLR module, aiming on a determination of the zenith path delay, as well as the utilzation of numerical wheather prediction data, which is applied to the estimation of horizontal refractivity gradients. Moreover a comparison of mapping functions with respect to their applicable spectrum of wavelengths is given using raytracing techniques.

1 Introduction

As the accuracy of today's SLR systems is improving towards the millimeter level, the atmposheric contribution to the ranging error budget plays a more and more vital role in the interpretation of laser ranging data. Being dependent on an atmospheric correction formula like the famous MARINI and MURRAY model [8], which permits an estimation of the refractive delay from surface meteorological data with centimeter accuracy, the entire SLR technique seems to be limited to this level.
There are multiple proposals to overcome this shortcoming by multi colour laser ranging [1] [3] [6] being capable to measure the refraction by the propagation delay imposed on laser pulses of different wavelengths. However due to the complexity of a two colour SLR system and the high accuracy required for the differential delay, which imposes stringent limits on the usage of such two colour data, there are only three SLR stations worldwide operating in two colour mode. Nontheless the technique has matured so far that a test of refractivity models by two colour measurements becomes feasible with sufficient accuracy, which is

presented in the following chapter.

The availability of tabulated weather forecast data, which supply tabulated meteorological parameters on a grid of approximately 20 km four times a day for the european region,

2 Testing mapping functions by two colour SLR

Unlike former aproaches to the two colour ranging technique (see [1]) and for reasons of data averaging required to beat down the scatter of individual measurements, we project the atmospheric delays on the zenith path parameter. By means of the mapping function under test, we obtain the Zenith Path Differential Delay (ZPDD) from all data acquired during one satellite pass. The obtained ZPDD can be compared with the ZPDD calculated from the meteorological surface data and the discrepancies of these values give an indication how well the mapping function under test corresponds to reality. Moreover this procedure, as described below, circumvents the necessity of echoes in both wavelength channels for each individual shot. As an example we have chosen the recently published mapping function of MENDES [7] for our tests, since the mapping function and the zenith path delay are explicitly seperated.

We write the atmospheric correction R dependent on wavelength λ_i as

$$\Delta R_{\lambda_i} = m(\theta)\frac{f(\lambda_i)}{f(\lambda_1)-f(\lambda_2)}\Delta^2 R_{\lambda_1,\lambda_2}, \tag{1}$$

i.e. a product of the mapping function m dependent on elevation angle θ, the dispersion factor expressed by the wavelength dependence of the group refractive index $f(\lambda_i)$ and the ZPDD $\Delta^2 R_{\lambda_1,\lambda_2)}$. Further we parameterize the residual of the i'th measurment $\mathcal{R}_\rangle$ as

$$\mathcal{R}_\rangle = \sum_{j=1}^{4}\frac{\partial F}{\partial p_j}p_j + \frac{\partial \Delta R_{\lambda_i}}{\partial \Delta^2 R_{\lambda_1,\lambda_2}}\Delta^2 R_{\lambda_1,\lambda_2} \tag{2}$$

where $\frac{\partial F}{\partial p_j}$ are the partial derivatives of the satellite trajectory with respect to four orbit parameters, namely timebias, radial error and their first temporal derivatives. Further the residuals are parameterized by the ZPDD, as indicated by equation 1.
This parameterization is used to estimate the metioned parameters from the obtained data. Figure **??** shows residuals of one of the three Lageos-1 observations, which were used to derive the ZPDD. The residual histograms indicate good agreement with the ab initio calculated signatures, where the model of NEUBERT [5] is used. The model is corrected for the appropriate return rate, which is kept at 10The operation at 20the SPAD detector shows at lower return rates for the wavelength of 847nm. The calculated signature for 20of the

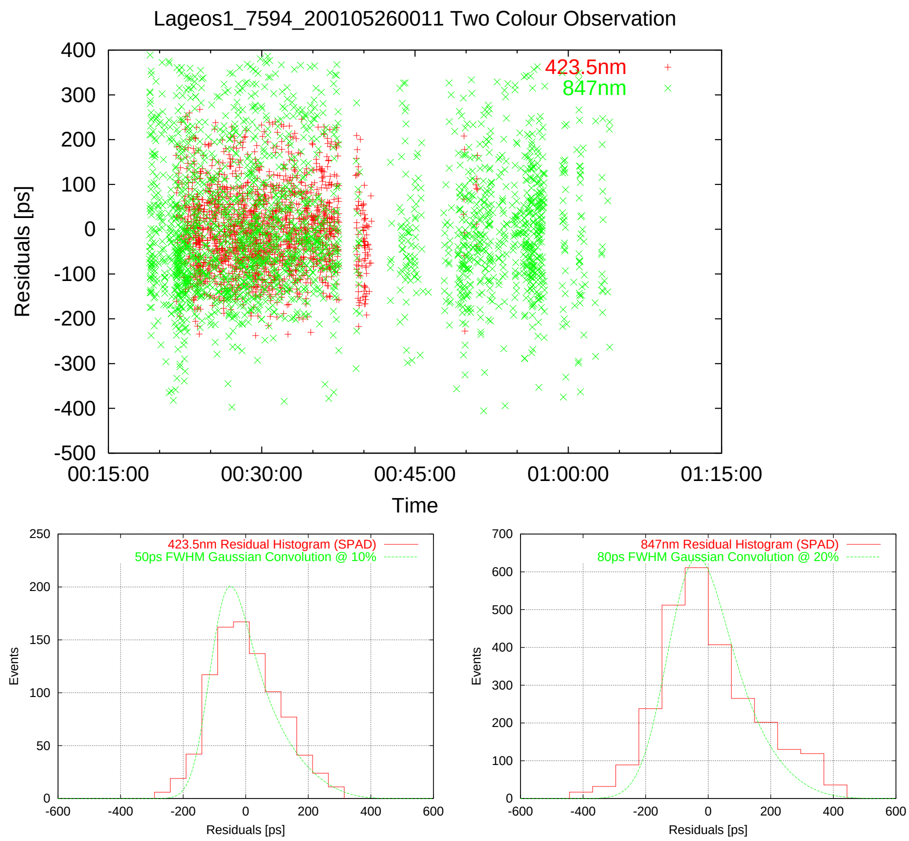

Figure 1: Residuals obtained from a two colour measurement to Lageos1 in the infrared and blue channel.

mean of the distribution with respect to the 10scintillation of the return signal strength in the infrared is about a factor 4 less than in the blue.

Table 1 presents the results for the ZPDD obtained from the three successful two colour measurements to Lageos-1. The deviations between the two colour ZPDD and the meteorological ZPDD are at the millimeter level and lie beyond the margin of the measurement accuracy, as the sigma column suggests. The reason for that is mainly due to the the incompliant treatment of dispersion in the zenith path delay formula used by the mapping function of MENDES [7] which is used here to calculate the ZPDD. Moreover one should keep in mind that the mapping function under test is tuned to the wavelength of $532nm$.
From top to bottom the number of echoes used for determination of the ZPDD raises from 1956 to 3904, so that the improved accuracy of the latter two measurments is due to the higher quantity of data. For the wavelength pair used in these measurements, i.e. 847nm and 423.5nm the dispersion factor is about 14. Therefore the entire zenith path delay for one wavelength can be determined

Date of observation	Differential Zenith Path Delay from 2 colour measurement/m	Sigma/m	Differential Zenith Path Delay from meteorology
2001-10-09 10:17	0.1689	0.0010	0.1671
2001-05-25 20:45	0.1665	0.0004	0.1672
hline 2001-05-26 00:11	0.1680	0.0003	0.1673

Table 1: Results for the differential zenith path delay obtained from two colour measurements vs. zenith path delay obtained from meteorological parameters taken at the SLR site.

with an accuracy of 4.2mm, for the best measurements obtained so far.

3 Comparison of mapping and zenith path delay functions

To compare different zenith path delay models and mapping functions, a raytracing procedure is used as a reference. The raytracing procedure uses the refractive index formula of OWENS [9] and an atmospheric model as described by MARINI and MURRAY [8]. The refraction delay models under test are

- MARINI and MURRAY [8],
- GARDNER [10], modified with a new dispersion formula with a seperate treatment of the wate vapour and updated dispersion for dry air, which enhances the validity in the UV-range,
- the recently developed mapping function of MENDES [7] which uses a zenith path delay formula of SAASTOMOINEN [11].

Figure 2 shows the result of the comparisons carried out for various wavelengths, latitudes, elevation angles of 90 and 15 degree and relative humidities of 0 and 50 percent. The comparisons are evaluated for a laser ranging station at sea level, a surface preesure of 100mb and 293 K ambient temperature.

Figure 2 indicates agreement at the millimeter level of all three models for the zenith path delay, low humidity and wavelengths from $0.5\mu m$ to $1.05\mu m$. The deviations in this wavelenght range can be explained by the usge of different refraction formulas. For wavelengths in the UV region the MARINI and MENDES model deviate from zero, which is due to their incorrect dispersion model. Going to humidity of 50 percent the deviation at larger wavelengths shifts from positive to negative sign for the MARINI and MENDES correction formula, but remains in the millimeter range. This behaviour can be attributed to the treatment of the dispersion of water vapour in these models, which is treated like that of dry air. This causes as well the higher deviation at shorter wavelengths in comparison to the dry air case.
For low elevations the qualitative behaviour of the deviations are the same as

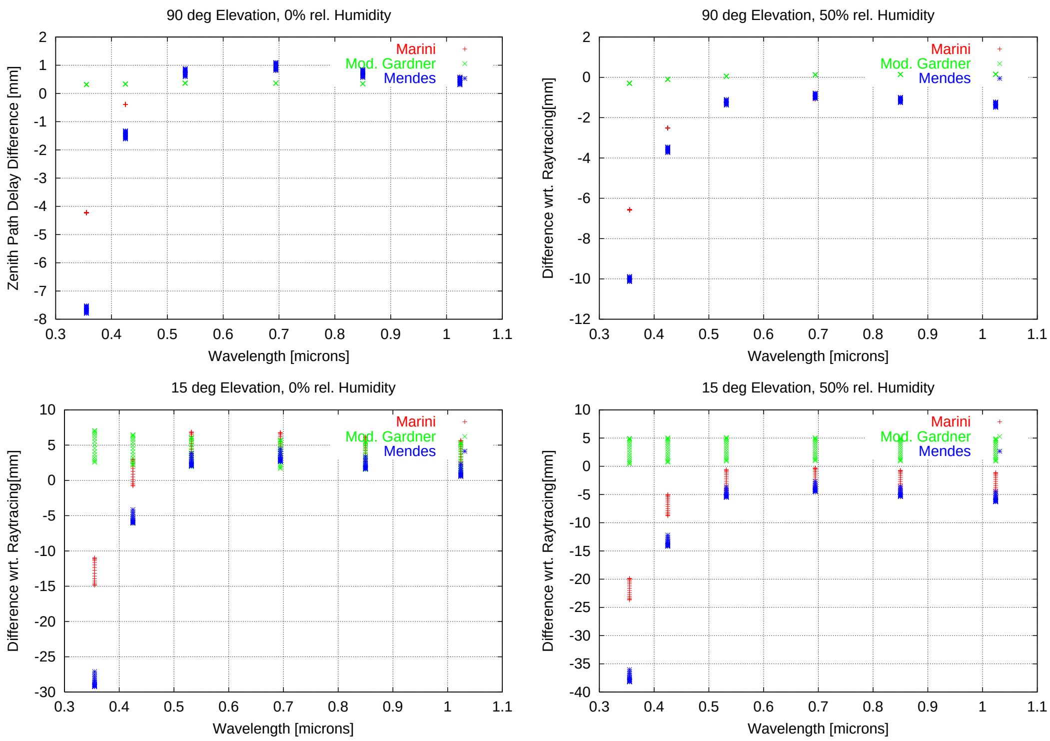

Figure 2: Comparison of 4 mapping functions and zenith path delay models. The reference is chosen to be a raytracing procedure using the model atmosphere for which the MARINI MURRAY model was derived. The mapping functions are evaluated for 8 latitudes from 0 to 90 degree for different wavelengths and elevation angles of 90 and 15 degree.

for the zenith path case but with higher amplitudes. That can be explained by the roughly secans behaviour of the atmospheric correction in terms of the elevation, so that the deviations at 15 degrees elevation are roughly 3.8 times the deviations at zenith angle.

It should be pointed out that the scatter wrt. latitude for the zenith path delay deviation is higher for the MENDES model. This is due to the improved latitude dependence of the MENDES mapping function coefficients, which are adjusted by raytracing through radio sound data from globally distributed locations. At 15 degrees elevation the scatter of the MENDES mapping function deviations is lower due to the very detailed modelling of the coefficients with respect to latitude.

4 Horizontal refractivity gradients

Considering millimeter accuracy, horizontal refractivity gardients are by far the most important contributors to the error budget of SLR. The effect of horizontal refractivity gradient has been investigated in the Haven Hop project [12] by the use of radio sounding from various locations forming a network for suppying meteorological parameters around a laser ranging site. The results indicated a more or less permanent north south gradient around the selected site with an amplitude in the range correction of 2.5cm at 10 degrees elevation. It was found that the sources for horizontal refractivity gradients can be attributed to the topographic features of the ambient terrain which distort the temperatire and preesure fields around a ranging site. To investigate this effect for a larger region, tabulated data from the german wheather forecast center, the DWD, was used. The data available offers three dimensaíonal gridded data of pressure and temperature up to a height of 23 km and covers the area shown in figure [?]. The grid size on the ground is approximately 20 times 20 km, so this suites very well the requirements for deriving refractive gradients. For every grid point the values for temperature and pressure were reduced to the sea level and numerical derivatives were formed. The data was input to a raytracing procdure capable to deal with meteorological gradient data and the difference to the conventional atmospheric correction for a spherical symmetric atmosphere was calculated.

Figure 4 displays the results for two example latitude sections. The diagrams show the correction due to the horizontal gradient correction (HGC) at the elevation angle of 20 and 15 degrees, as well as the topographic height. The amplitude of the gradient correction is highly correlated with the height profile, which is due to the topographic influences on pressure and temperature. In regions of high slopes there are peak values of up to 30 cm of the gradient correction at 15 degrees, talking in absolute values. In regions of plane topography or over the sea we find the gradient correction much smoother than in the alpine regions and the value distribute around zero with peak values of about 3 and 2 cm at 15 and 20 degrees elevation respectively. For the location of the SLR stations at 43.8 degree latitude for Grasse in the upper diagram and at 47.1

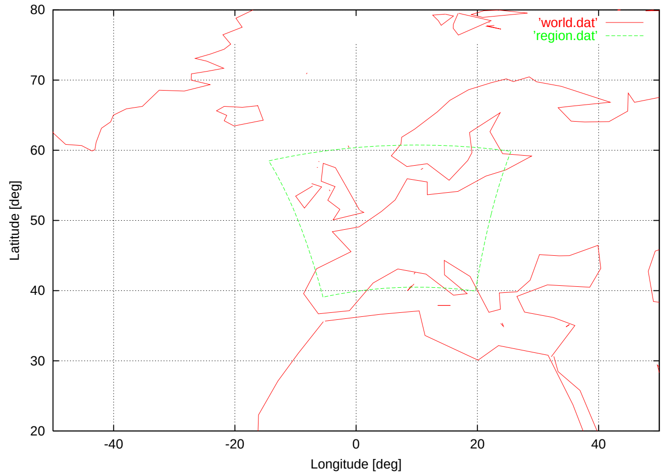

Figure 3: The region for which the "Deutsche Wetterdienst" provides gridded meteorological data of the numerical weather forecast.

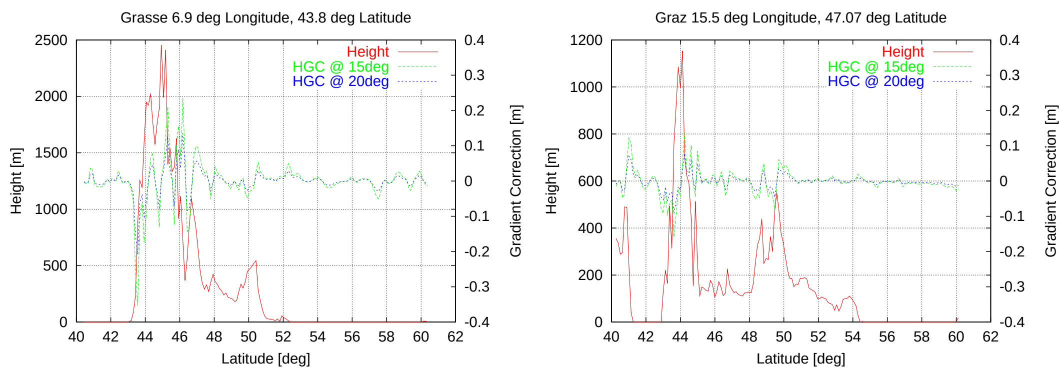

Figure 4: Two example latitude sections, one for the longitude of Grasse, one for the longitude of Graz, display the dependence of the refractive gradient correction with respect to the topography. The corrections were calculated for the data set of the 1'st of February 2001.

degree latitude for Graz in the lower diagram we find a gradient correction of -15cm and 2cm at 15 degrees elevation. At 20 degrees elevation the correction is -8 and 1 cm for the two stations.

5 Conclusion and Outlook

The conclusion and outlook of this paper can be summarized as follows:

- Two colour zenith path esitimations agree with MENDES zenith path values better than 1cm. It seems feasible to test the refractive zenith path delay at the millimeter level if one considers two colour tracking of high orbiting satellites and/or the use of higher repetition rates.

- The MENDES mapping function underestimates atmospheric correction at wavelengths smaller than 532nm. It reproduces more or less zenith path delay of MARINI and MURRAY with the same apparently erroneously behaviour with respect to a varying water vapour content. Respectively to zenith delays calculated from the GARDNER refraction model and the raytracing through a model profile, this causes a deviation of up to 1mm for low humidity down to approximately -2 mm for high humidity and wavelenghts larger than or equal to 532nm. The error muliplies as expected at smaller elevation angles.

- Numerical weather prediction data is used to evaluate horizontal refractivity gradients all over Europe. For the two test sites, for which the gradient correction was derived as an example, the analysis shows a highly variable gradient correction, presumably in regions with hill valley structures in the surrounding terrain, which cause ranging errors of up to 15cm at 15 degrees elevation.

- Aiming at one millimeter accuracy at 15 degrees elevation, the analysis of horizontal gradient effects supposes the necessity to be included into the SLR standard correction procedures. As proposed by [12] this can be achieved by a network of meteorolgical stations around the SLR sites.

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