

GPS Sequential Orbit Determination Using Sparse SLR Data

Richard S. Hujsak¹, G. Charmaine Gilbreath², Son Truong³

A coordinated Satellite Laser Ranging campaign was conducted in late 1996 to collect tracking data for Global Positioning System satellites GPS-35 and GPS-36. The tracking data collected is sparse, averaging one or two SLR tracks per day, with data gaps up to five days in duration. Typically this data density is insufficient for accurate orbit determination. Our objectives are to process this tracking data, determine the accuracy that is achievable with sparse data, both in a global sense, and locally, over the SLR sites, and demonstrate the utility of sparse SLR data for future applications. This investigation benefits from two innovations. The first is the addition of the Naval Research Laboratory laser ranging system at the USAF Starfire Optical Range, which provided enough additional tracking data to make precision orbit determination possible. The second is the use of a sequential filter-smoother to process the SLR data. A filter-smoother is used because it should provide the best combination of short arc and long arc orbit determination, with the best accuracy predictably over the SLR sites. Position accuracy better than 1.25 meters was achieved globally and better than 40 cm when measured over selected SLR sites. These results suggest additional applications for SLR data, where a few sites can support satellite programs seeking 1-10 meter accuracy.

1 INTRODUCTION

The Satellite Laser Ranging (SLR) community actively tracks two Global Positioning System (GPS) satellites, designated GPS-35 and GPS-36. These satellites were equipped with laser retroreflectors as a technology demonstration, intending that SLR data be used to independently validate GPS radiometric ephemeris accuracy. For a variety of reasons, the SLR tracking data for GPS satellites has always been sparse, never of sufficient density to determine definitive orbits for these satellites on a routine basis. As a consequence, routine, global, independent verification of GPS ephemeris accuracy has not been demonstrated and there probably will not be any more GPS satellites equipped with laser retroreflectors.

In this investigation, the task is to discover the achievable orbit accuracy with sparse SLR data, both in terms of global accuracy and accuracy over SLR sites. A GPS tracking

¹ Logicon, 760 Constitution Dr., Suite 30, Exton, PA, 19341, USA, Tel: (610) 458-3330, e-mail „dhujak@logicon.com“ or „rhujak@cwix.com“.

² US Naval Research Laboratory, Code 8123, 4555 Overlook Ave. SW, Washington DC 20375, USA, Tel: (202) 767-0170, e-mail: „gilbreath@rsd.nrl.navy.mil“.

³ 301 Colonial Dr., Exton, PA 19341, USA, Tel: (610) 524-8623, e-mail „son.truong@cwix.com“.

campaign was conducted to support the analysis and IGS ephemerides were selected as definitive references. If a reliable repeatable accuracy of 1-2 meters can be demonstrated with sparse SLR data, then low density SLR tracking becomes a viable alternative for satellite programs seeking 1-10 meter accuracy.

This investigation uses a sequential filter-smoother to determine the orbits for these GPS satellites using SLR data exclusively. This approach is based upon previous analyses by the authors in applying filter-smoother methods to SLR orbit determination for TOPEX, both with robust SLR data¹ and then with a minimal subset of SLR data². It was established in those studies that a filter-smoother could provide comparable accuracy to the NASA POE for TOPEX with dense SLR data. It was also established that the filter-smoother could be applied to sparse data and achieve precise three-dimensional position solutions over the SLR sites. For the current investigation, the objective is to determine the three-dimensional accuracy which can be achieved over the SLR ground sites with sparse SLR data. This work extends and improves upon a previous paper³ by the authors.

This investigation was enabled by the advent of the SLR capability integrated by the Naval Research Laboratory at the USAF Starfire Optical Range (NRL@SOR). This capability established an extremely robust link (300 mJ of doubled Nd-YAG with a 3.5 meter telescope), with precision and accuracy commensurate with highest ranking SLR sites worldwide^{4,5,6}. The availability of this SLR site helped motivate a coordinated campaign in the western U.S. and Pacific to sample the GPS-35 and GPS-36 orbits in a systematic way over the period of Oct. 28 through November 26, 1996.

The tracking data is very sparse, particularly when compared to low altitude geodetic satellite programs. GPS-35 received 56 SLR tracks in the 30 day span, while GPS-36 received 38 tracks. At most there might be three tracks per day, with data gaps up to five days long. After processing with the filter and smoother, the smoothed ephemeris was compared to the corresponding IGS ephemeris, and the differences accumulated as r.m.s. statistics in radial, intrack, and crosstrack directions. The accuracy achieved was:

	Radial r.m.s. (cm)	Intrack r.m.s. (cm)	Crosstrack r.m.s. (cm)
Global:			
GPS-35	18	102	23
GPS-36	15	123	36
Over Site:			
GPS-35	12	56	19
GPS-36	7	65	41

Global statistics improve to better than a meter when there is at least one track of SLR data per day, over consecutive days, and when there are two or three sites tracking on the same day. Accuracy statistics over the site improve to 40 cm where there is daily tracking.

The importance of these results is not in the GPS accuracy achieved, but in the capability implied to support other satellite programs, those seeking one-to-ten meter accuracy globally. These results suggest that very few SLR sites could be used to provide a

definitive ephemeris with a global accuracy on the order of one-to-two meters. The same subset of SLR sites could also be used to independently validate ephemeris accuracy at the sub-meter level by performing the validation over selected SLR sites.

2 CONTRIBUTING STATIONS

2.1 NRL@SOR

The Naval Research Laboratory, together with the Starfire Optical Range and NASA integrated NRL's SLR transmit/receive system into the 3.5 meter telescope at SOR. First returns from LAGEOS were obtained March, 1995. The NRL@SOR SLR system is comprised of a 300 mJ doubled Nd-YAG laser with a 250 ps pulsewidth, firing at a rate of 10 Hz. A photomultiplier tube with amplifier was placed in the exit pupil plane of the optics used to bring the light from the 3.5 meter collecting aperture to the receiver system. The system used polarization aperture sharing to transmit and receive along the same path. The system used chromatic aperture sharing to use white light, less the 532 nanometer wavelength, to do in-line, real-time acquisition and tracking for objects in terminator mode. System precision and accuracy were comparable to MOB LAS 7^{4,5}. The Energy-Area Figure-of-Merit of the system is 65 times that of the standard NASA MOB LAS site. Hence, obtaining unambiguous returns from the GPS spacecraft was possible in daylight as well as at night. For this campaign NRL@SOR was able to dedicate tracking to the spacecraft for optimal short arc sampling at the site. NRL@SOR did periodically point to LAGEOS and TOPEX for calibration purposes.

2.2 STANDARD SLR STATIONS

In addition to NRL@SOR, there were four SLR stations participating in the November 1996 tracking campaign. These are Haleakala Laser Ranging System, HI (HOLL), Monument Peak, CA (MNPE), McDonald, TX (MCDO), and Simosato Hydrographic Observatory, Japan (SHO).

There were 56 tracks for GPS-35 and 38 tracks for GPS-36, collected between Oct 28 and Nov 30.

Three of the SLR sites, MNPE, NRL@SOR, and MCDO, all lie in southwestern continental US (CONUS), and provide a relatively weak geometric baseline to use in tracking satellites at slant ranges greater than 20,000 km. When any of these sites are combined with HOLL or SHO the geographical baseline improves significantly.

3 ANALYSIS SOFTWARE

3.1 FILTER AND SMOOTHER DESCRIPTION

The sequential filter used in this analysis is a locally-linearized Extended Kalman Filter, with some modifications. The process noise model is derived from physical error sources in the same sense as consider parameters in least squares applications. A modification employed here includes the second order terms in the range measurement error variance (see Gelb⁷, pp. 191-192). State space consists of equinoctial orbit elements for the GPS satellite, a solar pressure scale factor correction and a solar pressure V-bias correction (see Section Force Models), and a pass-dependent Gauss-Markov range bias for each tracking station.

The smoother is a modified, extended Rauch Tung Striebel⁸ smoother, with the linear transition of the orbit sub-state replaced by a non-linear integration of the equations of motion and the introduction of process noise into the smoother formulation.

The analysis strategy is to filter all of the tracking data for thirty five days with the forward-running filter. The filter states and covariances are stored on a fifteen minute grid (and on the measurement grid) and passed to the smoother. The smoother processes the filter outputs in reverse chronological order, constraining the estimates and covariances to be smoothed over the entire thirty five day span. The last five days are discarded, to allow for smoother convergence, and the first thirty days of smoothed ephemeris are used in this report.

3.2 FORCE MODELS

The GPS orbits are propagated by numerically integrating gravitational accelerations due to the Earth, Moon, and Sun, together with the accelerations due to solar pressure. The gravity model is a 20×20 subset of the JGM-2 model. The positions of the Earth, Moon, and Sun are taken from the JPL DE200 ephemeris. The Bar-Sever solar pressure model⁹ is used, with the traditional Y-bias parameter replaced by a V-bias parameter. Smaller forces such as Earth's albedo are neglected in this analysis.

3.3 LASER MEASUREMENT MODELS

Two-way laser range is computed from the time tagged round-trip time from laser firing to photon detection. These data are corrected for the optical path length inside the telescope and mount before⁷ being processed for orbit determination. In this case the filter-smoother applies corrections for atmospheric delays, the offset of the laser retroreflector from the center-of-mass of the GPS satellite, and for solid earth tides which influence station locations. (A simplified model for GPS attitude is used to correct for the retroreflector offset from satellite center of gravity.) In addition, the filter estimates a range bias for each SLR station, and that bias is applied as a correction. Smaller effects such as ocean tidal influence on station locations are omitted.

4 ACCURACY ANALYSIS

4.1 GENERAL ORBIT ACCURACY FOR GPS-35

Accuracy is measured by differencing the smoother post-fit ephemeris with the definitive IGS reference obtained from NASA's archive. Figure 1 and Figure 2 provide radial and intrack position differences and sigma values for GPS-35. Ephemeris differences are graphed as bold lines, while the symmetric one-sigma error bounds are lighter in appearance. The one-sigma values are computed dynamically by the smoother as an integral part of the orbit determination process. Tracking data histories are also provided as dots (the length of each track is not evident on this scale). The covariance is included to clarify the relationship between tracking data distribution and orbit accuracy.

The covariance exhibits a natural periodicity, varying once-per-rev in the intrack direction and twice per rev in crosstrack and radial directions. The smallest covariance values align with SLR tracking events. This behavior is characteristic of the filter-smoother in sparse data applications.

For GPS-35, there are four gaps in the tracking data of more than one day. The most dramatic growth in the covariance occurs when there is no tracking data. The radial one-sigma bounds in Figure 1 collapse with every SLR track to less than 25 cm, one-sigma, in most cases. This is as expected, since the laser range measures the radial position component directly.

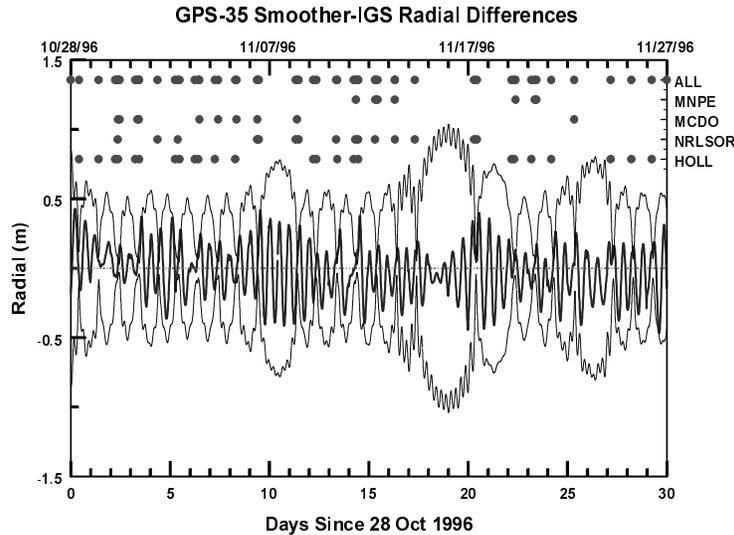


Figure 1 GPS-35 Radial Position Error, One-Sigma Bounds & Tracking Schedule

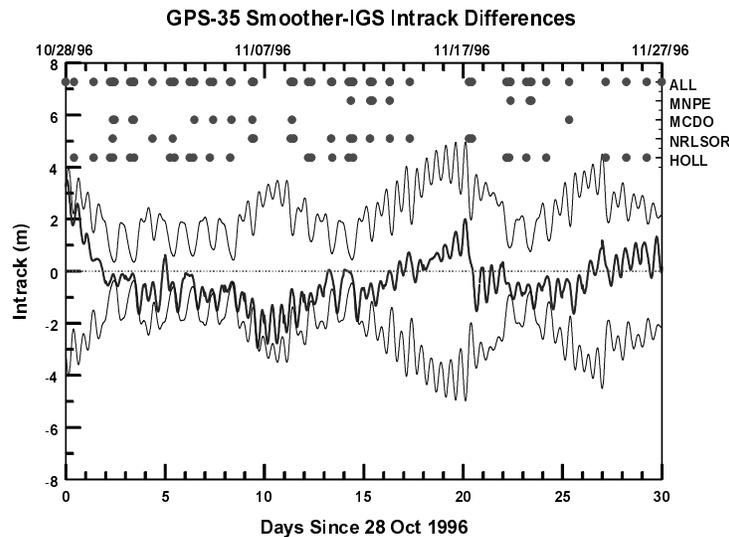


Figure 2 GPS-35 Intrack Position Error, One-Sigma Bounds & Tracking Schedule

Figure 2 illustrates many of the classical „rules of thumb“ of orbit determination, better accuracy is achieved with more SLR tracks and longer baselines between stations. Note how the best accuracy is achieved when HOLL is paired with any of the CONUS stations

on the same day and degrades when there is only one SLR site or when two CONUS sites are tracking at the same day (e.g. MNPE and NRL@SOR).

The differences graphed above were collected as r.m.s. statistics, using differences on a uniform grid of one point per 3 minutes, without regard to data gaps. This calculation gives a global accuracy of 18 cm, 102 cm, and 23 cm, r.m.s., in radial, intrack, and crosstrack position components, respectively.

4.2 GPS-35 ACCURACY STATISTICS OVER SLR SITES

To visualize ephemeris accuracy over the SLR sites, Figure 1 and Figure 2 were redrawn with the covariance omitted and the ephemeris differences plotted only when SLR tracking data was reported. The result is Figure 3 and Figure 4.

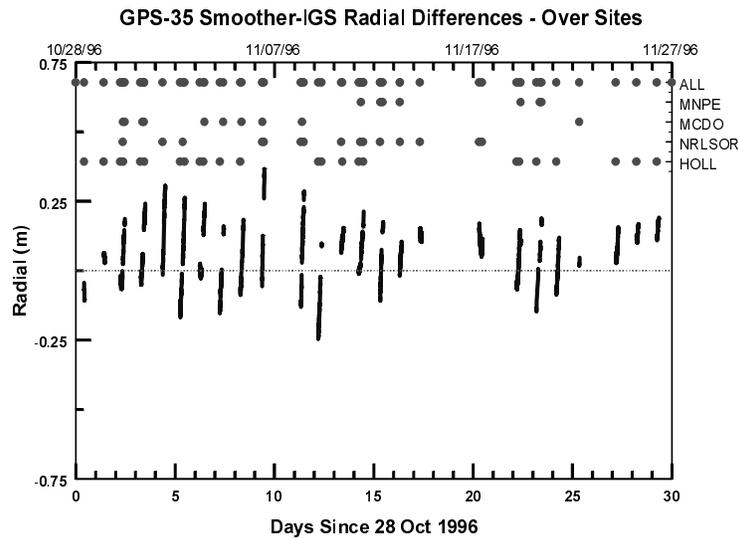


Figure 3 Radial Differences Over SLR Sites

A statistical summary of these graphs is formed by constructing the r.m.s. of the over-site errors in each component, accumulated at the density of the tracking data itself. These statistics are collected without regard to the number of tracks per day, or the number of tracks on adjacent days, or length of track. This calculation gives an accuracy over the SLR stations of 12 cm, 56 cm, and 19 cm, r.m.s., in radial, intrack and crosstrack components, respectively.

If accuracy over the site is an objective in a sparse data environment, then it is likely that accuracy can be measured over selected SLR tracks. Recomputing accuracy over the site and omitting tracks bounding data gaps reduces the intrack r.m.s. to 30 cm.

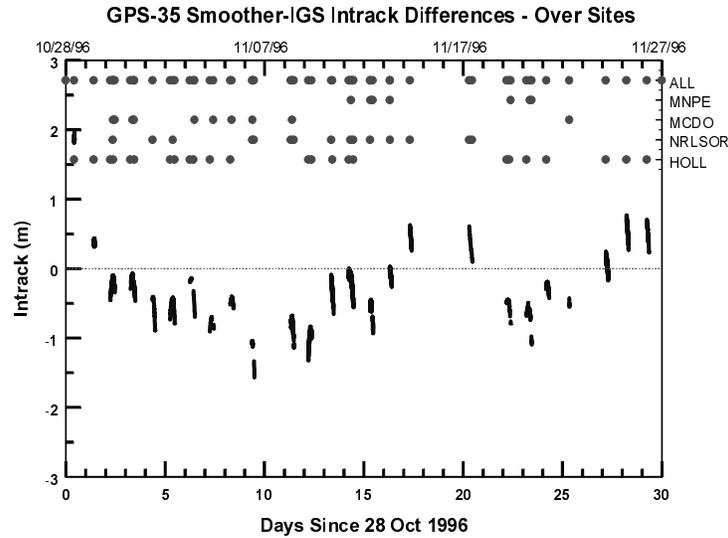


Figure 4 Intrack Differences Over SLR Sites

4.3 GENERAL ORBIT ACCURACY FOR GPS-36

Global accuracy for GPS-36 is similar to GPS-35, except that GPS-36 has fewer tracks, larger data gaps, and fewer days where there are multiple tracks (see Figure 5).

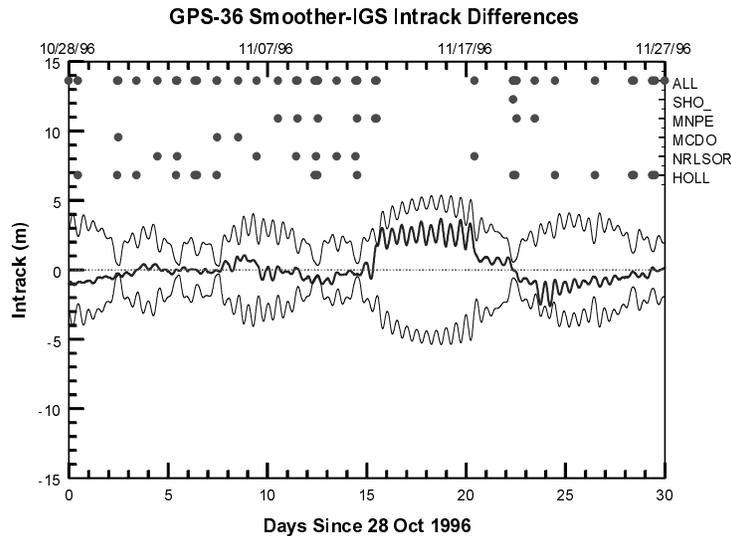


Figure 5 GPS-36 Intrack Position Error, One-Sigma Bounds & Tracking Schedule

Global accuracy, computed as an r.m.s., regardless of data gaps, is 15, 123, and 36 cm, in radial, intrack, and crosstrack position components, respectively. These statistics could be improved dramatically if the data gap from Nov 12-17 could be excluded.

4.4 GPS-36 ACCURACY OVER THE STATION, THREE STATIONS PER REV

Figure 6 is provided as an example of the way filter-smoother accuracy improves when there are three SLR sites contributing. This is the case with the best geographic coverage,

with SHO (Japan) tracking GPS-36 on the same rev as HOLL and MNPE. Again, the bold line is ephemeris differences and the faint lines are covariance values.

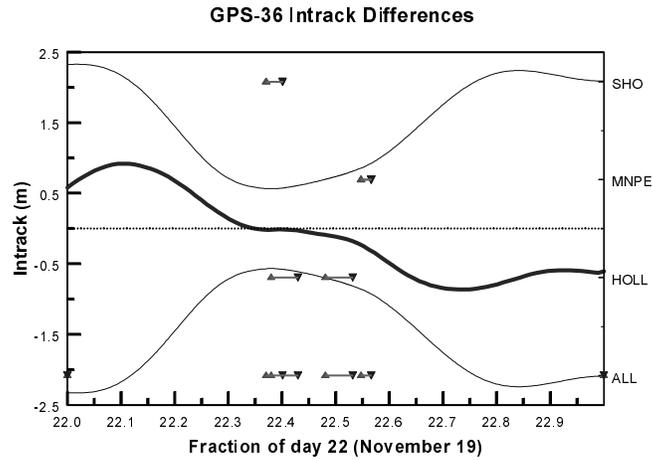


Figure 6 GPS-36 Intrack Position Error, With SHO

4.5 GPS-36 CUMULATIVE ACCURACY STATISTICS OVER THE STATION

Figure 7 and Figure 8 depict radial and intrack ephemeris differences over the SLR sites:

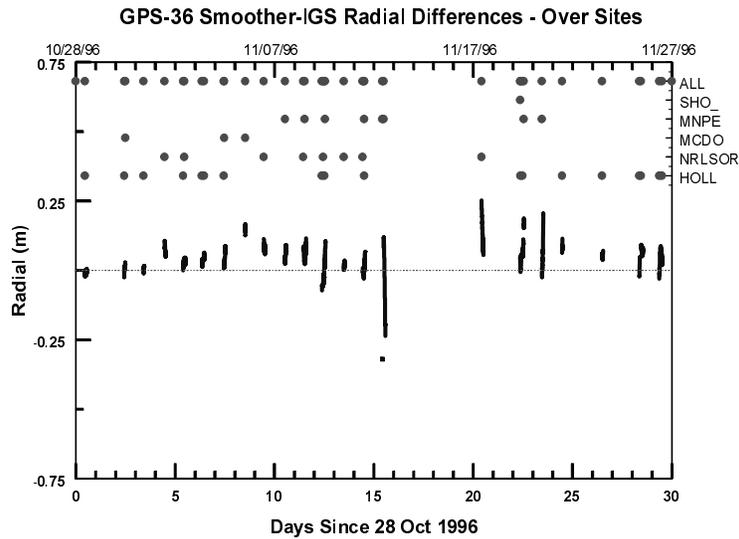


Figure 7 Radial Differences Over SLR Sites

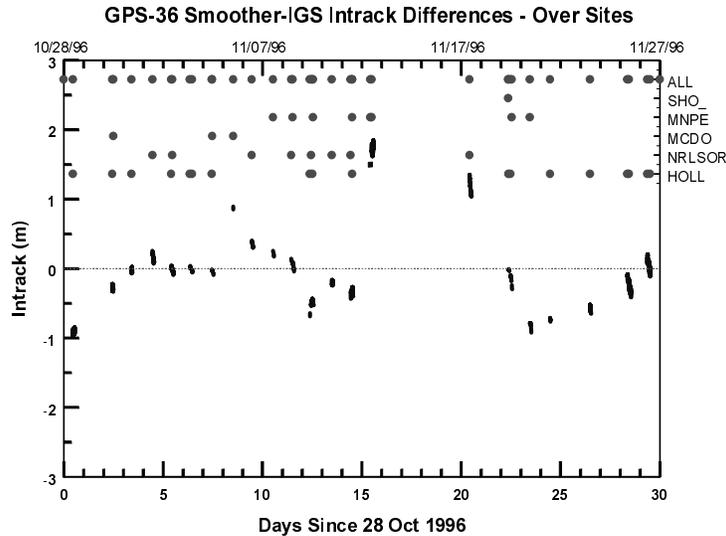


Figure 8 Intrack Differences Over SLR Sites

The over-site statistics were accumulated as for GPS-36, on the measurement grid, without regard for track length, number of SLR stations, or proximity to the data gap. This calculation gives an over-site accuracy of 7 cm, 65 cm, and 41 cm, r.m.s., in radial, intrack, and crosstrack position components, respectively. These statistics could be improved intrack if the two passes on either side of the 5-day data gap are omitted.

5 CONCLUSIONS

The SLR campaign in the autumn of 1996 has yielded sufficient data to compute SLR-derived ephemerides which agree with the IGS ephemerides at the meter level globally and at a sub-meter level over the SLR sites. It is believed that this is the first time that an SLR-only solution has been generated to independently calculate GPS ephemerides and achieve this level of accuracy and precision.

The best SLR ephemeris accuracy is achieved when there is at least one SLR pass per day, day after day, and within that sequence when there is good geometric coverage from two or three stations.

The accuracy of the SLR-derived ephemerides was measured in two ways, globally and over the SLR sites. Those statistics are summarized in the Introduction.

While these results are new, they probably will not motivate the GPS community to adopt satellite laser ranging as the IV&V method for future GPS missions. GPS tracking systems have matured and alternative methods of verifying accuracy have been developed in the absence of definitive SLR ephemerides.

The importance of these results might be in the prospect for applying Satellite Laser Ranging to new satellite missions, missions which may require accuracy on the order of 1-10 meters. For most of these programs the operational tracking system will probably be radiometric. However many of these programs will need to verify the accuracy of the radiometric ephemeris, and SLR is a viable candidate for that mission, provided that costs can be minimized. In general, cost is proportional to the number of tracking stations

employed, so the challenge is to use a minimum number of stations. This paper establishes that a few laser sites can provide global accuracy on the order of 1-2 meters. This paper also establishes that sub-meter accuracy can be obtained by using the solutions obtained over the SLR sites.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

1. R. S. Hujsak and G. C. Gilbreath, „Filtering TOPEX SLR Data,“ AAS 95-230, presented at AAS/AIAA Spaceflight Mechanics Meeting, Feb. 1995.
2. R. S. Hujsak and G. C. Gilbreath, „Using a Single SLR Site to Validate GPS Receiver Navigation,“, *Advances in the Astronautical Sciences*, Vol. 90, pp. 1173-1185, 1995.
- 3 R. S. Hujsak and G. C. Gilbreath, „Sequential Orbit Determination For GPS-35 And GPS-36 Using SLR Data From NRL@SOR“ *SPIE Proceedings*, 3380 (32), April 1998.
4. G. C. Gilbreath, M. A. Davis, P. Rolsma, T. Meehan, R. Eichinger, J. M. Meehan, „NRL@SOR: Satellite Laser Ranging with Robust Links,“ *SPIE Proceedings*, 3065 (51), pp. 116-130, 1997.
5. M. A. Davis and G. C. Gilbreath, „NRL Ranging Accuracy Analysis for NRL@SOR SLR Capability,“ *SPIE Proceedings*, 3380 (29), April 1998.
- 6 M. A. Davis, G. C. Gilbreath, P. Rolsma, J. Georges, G. Snyder, R. Eichinger, „NRL@SOR SLR Observation Techniques Using LAGEOS“, Proceedings 11th International Workshop on Laser Ranging, Sept 1998.
7. A. Gelb, *Applied Optimal Estimation*, The M. I. T. Press, Massachusetts Institute of Technology, Boston, 1974.
8. H. Rauch, „Solutions to the Linear Smoothing Problem,“ IEEE, *Trans. Autom. Control*, Vol. AC-8, pp. 371, 1963.
- 9 Y. E. Bar-Sever, „New and Improved Solar pressure Radiation models for GPS Satellites based on Flight Data,‘ Jet Propulsion Laboratory Report, (for Air Force Material Command), April 12, 1997.