The Center of Mass Correction (CoM) for Laser Ranging Data of the CHAMP Reflector

prepared by Reinhart Neubert, formerly with GFZ Potsdam, mailto: reinhart.neubert@web.de

The compact retro reflector of the CHAMP satellite is suited for very high precision SLR because of it's negligible pulse spreading (signature) [1] . Identical reflectors have been adopted for the GRACE, TERRA-SAR-X and other missions.
It has been shown [1], that an additive constant is sufficient to refer the SLR measurements to the reference center of the reflector, if an error of up to 5 mm can be tolerated.
For higher requirements, for instance multi wavelength ranging, we compile here the equations to achieve an accuracy of better than 0.6 mm.

Let us define the following satellite fixed reference system:

Origin: Center of Mass of the satellite
X - nominal flight direction
Y- forming a right handed system with X,Z
Z- nominal Nadir direction

We denote by **S** the vector from the center of mass to the reference point*) of the reflector array. This vector has in the given coordinate system the following components (unit: mm)

	CHAMP	GRACE	TerraSAR-X
Sx	0	+/- 600	-1307.7
Sy	0	+/- 327.5	-212.1
Sz	249.8	217.8	947.6

In the case of GRACE ,the positive signs of Sx and Sy are valid for the body in front and the negative signs are valid for the follower.

Let us further define the following unit vectors:

C : unit vector from the satellite to the station
N(k) : unit vector from the array reference point to the center of the front face of prism No. k (k=1...4)

then the center of mass correction of prism k can be written

$$\mathrm{CoM}(k) = (\mathbf{S}\cdot\mathbf{C}) + \Delta R(k) \qquad \text{Eq. 1}$$

The first term is the eccentricity correction whereas the second term is the reduction to the reflector array reference point:

$$\Delta R(k) = D\cdot(\mathbf{C}\cdot\mathbf{N}(k)) - L\cdot\sqrt{ng^2 + (\mathbf{C}\cdot\mathbf{N}(k))^2 - 1} \qquad \text{Eq. 2}$$

where
ng = 1.4855 @ 532nm is the group index of refraction of the quartz prisms **).
D = 47.1mm is the distance of the prism front faces from the array reference point
L = 28mm is the vertex length of the prisms

In the coordinate system of the satellite, the direction vectors of the prisms are constant and may be written:

$$\mathbf{N}(k) = \begin{bmatrix} \sin\left(\frac{\pi}{4}\right)\cdot\cos\left[\frac{\pi}{2}\cdot\left(k-\frac{1}{2}\right)\right] \\ \sin\left(\frac{\pi}{4}\right)\cdot\sin\left[\frac{\pi}{2}\cdot\left(k-\frac{1}{2}\right)\right] \\ \cos\left(\frac{\pi}{4}\right) \end{bmatrix} \qquad \text{Eq.3}$$

The CHAMP reflector is designed using a minimum number of cube corners, so that for most orientations only one prism is contributing to the signal. Therefore the range correction due to the nearest prism is a good approximation for all angles. This is the prism with the smallest angle of incidence (or with maximum $\Delta R(k)$). As shown in appendix A, the error of this approximation is always smaller than 0.6mm.
ΔR varies weakly with orientation between –3mm and 6mm with an average of about 3mm. Therefore it may be replaced by a constant value, if 5mm precision is sufficient:

$$\Delta R = 3mm(\pm 3mm)$$

On the other hand, the first term of Eq.1 (the eccentricity term) is very sensitive to attitude changes of the satellite. To achieve 1mm accuracy, the satellite attitude must be known to about 15 arc minutes so that the use of measured attitude data might be necessary.

---

*) The reflector array reference point is defined by the intersection of the optical axes of the individual cube corners

**) Helpful discussions with Stefan Riepl on the group index of refraction is greatly acknowledged. Numerical information on the dispersion of quartz glass can be found in Appendix B.

References:

[1] R. Neubert, L. Grunwaldt, J. Neubert: The Retro-Reflector for the CHAMP Satellite: Final Design and Realization,
Proc. 11th Workshop on Laser Ranging, Deggendorf Sept. 1998, pp. 260-270

[2] L. Grunwaldt, R. Neubert: First Results of Laser Ranging to the CHAMP Retroreflector
Proc. 12th Workshop on Laser Ranging, Matera Nov. 2000

Appendix A:
Detailed Description, Numerical Data and Error Estimation

Description of the Retro reflector

The retro reflector of the CHAMP satellite has 4 cube corner prisms mounted on a regular pyramid. The individual cube corner prisms of the reflector are made from fused quartz and the reflecting surfaces are metal coated. Two of the dihedral angles are rectangular to high precision whereas one angle has an offset to produce a two-spot far field pattern. The front faces are slightly spherical.

Specifications :

Vertex length (L):	28.0 mm
Free aperture of the front face:	38.0 mm
Dihedral angle offset	-3.8 arc sec (smaller than 90°)
Radius of curvature of the front face	+500 m (convex)
Index of refraction	see Appendix B
Distance of the two peaks of the far field:	24 arc sec
width of the peaks (FWHM)	6 arc sec
widths at 20% of the maximum	10 arc sec

Fig.1
Photograph of the retro reflector
The arrow indicates the nominal flight direction

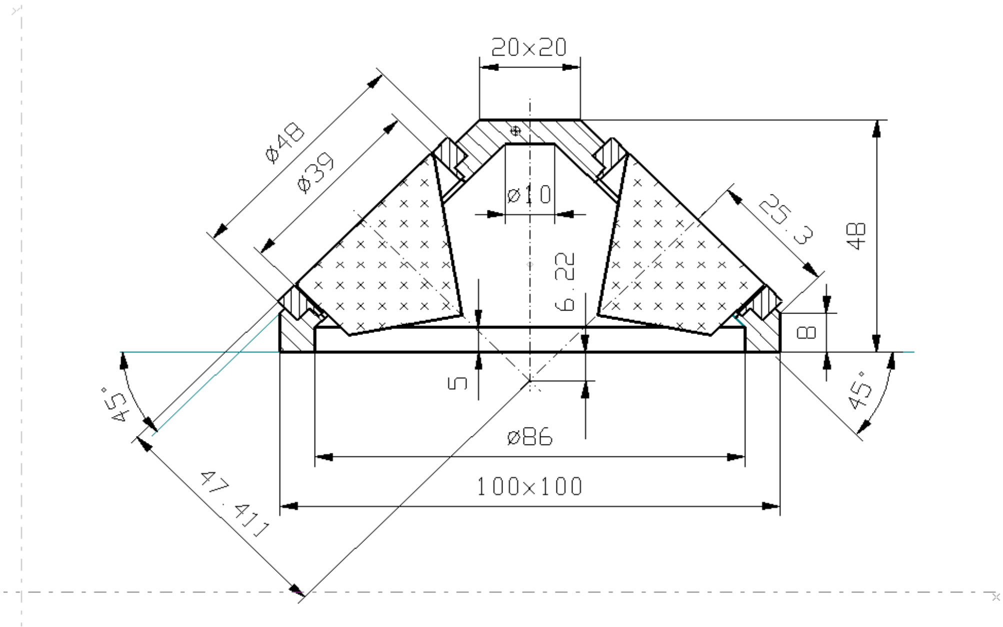


Fig.2:
Cross section showing
the localization of the Reference Point

Calculating the Center of Mass Correction

As a reference point of the array we are using the crossing point of the optical axes of the cube corner prisms. The range correction of a single cube corner referred to this point is given by the Eqs.1 and 2 on page 1:
If more than one prism is contributing to the signal and if the signal is kept at single photoelectron level the average range correction will be:

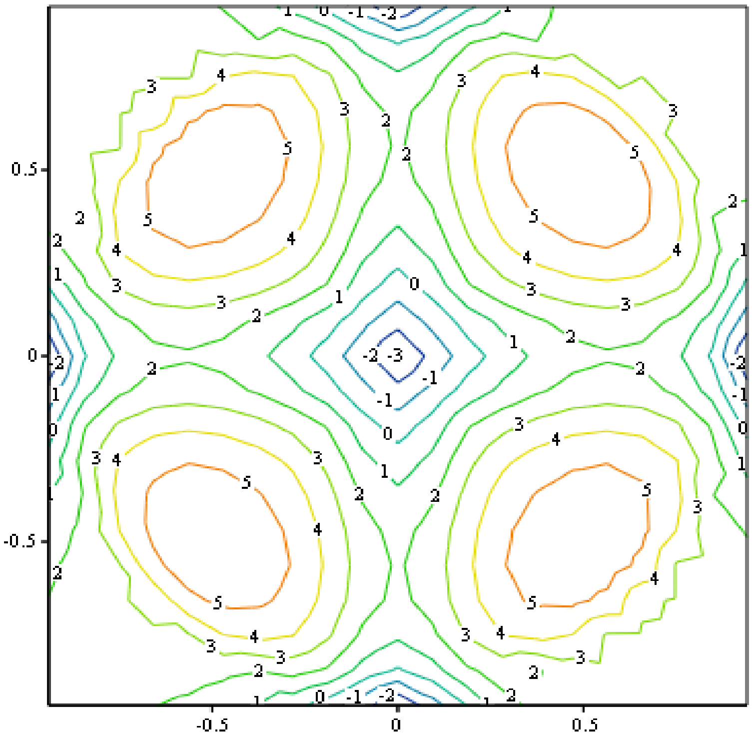
$$\langle \Delta R \rangle = \frac{\sum_k S_k \cdot \Delta R_k}{\sum_k S_k} \qquad \text{Eq. 1A}$$

The relative intensities S_k of the contributing prisms are determined by the location of the receiving telescope in the far field diffraction patterns. This patterns are complex and depend on production errors, thermal distortions etc. For an order of magnitude estimation we are using the simple approximation:

$$S_k(\gamma_k) = \left(1 - \frac{\gamma_k}{0.85}\right)^2 \qquad \cos(\gamma_k) = (\mathbf{C} \cdot \mathbf{N}(k)) \qquad \gamma_k \le 0.85 \qquad \text{Eq. 2A}$$

Note that this rough approximation for the intensities is sufficient to attain 1 mm accuracy of the range correction, as shown below.
To present numerical results, we express the vector **C** by angular coordinates:

$$\mathbf{C}(\alpha, \delta) = \begin{pmatrix} \sin(\delta) \cdot \cos(\alpha) \\ \sin(\delta) \cdot \sin(\alpha) \\ \cos(\delta) \end{pmatrix} \qquad \text{Eq. 3A}$$



(X, Y, Z)

Fig.3: Contour map of the array range correction for λ=532nm .

The coordinate system for the the map is uniquely related to the angles α ,δ :

X=sin(δ)cos(α)
Y=sin(δ)sin(α)

Z= <ΔR>
Average range correction (Millimeters) to the array reference point

From the symmetry of the array it is clear, that the following relations hold:

$$\langle \Delta R \rangle \left(\alpha + \frac{\pi}{2} \cdot k \right) = \langle \Delta R \rangle (\alpha) \qquad k=1...4 \qquad \text{Eq. 4A}$$

$$\langle \Delta R \rangle \left(\frac{\pi}{2} - \alpha \right) = \langle \Delta R \rangle (\alpha) \qquad \text{Eq. 5A}$$

Therefore, it is sufficient to tabulate the range correction in the interval $\alpha = 0....\pi/4$ (0 to 45°). All numerical data are in Millimeters.

Table1: The range correction at λ=532nm according to Eq.A1 for different orientations

α=	0	5	10	15	20	25	30	35	40	45
δ= 0	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1
5	-1.8	-1.8	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7
10	-0.65	-0.63	-0.57	-0.47	-0.35	-0.21	-0.061	0.055	0.13	0.15
15	0.25	0.3	0.42	0.61	0.84	1.1	1.3	1.6	1.7	1.8
20	0.96	1	1.2	1.5	1.9	2.2	2.6	2.8	3	3.1
25	1.5	1.6	1.9	2.3	2.7	3.2	3.6	3.9	4	4.1
30	1.8	1.9	2.3	2.8	3.4	4	4.4	4.7	4.8	4.8
35	1.9	2.1	2.6	3.2	3.9	4.5	4.9	5.2	5.3	5.4
40	1.8	2	2.7	3.5	4.2	4.9	5.3	5.5	5.6	5.7
45	1.5	1.8	2.6	3.5	4.3	4.9	5.3	5.6	5.8	5.8
50	1	1.4	2.3	3.4	4.2	4.8	5.2	5.5	5.6	5.7
55	0.34	0.82	1.9	3	3.8	4.4	4.8	5.1	5.3	5.4
60	-0.55	0.095	1.4	2.4	3.2	3.8	4.2	4.6	4.8	4.8
65	-1.6	-0.75	0.6	1.5	2.3	2.9	3.4	3.8	4	4.1
70	-2.9	-1.7	-0.56	0.4	1.2	1.9	2.4	2.8	3	3.1
75	-4.4	-3.1	-1.9	-0.92	-0.067	0.63	1.2	1.6	1.8	1.9
80	0	0	-3.5	-2.5	-1.6	-0.84	-0.27	0.13	0.38	0.46
85	0	0	0	-4.2	-3.3	-2.5	-1.9	-1.5	-1.3	-1.2
90	0	0	0	0	0	-4.5	-3.8	-3.4	-3.1	-3.1

If the inclination angle is greater then the cutoff angle 0.85 rad, then the range correction has been set to zero in Table 1.

In the following Table 2 we print the differences of the Range correction of the nearest prism (dominating prism) to the values of Table 1.
As can be seen , the contribution of the other prisms shifts the range correction by less than 1 mm. Zero in table 2 means that only one cube corner contributes to the signal or the contributing cubes are equivalent. The latter is obviously the case if α=0 or δ=0.

Table 2: Difference of the dominating prism approximation to Table 1 (λ=532nm)

α= 0	0	5	10	15	20	25	30	35	40	45
δ= 0	0	0	0	0	0	0	0	0	0	0
5	0	0.11	0.21	0.29	0.35	0.4	0.43	0.46	0.47	0.48
10	0	0.2	0.34	0.42	0.45	0.44	0.39	0.35	0.32	0.31
15	0	0.29	0.46	0.53	0.53	0.46	0.36	0.24	0.14	0.1
20	0	0.36	0.55	0.6	0.54	0.42	0.28	0.15	0.048	0.0031
25	0	0.43	0.62	0.62	0.5	0.34	0.18	0.062	0.0029	0
30	0	0.48	0.65	0.6	0.43	0.24	0.092	0.0093	0	0
35	0	0.53	0.66	0.55	0.34	0.14	0.025	0	0	0
40	0	0.56	0.65	0.47	0.23	0.059	0	0	0	0
45	0	0.58	0.6	0.36	0.12	0.0055	0	0	0	0
50	0	0.58	0.52	0.23	0.03	0	0	0	0	0
55	0	0.55	0.38	0.086	0	0	0	0	0	0
60	0	0.47	0.18	0	0	0	0	0	0	0
65	0	0.3	0.006	0	0	0	0	0	0	0
70	0	0.023	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0

Appendix B:
The Group Refractive Index of Quartz Glass. Application to Two Wavelength Ranging

The cube corner prisms for the CHAMP satellite were produced from Heraeus Suprasil whereas for the GRACE reflectors Homosil have bee used. The differences of the refractivity of both materials are very small.
Using the refractivity data given by the manufacturer the following table for the most important SLR wavelengths can be obtained:

	Suprasil (CHAMP)		Homosil (GRACE)	
λ	n	ng	n	ng
0.355	1.4761	1.5329	1.4762	1.5333
0.4235	1.4678	1.5064	1.4679	1.5067
0.532	1.4607	1.4853	1.4608	1.4855
0.85	1.4525	1.4657	1.4526	1.4658
1.064	1.4496	1.4624	1.4497	1.4625

In this table n is the phase index and ng the group index computed using the Sellmeier equation:

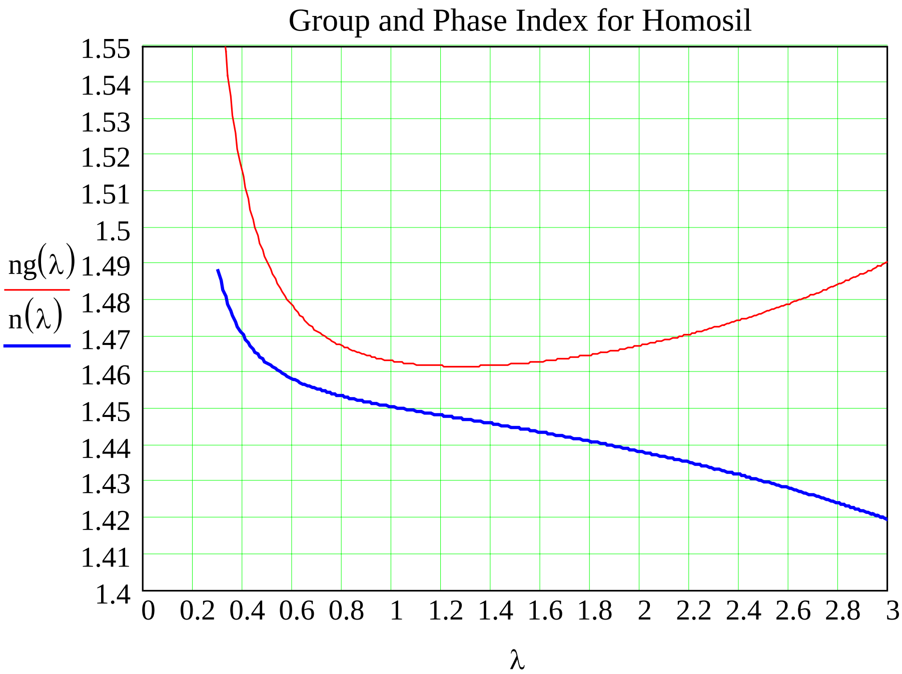
$$n^2 - 1 = B_1 \cdot \frac{\lambda^2}{\lambda^2 - C_1} + B_2 \cdot \frac{\lambda^2}{\lambda^2 - C_2} + B_3 \cdot \frac{\lambda^2}{\lambda^2 - C_3}$$

and the well known formula:

$$ng = n - \lambda \cdot \frac{dn}{d\lambda}$$

For convenience, we reproduce here the constants from the data sheet of the manufacturer (λ in µm) :

Suprasil		Homosil	
B	C	B	C
0.473115591	0.012995717	0.47652307	0.00284888095
0.631038719	0.0041280922	0.627786368	0.0118369052
0.906404498	98.7685322	0.872274404	95.6856012



In the following Table the difference of the range corrections between λ=423.5 nm and 850 nm is given:

Table 3: Differential range correction for the pair 850 / 423.5 nm

	α= 0	5	10	15	20	25	30	35	40	45
δ= 0	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
10	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2
15	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
20	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
25	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
30	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
35	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
40	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1
45	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1
50	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1
55	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
60	1.3	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
65	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
70	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2	1.2	1.2
75	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.2
80	0	0	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2
85	0	0	0	1.3	1.3	1.3	1.3	1.3	1.3	1.3
90	0	0	0	0	0	1.3	1.3	1.3	1.3	1.3

It can be seen that the differential range correction is almost independent of the orientation within 0.1 mm
For the given wavelength pair it can be replaced by (1.2 +/- 0.1) mm for correcting the measured range difference.
For the wavelength pair 1064 / 532 nm we get the correction (0.7 +/- 0.1) mm.
This correction is important however for the determination of atmospheric refraction because its influence on the result is at the cm level.