Earth Orientation and Relativity Parameters Determined from LLR Data

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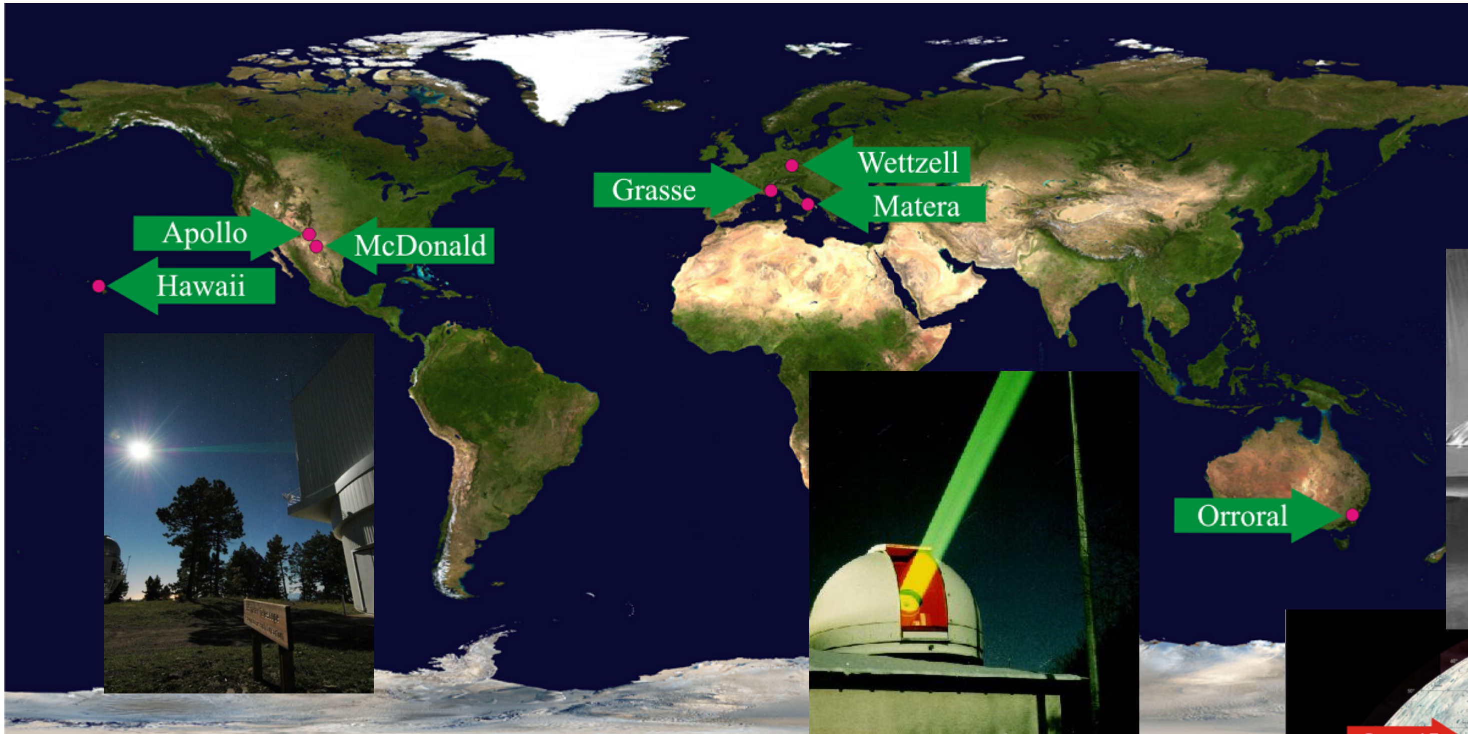
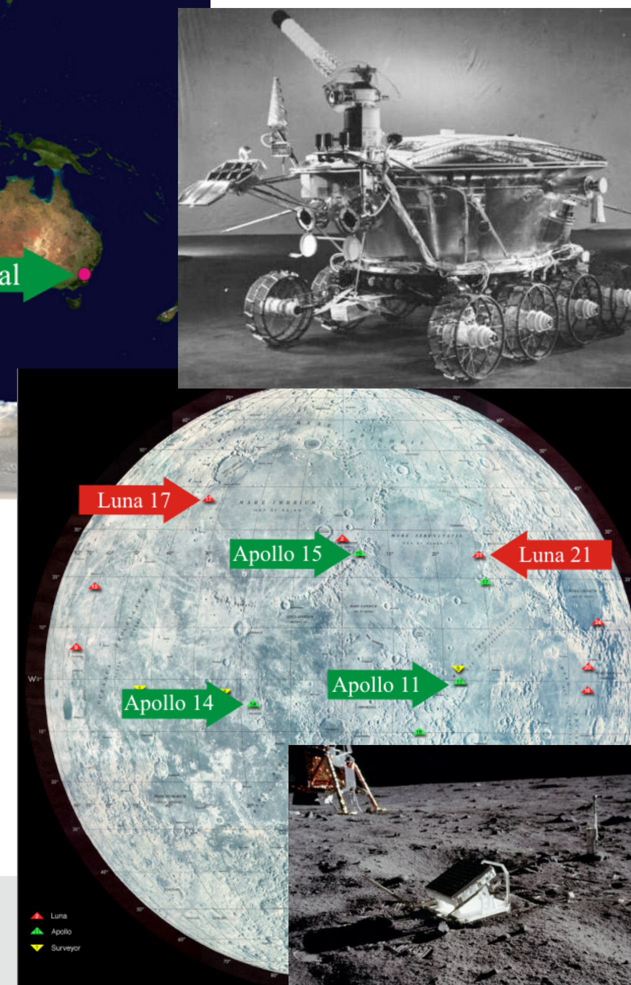
Lunar Laser Ranging – in general





- 45 years of observations
- Post-Newtonian model at cm level
- high long-term stability (orbit, reference frames, **Earth orientation**)
- **relativity tests**

LLR parameter fit

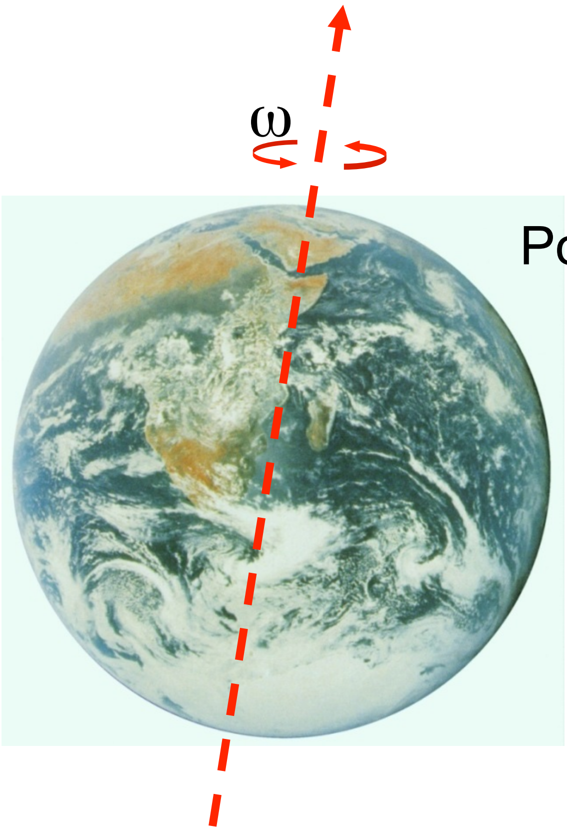
Analysis

- model based upon
  - Einstein's theory (orbit, rotation, signal propagation, time scales, reference systems, etc.)
  - Consistent application of IERS Conventions
- least-squares adjustment
- parameter estimation (about 200 unknowns, without EOPs) applying various constraints

Results (this talk)

- Pole coordinates x_p, y_p and Earth rotation phase ΔUT
- Nutation coefficients for selected periods
- Relativity test: Example $\dot{G}/G$, …

Components of Earth Orientation

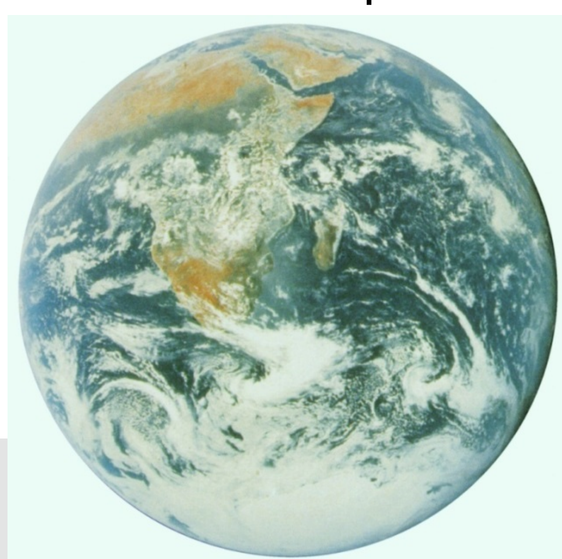


Polar motion

Precession

nutation

Earth rotation phase ΔUT



1 ms = 50 cm
at equator

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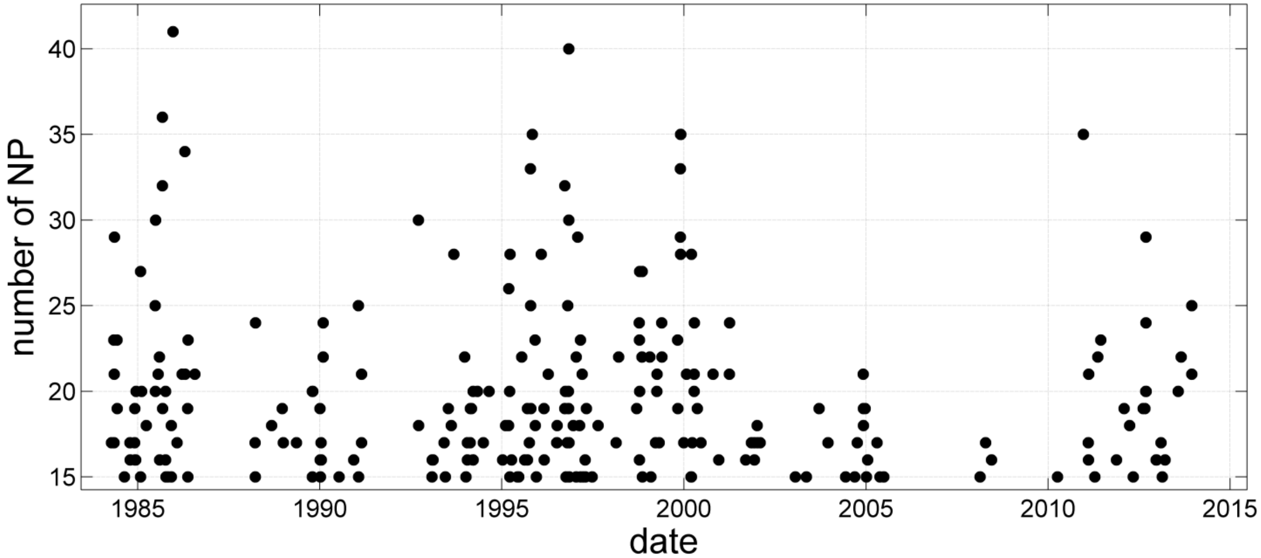
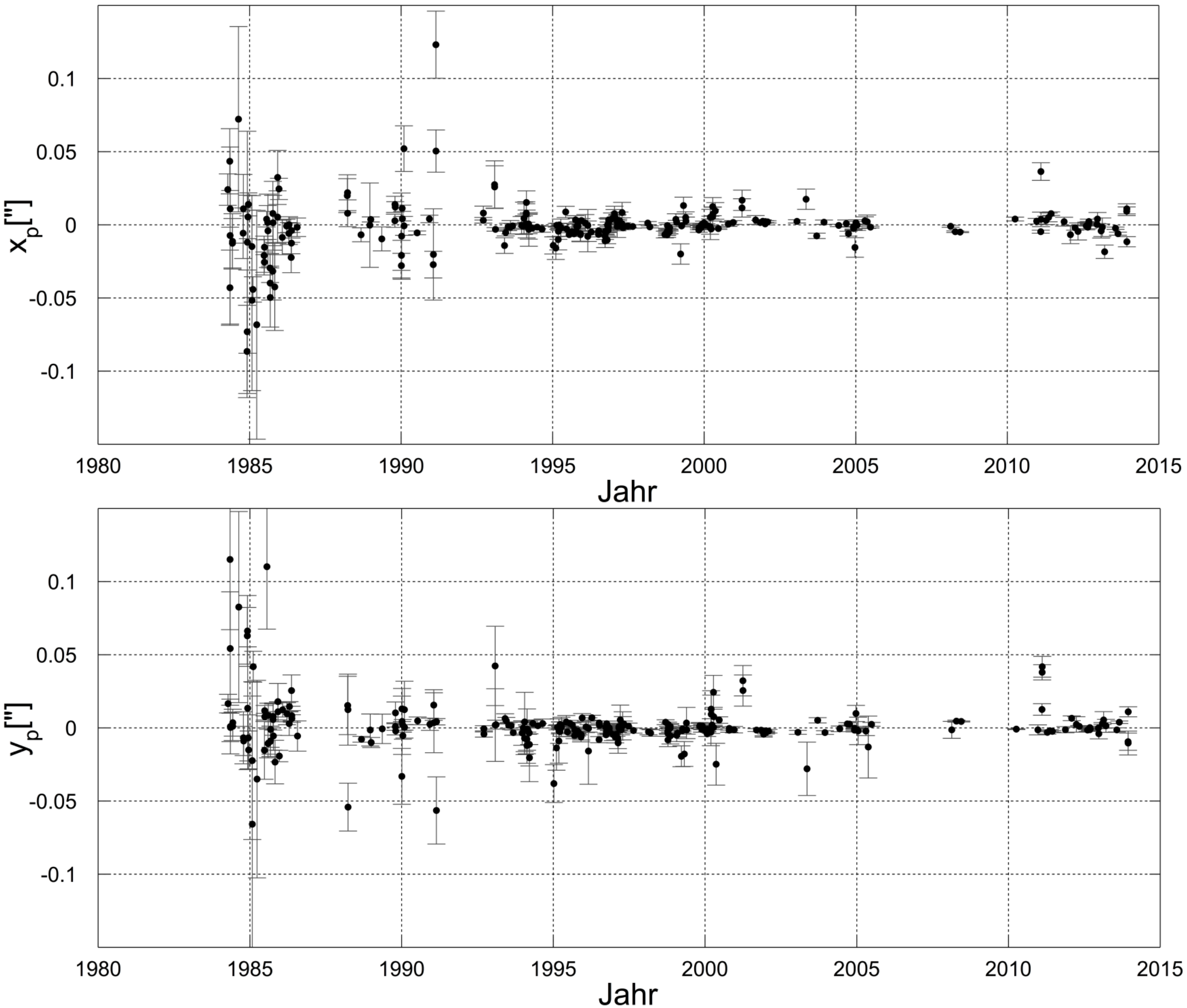
Pole coordinates (x_p, y_p) from LLR – all sites

Characteristics of the fit

- 15 NPs per night for the time span 4/1984 – 12/2013 (247 n.)
- Simultaneous determination of one component of the pole coordinates and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered



Pole coordinates (x_p, y_p) from LLR – all sites



Accuracy
1-14 mas

Correlation with coordinates of the observatories up to 10%

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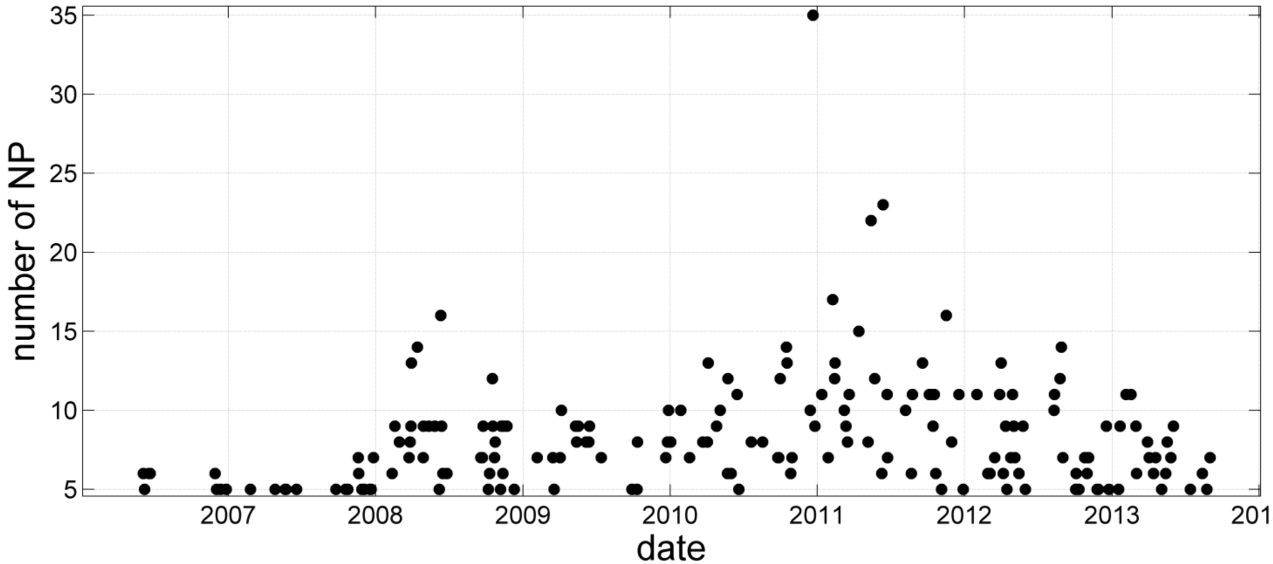
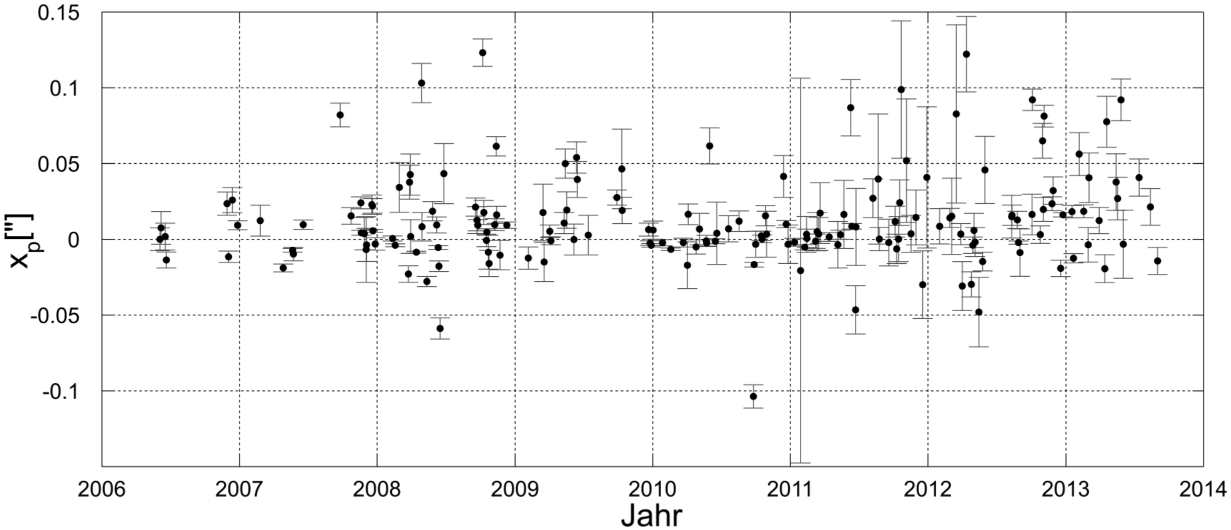
Pole coordinates (x_p, y_p) – only APOLLO

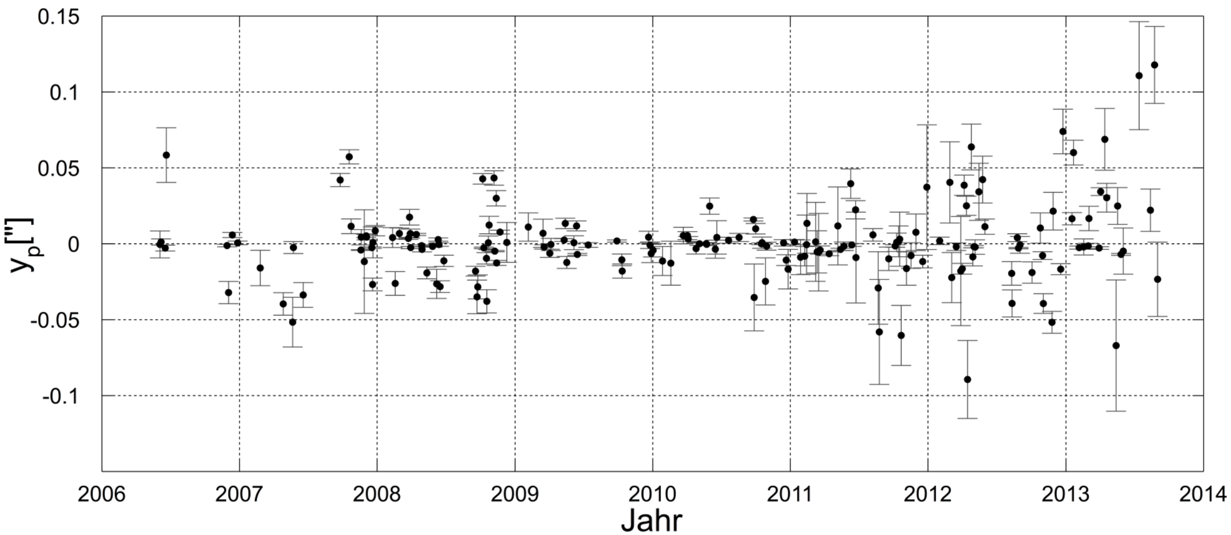
Characteristics of the fit

- 5 NPs per night for the time span 6/2006 – 9/2013 (182 n.)
- Simultaneous determination of pole coordinates and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered



Pole coordinates (x_p, y_p) – only APOLLO





Accuracy
0.5 – 40 mas,
less after 12/2010

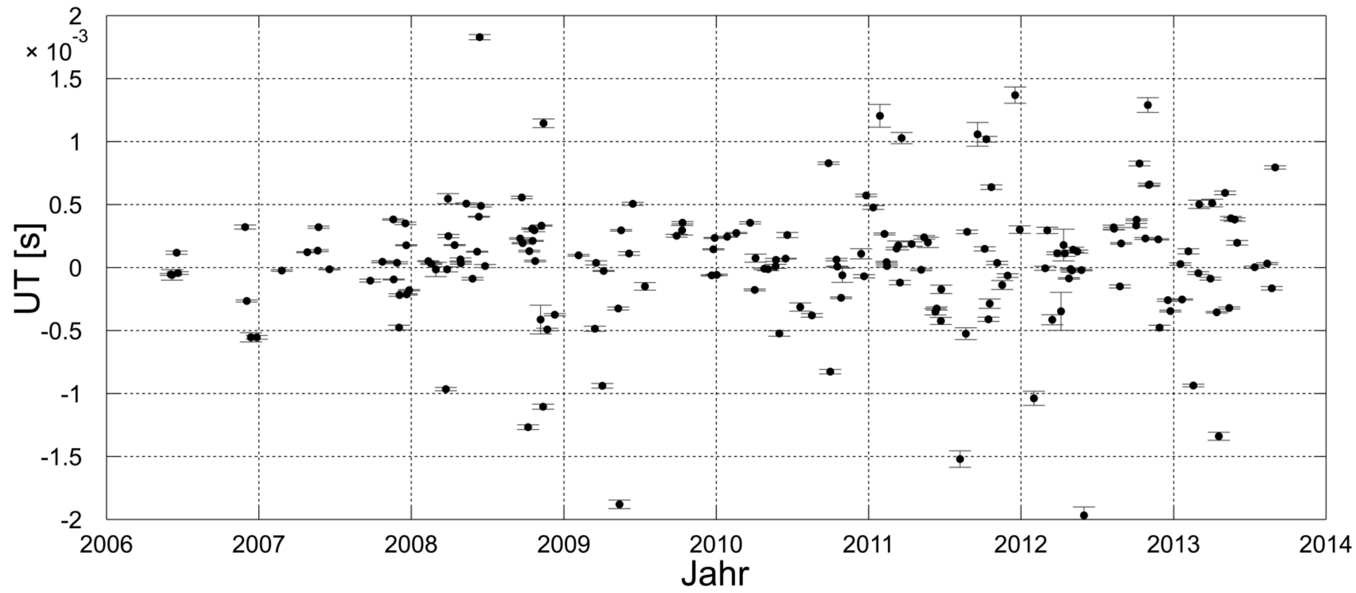
Correlation
with coordinates of the observatories up to 20-40%,
with each other 20-40%

ΔUT from LLR – only APOLLO

Characteristics of the fit

- 5 NP per night for the time span 6/2006 – 9/2013
- Simultaneous determination of ΔUT and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered

ΔUT from LLR – only APOLLO



Accuracy 0.003 – 0.05 ms
less after 12/2010 due to reduced NP accuracy
Correlation
with coordinates of the observatories up to 50%,
with other ΔUT values 20 – 40%
→ best LLR results can be used to validate VLBI

Nutation determined from only LLR data

Initials: precession and nutation according to IAU Resolution 2006 and IERS Conventions 2010

Use of **different realizations** of precession/nutation for ICRS-ITRS transformation

Fit of luni-solar nutation coefficients from 44 years of LLR data for nutation periods of 18.6 years, 9.3 years, 1 year, 182.6 days, (13.6 days), i.e. estimation of various A', A'', B', B'' of

$$\Delta\psi = \sum_{i=1}^{N} \left(A_i + A_i' t\right) \sin\left(ARG\right) + \left(A_i'' + A_i''' t\right) \cos\left(ARG\right)$$

$$\Delta\varepsilon = \sum_{i=15}^{N} \left(B_i + B_i' t\right) \cos\left(ARG\right) + \left(B_i'' + B_i''' t\right) \sin\left(ARG\right)$$

$$ARG = \sum_{i} N_j F_j$$

N_j : multiplier, F_j : Delaunay parameters

Results - example

Period		MHB2000 [mas]	Difference to MHB2000 for case LLR 1 [mas]	LLR 2 [mas]
18,6 years	A	-17206,42	2,70 ± 0,20	5,21 ± 0,25
	B	9205,23	-0,48 ± 0,10	-1,32 ± 0,11
	A"	3,34	-4,62 ± 0,12	-3,46 ± 0,21
	B"	1,54	-2,29 ± 0,09	-2,19 ± 0,10
182,6 days	A	-1317,09	-2,38 ± 0,08	-1,69 ± 0,11
	B	573,03	0,25 ± 0,05	0,15 ± 0,05
	A"	-1,37	1,80 ± 0,07	1,85 ± 0,09
	B"	-0,46	0,23 ± 0,05	0,22 ± 0,05
9,3 years	A	207,46	0,45 ± 0,11	0,85 ± 0,18
	B	-89,75	-0,15 ± 0,07	-0,13 ± 0,08
	A"	-0,07	-1,50 ± 0,12	-0,97 ± 0,20
	B"	-0,03	-0,87 ± 0,08	-1,35 ± 0,09
365,3 days	A	147,59	-2,91 ± 0,10	-0,51 ± 0,16
	B	7,39	0,55 ± 0,06	0,01 ± 0,07
	A"	1,12	-2,30 ± 0,09	-0,06 ± 0,11
	B"	-0,19	-0,29 ± 0,05	-0,02 ± 0,05

LLR 1: precession according to Fukushima (2003) and Williams (1994)
LLR 2: precession according to P03, Capitaine et al. (2003)

Discussion of Nutation results

Differences in estimated corrections to the MHB2000 nutation model also depend on the implementation of precession/nutation (correlations change …)

Realistic **accuracy** of nutation coefficients from LLR about 0.1 - 0.3 mas in obliquity and 0.2 - 0.5 mas in longitude

Largest **differences** in longitude components

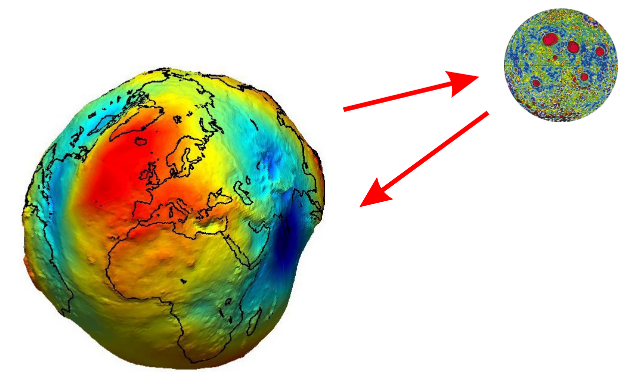
Large **correlation** of 18.6 and 9.3 year periods

Major **problems** in LLR are the unevenly distributed data (gaps in time series, orbit coverage, only few sites, weather, less accuracy in early years, …)

Future plan: Joined analysis of LLR and VLBI

Variation of the gravitational constant G

Use ansatz $G = G_0 + \dot{G}\Delta t + \frac{1}{2}\ddot{G}\Delta t^2$

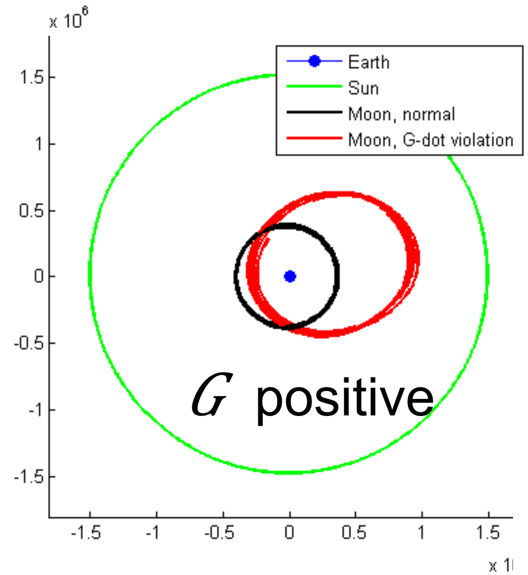


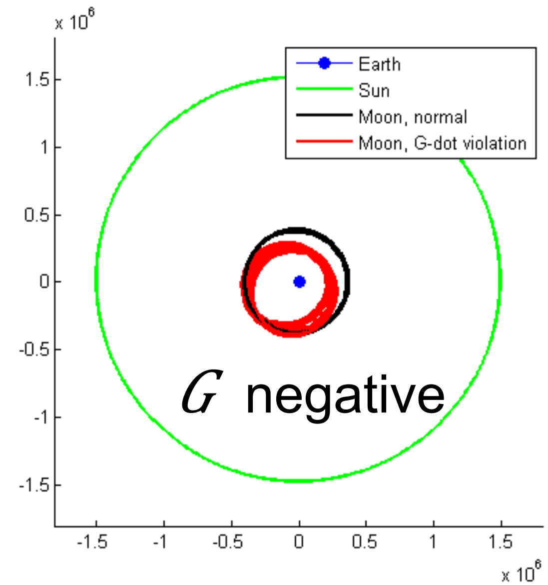
in equations of motion $\ddot{r}_{EM} \approx -\frac{GM_{E+M}}{r_{EM}^2} \ldots$

$$\frac{\dot{G}}{G} = \left(1.2 \pm 1.5\right) \times 10^{-13}\,\mathrm{yr}^{-1}$$

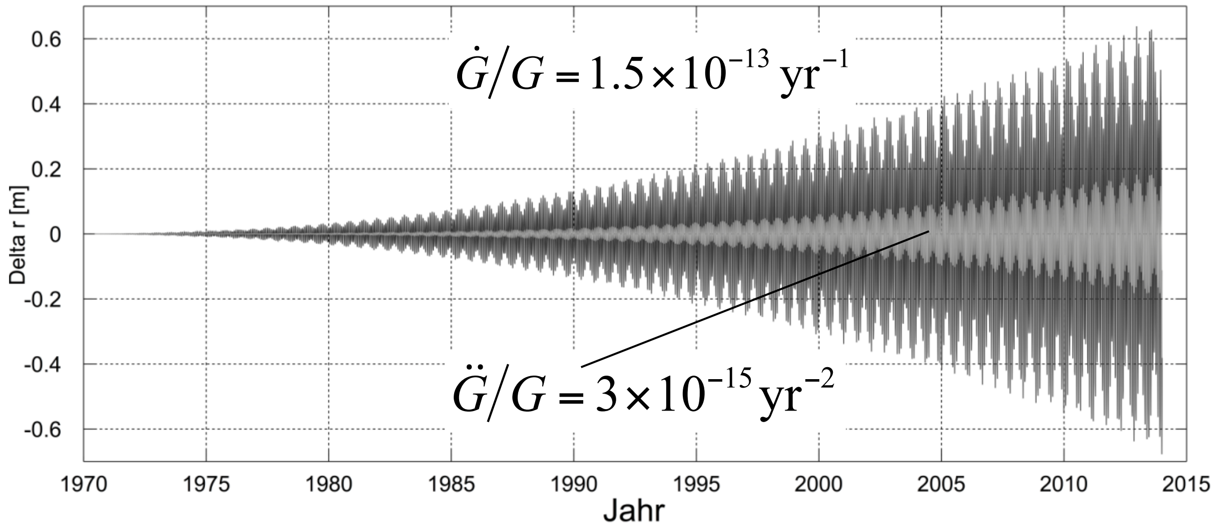
$$\frac{\ddot{G}}{G} = \left(1.4 \pm 3.0\right) \times 10^{-15}\,\mathrm{yr}^{-2}$$

correlation of almost 100 % with $k_2\delta$

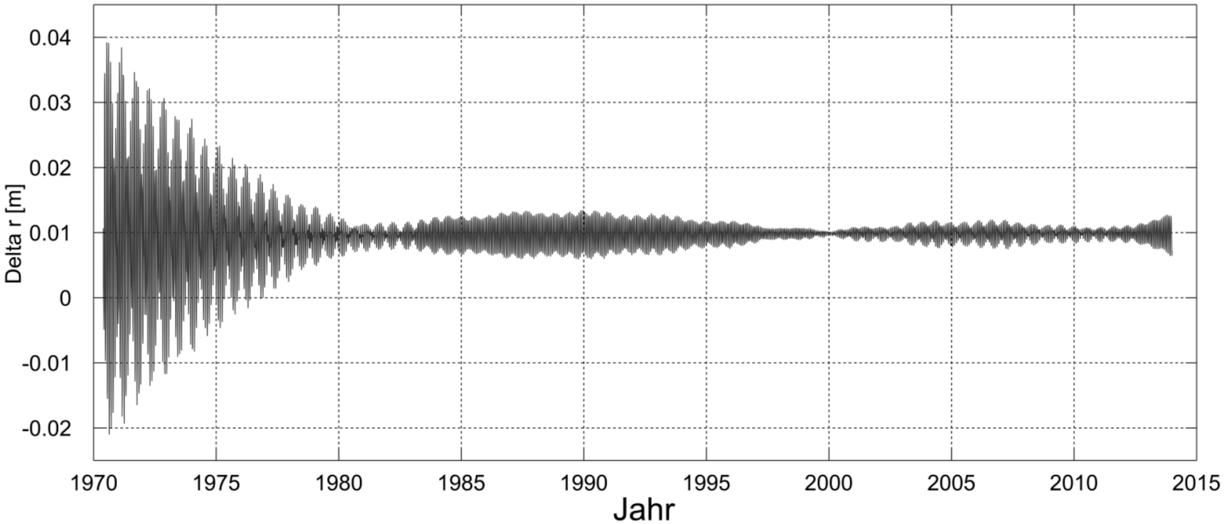




Effect of $\dot{G}/G$ perturbation on r_{EM}

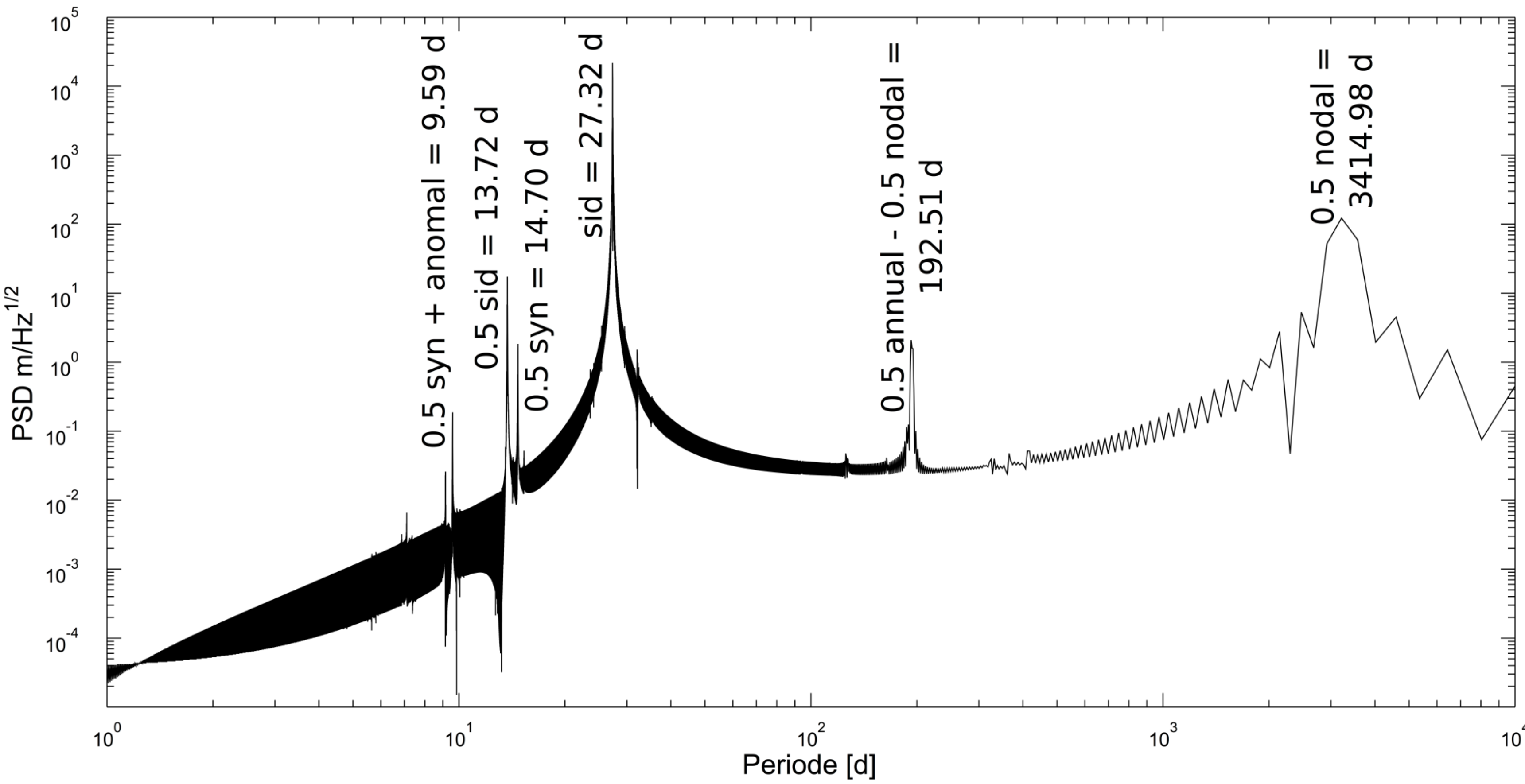


$$\ddot{r}_{EM} \approx -\frac{GM_{E+M}}{r_{EM}^2} \ldots$$



Here, the effect from the orbit has been reduced by estimating the initial values

Power spectrum of $\dot{G}/G$ perturbation



Conclusions

- Nutation coefficients from LLR partly well determined (e.g. annual), results affected by the uneven data distribution →combination with VLBI
- Pole coordinates and ΔUT can be obtained with high accuracy for those nights where good data are available
- LLR is a unique tool for studying the Earth-Moon system and testing general relativity, e.g. gravitational constant

$$\frac{\dot{G}}{G} = (1.2 \pm 1.5) \times 10^{-13}\,\mathrm{yr}^{-1}$$

$$\frac{\ddot{G}}{G} = (1.4 \pm 3.0) \times 10^{-15}\,\mathrm{yr}^{-2}$$

- Good results are only possible because of fantastic long-term lunar tracking by observatories (> 45 years of data). Thanks!

Acknowledgement

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FOR1503 Reference Frames

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