Intra-technique Combination and Its Precision Evaluation based on SHAO SLR SINEX solutions

Xiaoya Wang, Fan Shao, Xiaogong Hu, Hao Yang

Shanghai Astronomical Observatory, Chinese Academy of Sciences,

Shanghai 200030, China

中国科学院上海天文台

Shanghai Astronomical Observatory
Chinese Academy of Sciences

Contents:

1. Introduction
2. SHAO SLR SINEX solution
3. SLR Intra-technique Combination
4. Results and analysis
5. Conclusion

1.Introduction

As we know, Satellite Laser Ranging(SLR) play an important role for the determination of Terrestrial Reference Frame(TRF) and EOP. It is only technique for the origin of TRF or ITRF and works together with Very Long Baseline Interferometry(VLBI) to determine the scale factor.

Table. Datum definition of ITRF

Datum definition	
Origin	Defined by SLR
Scale	Defined by SLR and VLBI
Orientation	Align to ITRF2008's orientation(core sites)

1.Introduction

Now, there are 7 ILRS Analysis Centers(ACs). Because of the different models, strategies, methods and other reasons used in the data processing, their products are not completely same. Which is the best? They are often evaluated by combination. In general, the combination solution is the best one. The solution closer to the combination solution is better.

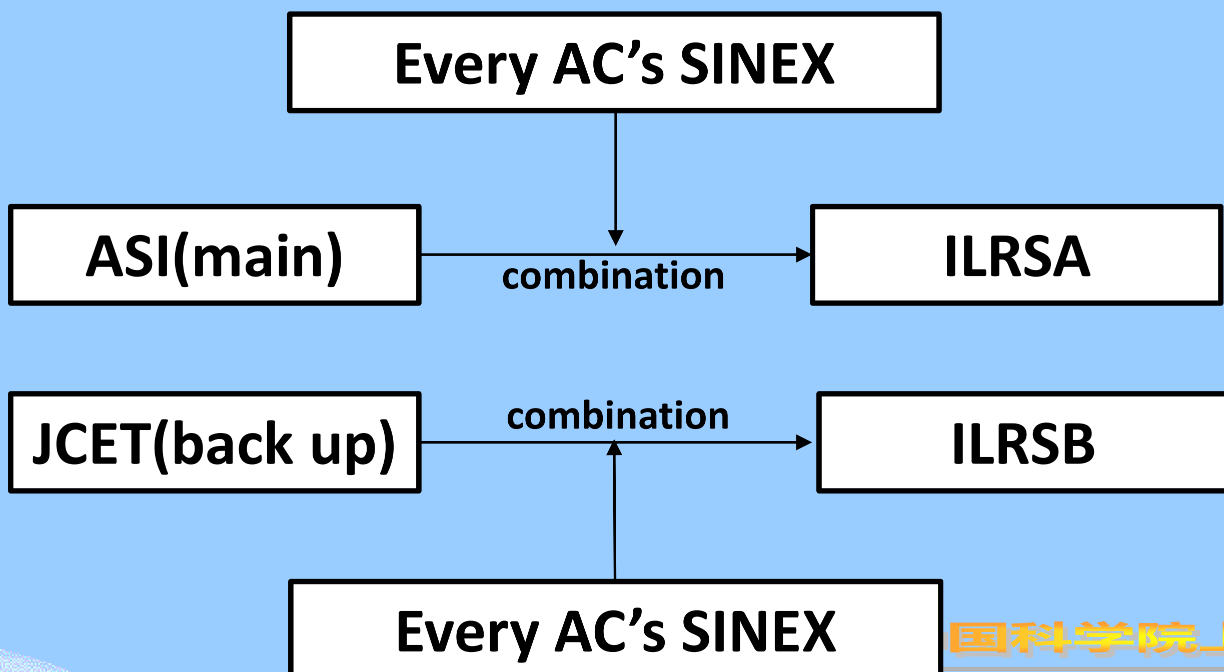
Table. List of 7 ACs

Analysis Centers(AC)	
ASI	Italian Space Agency
BKG	Bundesamt für Kartographie und Geodäesie
GFZ	Helmholtz Centre Potsdam German Research Centre for Geosciences
DGFI	Deutsches Geodätisches Forschungsinstitut
JCET	Joint Center for Earth System Technology
NSGF	NERC Space Geodesy Facility
GRGS	Groupe de Recherche de Géodésie Spatiale

SHAO

1.Introduction

Now, there are 2 combination centers(CCs). They use weekly SINEX solutions from each AC as input and generate ILRS combination solution ILRSA and ILRSB. Which combination solution is better?



1.Introduction

ShangHai astronomy observatory provided SLR products to IERS since 1985 and became associate AC since 1999, providing global SLR data quality report including time and range bias and navigation satellite orbit to ILRS weekly. However, weekly SINEX solutions and combination products with other ACs have not been provided. So, we investigate our SINEX solutions and the combination solutions with other SLR ACs. Our SINEX solutions is named SHAO and our combined solutions is named ILRSC for comparison with other ACs and CCs. By comparison we have analyzed the accuracy and stability of our products.

```
  Residuals are summarized for the following 3-day arcs: wtd rms
                                                           (cm)

  LAGEOS-1 3-DAY ARC2476: 19/06/25 00:00 - 19/06/28 00:00 1.2
  LAGEOS-1 3-DAY ARC2477: 19/06/28 00:00 - 19/07/01 00:00 1.0
  LAGEOS-1 3-DAY ARC2478: 19/07/01 00:00 - 19/07/04 00:00 1.3
  LAGEOS-1 3-DAY ARC2479: 19/07/04 00:00 - 19/07/07 00:00 1.3
  LAGEOS-1 3-DAY ARC2480: 19/07/07 00:00 - 19/07/10 00:00 1.3
                          long   obs edited range  time   raw  prec sat
S-ID   starting time     /pass  /pass  obs   bias  bias   rms   est
     year/ m/ d  h:mi    min                  mm    us     mm    mm
1868 (KOMSOM)
1868 2019/ 6/28 13:45    9.0      6  1         32     3    29     1  L1
1868 2019/ 6/30 14:19   10.6      7  1         55    16    19     3  L1
1868 2019/ 7/ 3 14:11    9.0      6  0         41    -6    27     1  L1

1873 (SIMEIZ)
1873 2019/ 6/27 22:43    5.0      8  6 DEL   9999  9999    40     0  L1
1873 2019/ 7/ 1 20:42   10.3      6  0 DEL    170    -6   161    38  L1
1873 2019/ 7/ 2 19:36    7.5      4  0 DEL    257   -49   140    26  L1
```

As associate AC , SHAO provided BIAS report with residual RMS about 1cm.

中国科学院上海天文台
Shanghai Astronomical Observatory
Chinese Academy of Sciences

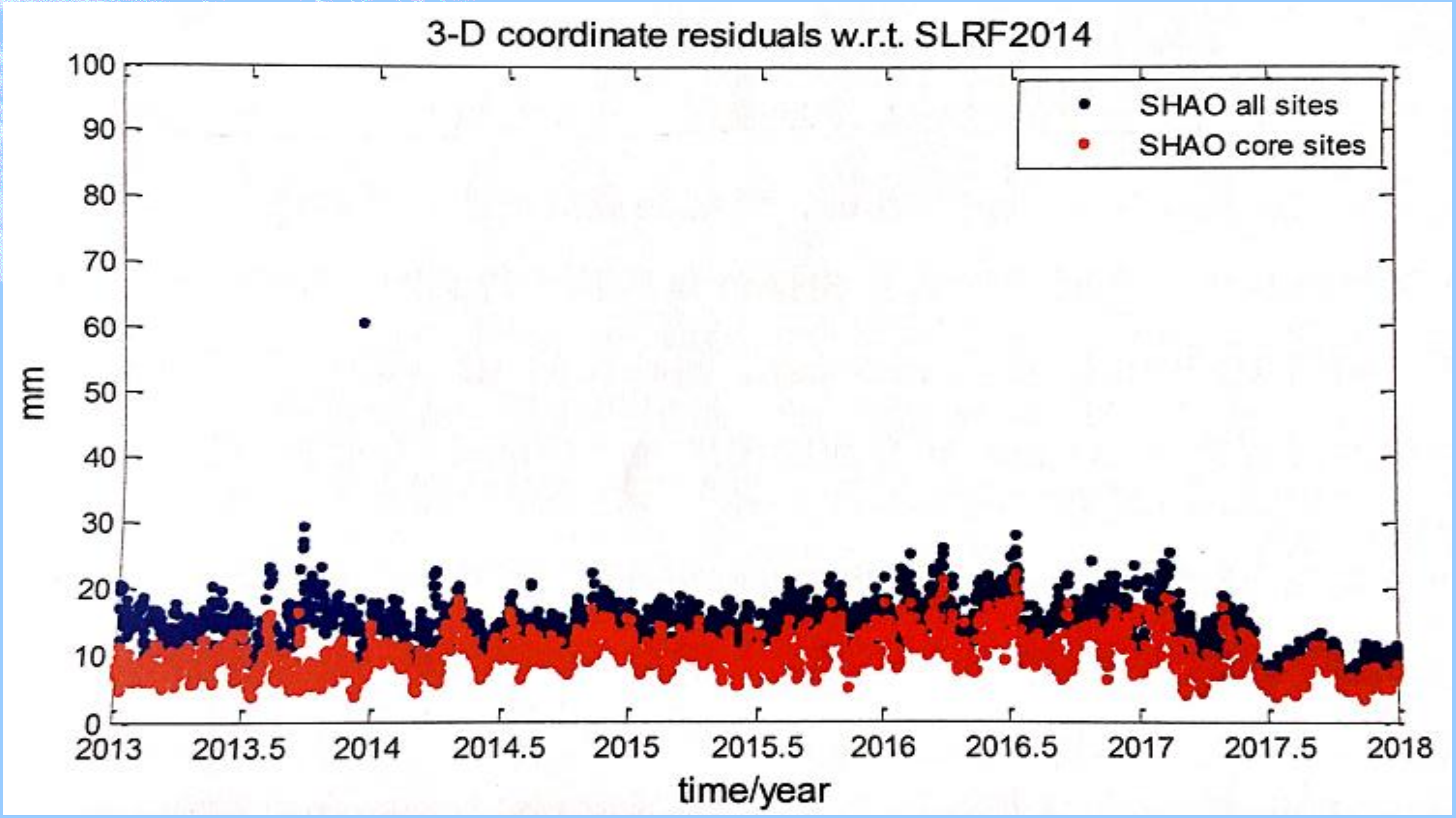
2. SHAO SLR SINEX solution

2.1 Processing model of SHAO SINEX solution

MODELS	TRF	ITRF2014 as a prior coordinate
	Troposphere	Mendes-Pavlis zenith delay model
	COM	ILRS station dependent CoM model
	Relativity	point-mass accelerations, Lense-Thirring effect, Coriolis force
	Precession	IERS2010
	Nutation	IAU 2006+IERS
	Geopotential	EGM2008 (static terms 70x70, C(2,0), C(2,1) ,S(2,1) time dependent)
	Tidal forces	solid earth tides: IERS 2010 Conventions model Ocean tides: FES2004
	Third-body	ephemeris: JPL DE421
ESTIMATED PARAMETERS	Station coordinate	a priori values: SLRF2014/1.0m sigma constraints
	EOP	a priori values: IERS 14 C04/xp,yp :20 mas ;lod:2 ms sigma
	Range bias	Estimation for some non core station, sigma:1.0m

2. SHAO SLR SINEX solution

2.2 SHAO SINEX solutions



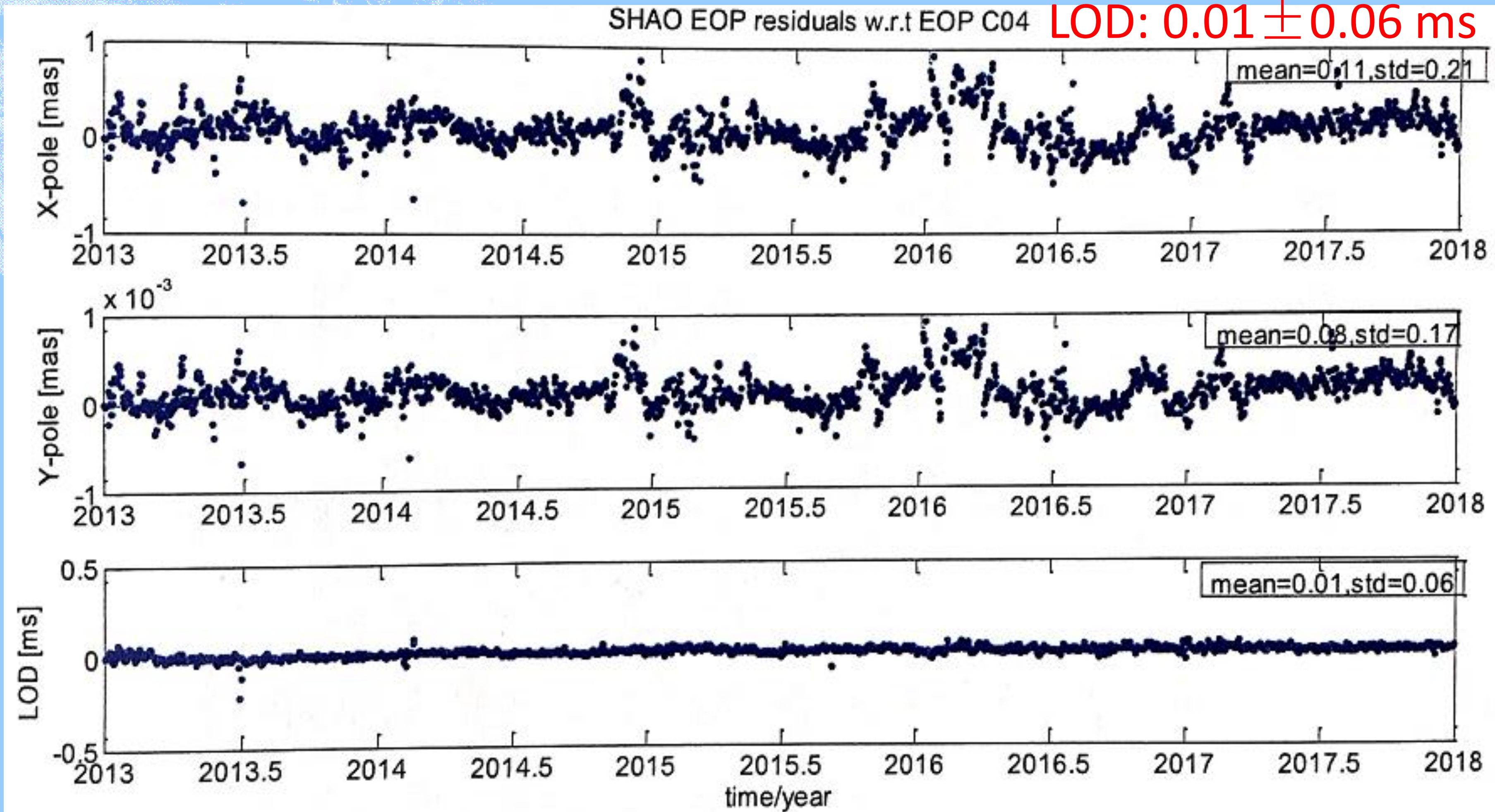
3-D coordinate residuals w.r.t SLRF2014 (Lageos1/2 + Etalon1/2)

中国科学院上海天文台
Shanghai Astronomical Observatory
Chinese Academy of Sciences

2. SHAO SLR SINEX solution

2.2 SHAO SINEX solutions

EOP accuracy
Xp: 0.11±0.21 mas
Yp: 0.08±0.17 mas
LOD: 0.01±0.06 ms



EOP residuals w.r.t EOP C04 (Lageos1/2 + Etalon1/2)

3.SLR Intra-technique Combination

3.1 Processing of a priori constraints

As for the constraints of station coordinates and EOP, there are two methods to deal with the a priori constraints for SLR intra-technique combination. First is straightforward method that the combined SINEX solution is directly based on SINEX solutions from various ACs due to the loosely constrained solutions. Another is minimal constraints method. We firstly remove the loose constraints of provided SINEX solutions and then impose minimal constraints on them. Finally, the SLR combined weekly solution is obtained with minimal constraint as well.

3.SLR Intra-technique Combination

3.1 Processing of a priori constraints

Table. Comparison of two methods for a priori constraint processing

method	advantages	disadvantages
straightforward method	convenient in calculation; a priori information is not required	orientation is random
minimal constraints method	orientation unified as certain reference frame	causing rank deficient of normal equation when remove loose constraints

3.SLR Intra-technique Combination

3.2 Determination of relative weight factors

There are mainly two approaches to determine the relative weight factors in SLR intra – technique combination.

Determination of weight factors

1. restraint condition iteration
2. variance component estimation

3.SLR Intra-technique Combination

3.2 Determination of relative weight factors

The first approach is the iterative calculation of weight factors based on the formulas (1) and (2). This method is assuming that the final combined residual $\chi^2 = 1$ and the contribution of every AC to the combined residual is the same. The weight factors of every AC is determined through continuous iteration and the termination condition of iteration is:

$$R_1^T(\sigma_1\Sigma_1)^{-1}R_1 = \cdots = R_n^T(\sigma_n\Sigma_1)^{-1}R_n \quad \textbf{(1)}$$

$$\chi^2 = R_1^T\Sigma_1^{-1}R_1 + \cdots + R_i^T\Sigma_i^{-1}R_i = 1 \quad \textbf{(2)}$$

3.SLR Intra-technique Combination

3.2 Determination of relative weight factors

Another method for relative weight factors is based on variance component estimation. In this method, the initial weight factors of each AC are set to 1 and the new iteration's weight matrix is determined by multiply the weight factor from last iteration by the weight matrix. When the difference value between two successive iteration is less than 0.001, the iteration is stopped.

$$\varepsilon_i = \frac{v_i^T P_i v_i}{n_i - tr(N^{-1} A_i^T P_i A_i)}$$

3.SLR Intra-technique Combination

3.3 ILRSC intra-technique combination method

Here, we list the methods of ILRSA, ILRSB and our combination products named ILRSC

Table. Different methods of every combined solution

combination method	processing of a priori constraints	determination of weight factors
ILRSA	straightforward method	restraint condition iteration
ILRSB	minimal constraints method	variance component estimation
ILRSC	straightforward method	variance component estimation

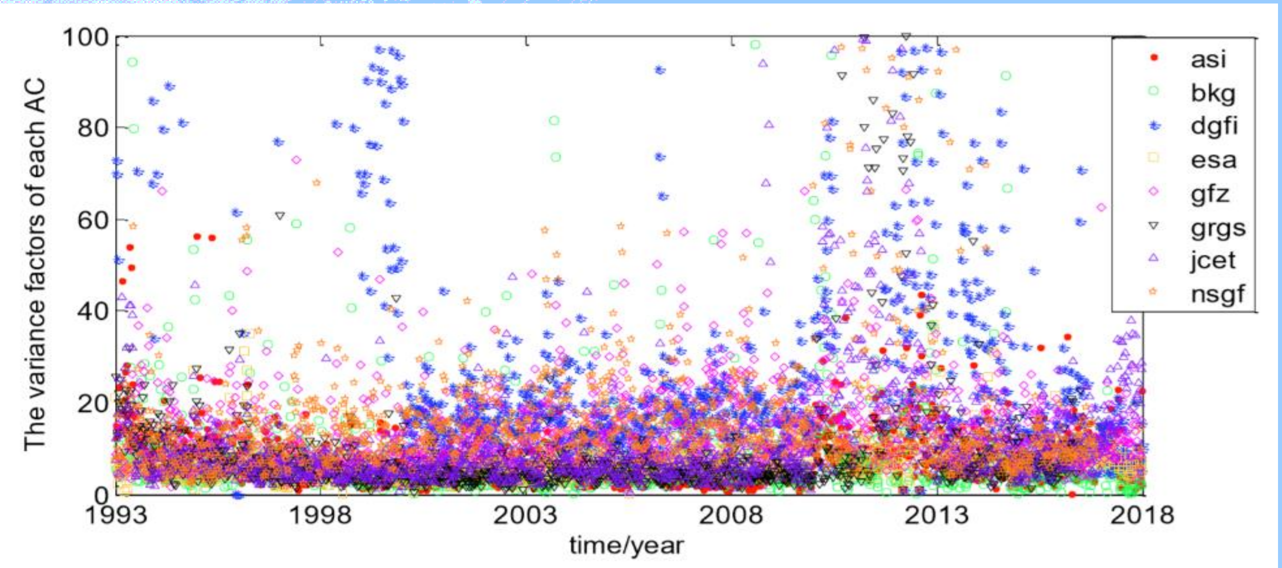
4.Results and analysis

Based on the method mentioned above, we combined the SINEX solutions that 8 ILRS ACs provide during Jan 1 1993 and Dec 31, 2017. The combined solution is analyzed by comparing our products with ILRS corresponding products. The analysis is performed according to the following aspects:

① Relative weight factors of each AC
② Accuracy analysis of station coordinates and EOP
③ Analysis of translation parameters and scale factor
④ Difference between SLRF2008 and SLRF2014
⑤ Evaluation of SHAO SINEX solutions

4.Results and analysis

4.1 Relative weight factors of each AC



ASI performs the best and then ESA, GRGS, BKG, JCET, GFZ, NSGF, DGFI (ASI is supposed as unit weight 1.00)

Table/Figure the variance factors comparison of each ACs in ILRSC combination weekly solution

combined solution	ASI	BKG	DGFI	ESA	GFZ	GRGS	JCET	NSGF
ILRSA	1.00	1.51	4.06	1.20	1.45	1.80	2.03	2.44
ILRSB	1.00	1.07	2.06	1.16	1.93	1.22	1.36	2.36
ILRSC	1.00	1.13	3.19	1.05	1.94	1.10	1.61	2.17

中国科学院上海天文台
Shanghai Astronomical Observatory
Chinese Academy of Sciences

4.Results and analysis

4.2 Accuracy analysis of station coordinates and EOP

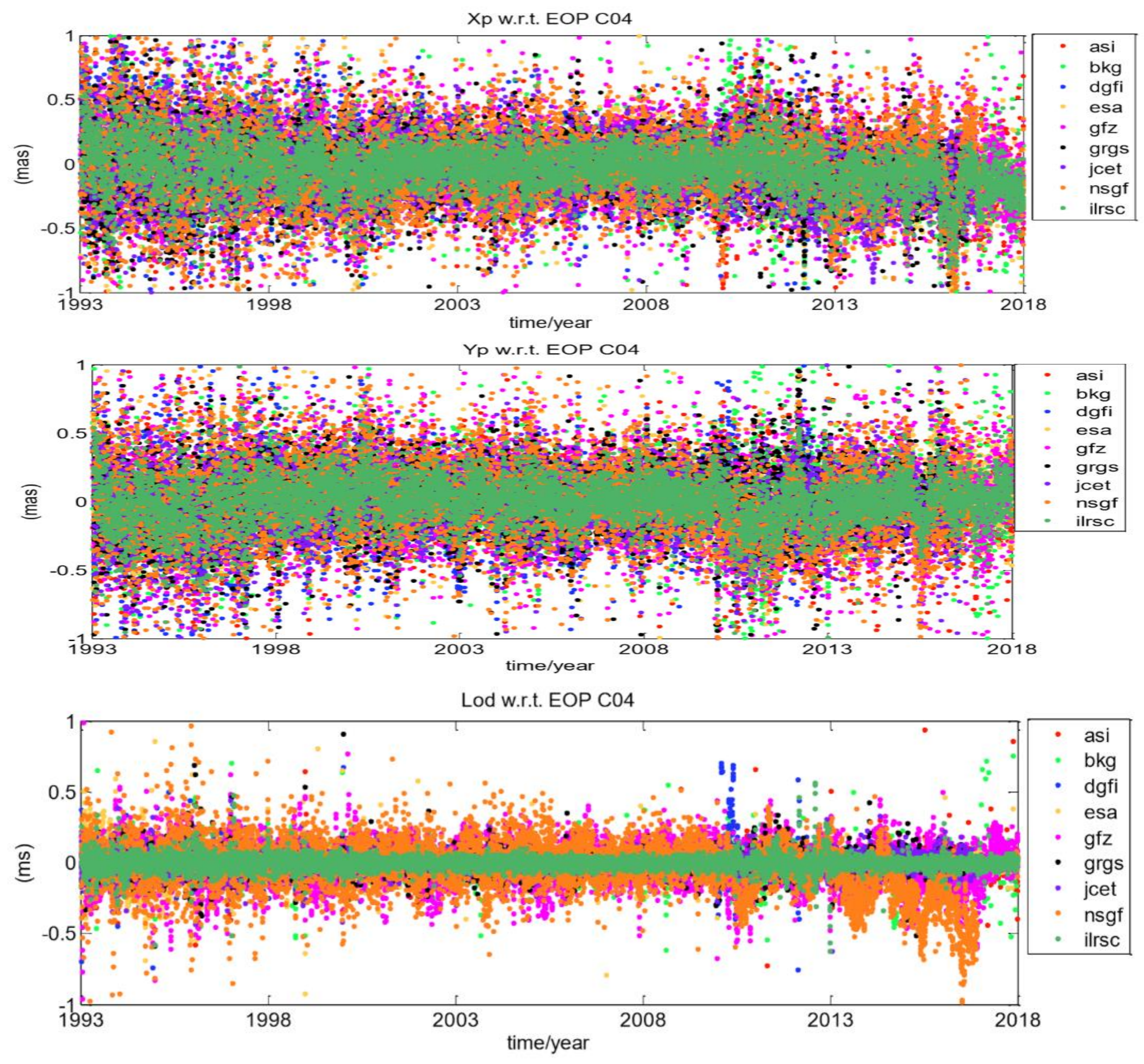
To assess the accuracy of the combination station coordinates and EOP the weekly combined solution is compared with SLRF2008 and IERS EOP C04. We use HELMERT seven parameters to transform weekly reference frame to SLRF2008, the transformation relationships are as follows:

$$\begin{cases} X_r^i(t_0) = X_w^i - \left(t_j - t_0\right) \cdot \dot{X}_r^i + T_1 + D \cdot X_w^i - R_3 Y_w^i + R_2 Z_w^i \\ Y_r^i(t_0) = Y_w^i - \left(t_j - t_0\right) \cdot Y_r^i + T_2 + D \cdot Y_w^i - R_1 Z_w^i + R_3 X_w^i \\ Z_r^i(t_0) = Z_w^i - \left(t_j - t_0\right) \cdot \dot{Z}_r^i + T_3 + D \cdot Z_w^i + R_1 Y_w^i + R_2 X_w^i \end{cases}$$

$$\begin{cases} X_r^P = X_w^p + R_2 \\ Y_r^P = Y_w^p + R_1 \\ lod_r = lod_w \end{cases}$$

4.Results and analysis

4.2 Accuracy analysis of station coordinates and EOP



Time series of weekly 3-D residuals for Xp, Yp, LOD with respect to EOP C04 are given for individual AC solutions and the combined ILRSC solution after transformation.

The results show that polar motion of Xp is 0.1875mas, Yp is 0.1759mas and LOD is 0.0485ms. The accuracy of combined solution is obvious better than that of individual AC solution. It's almost the same as ILRSA.

4.Results and analysis

4.2 Accuracy analysis of station coordinates and EOP

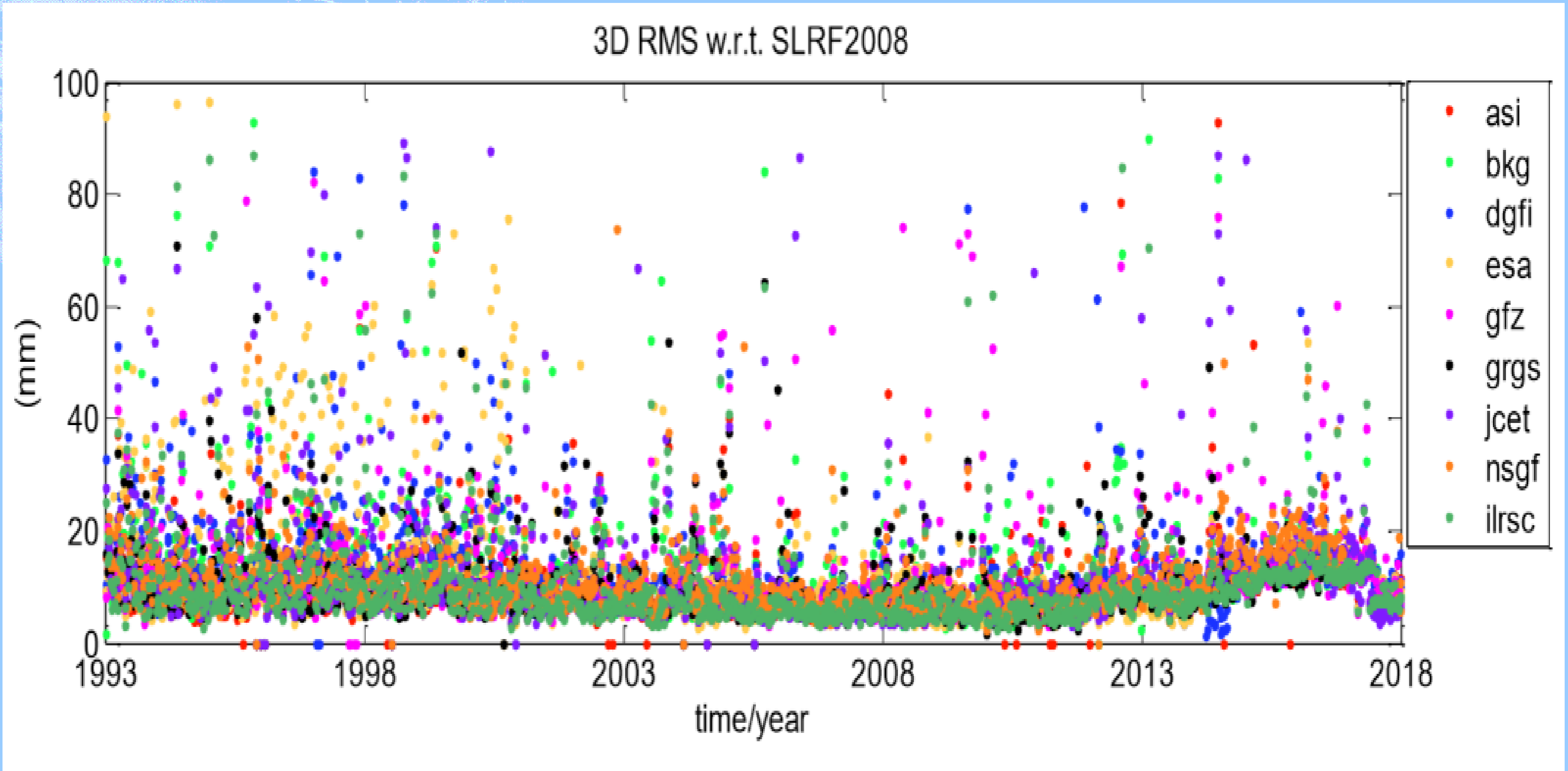


Figure. Time series of weekly 3-D residuals with respect to SLRF 2008 for ILRS core sites are obtained for individual AC solution and the combined ILRSC solution. The results show that 3D accuracy of combined station coordinates is 4.33mm

4.Results and analysis

4.2 Accuracy analysis of station coordinates and EOP

	POS-3D(mm)	Xp(mas)	Yp(mas)	LOD(ms)
ASI	7.61(±5.29)	-0.034(±0.243)	-0.011(±0.229)	-0.006(±0.062)
BKG	10.01(±7.38)	-0.041(±0.254)	0.012(±0.240)	-0.002(±0.071)
DGFI	10.71(±6.89)	0.026(±0.254)	-0.046(±0.247)	0.001(±0.074)
ESA	10.67(±6.89)	-0.014(±0.240)	0.025(±0.217)	-0.009(±0.085)
GFZ	9.42(±6.60)	-0.015(±0.288)	0.008(±0.279)	-0.012(±0.139)
GRGS	7.84(±5.46)	-0.040(±0.236)	0.006(±0.226)	-0.001(±0.062)
JCET	10.18(±9.00)	-0.046(±0.238)	0.006(±0.224)	-0.002(±0.055)
NSGF	9.20(±5.03)	-0.015(±0.303)	0.001(±0.291)	-0.035(±0.189)
ILRSA	5.51(±4.38)	0.012(±0.209)	-0.004(±0.202)	-0.002(±0.048)
ILRSB	5.43(±4.73)	*	*	*
ILRSC	5.67(±4.33)	-0.035(±0.187)	0.002(±0.176)	-0.001(±0.048)

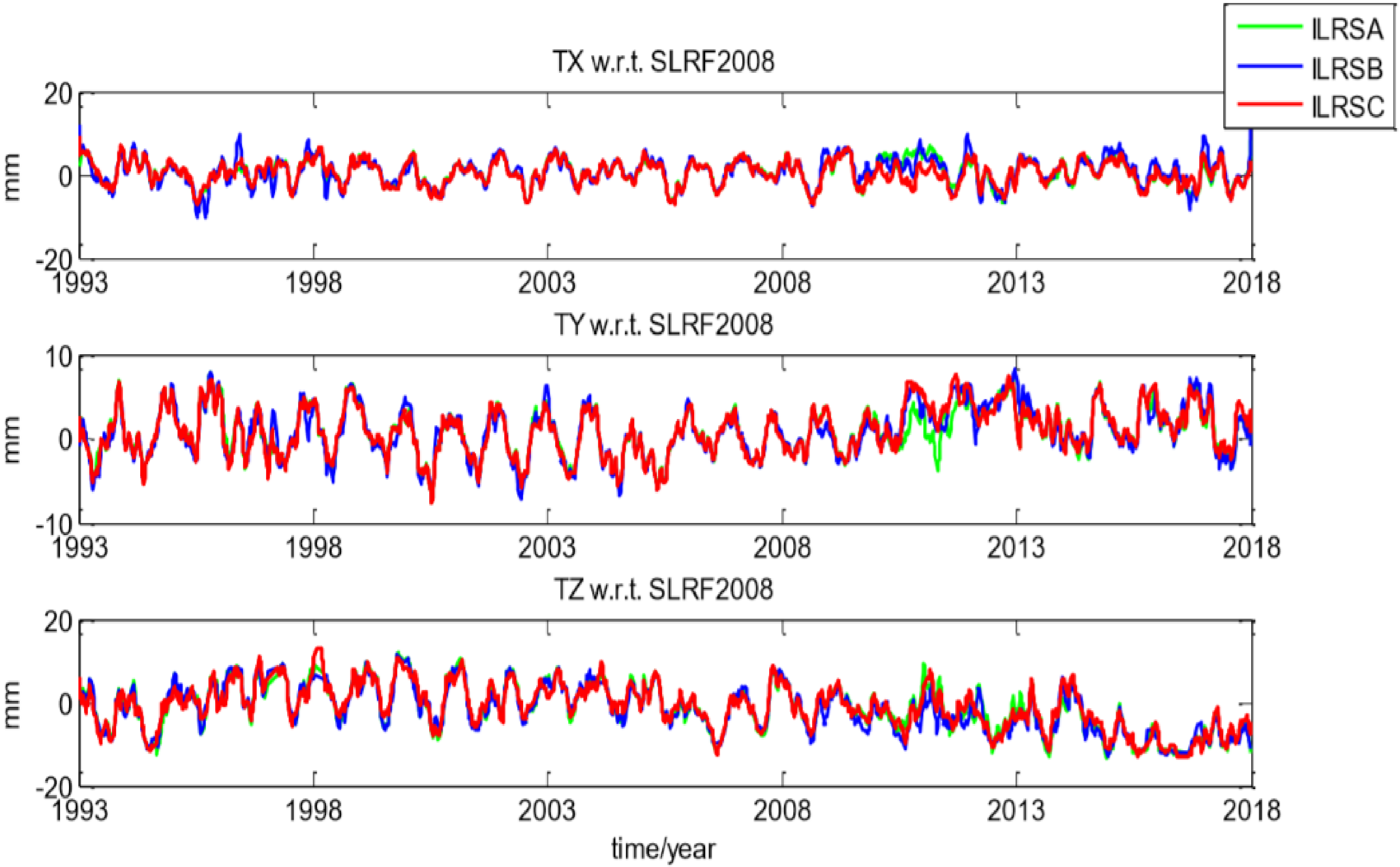
4.Results and analysis

4.3 Analysis of translation parameters and scale factor

we use HELMERT 7 parameters to transform the combined weekly solutions to SLRF2008. Three combined solutions show consistency to some extent. But from 1993 to 2014, ILRSC translation parameters are more consistent with ILRSA. From 2005 to 2017, scale parameters are more identical with ILRSB. The reason is not clear. But this means the existence of the third CC is necessary.

4.Results and analysis

4.3 Analysis of translation parameters and scale factor



4.Results and analysis

4.3 Analysis of translation parameters and scale factor

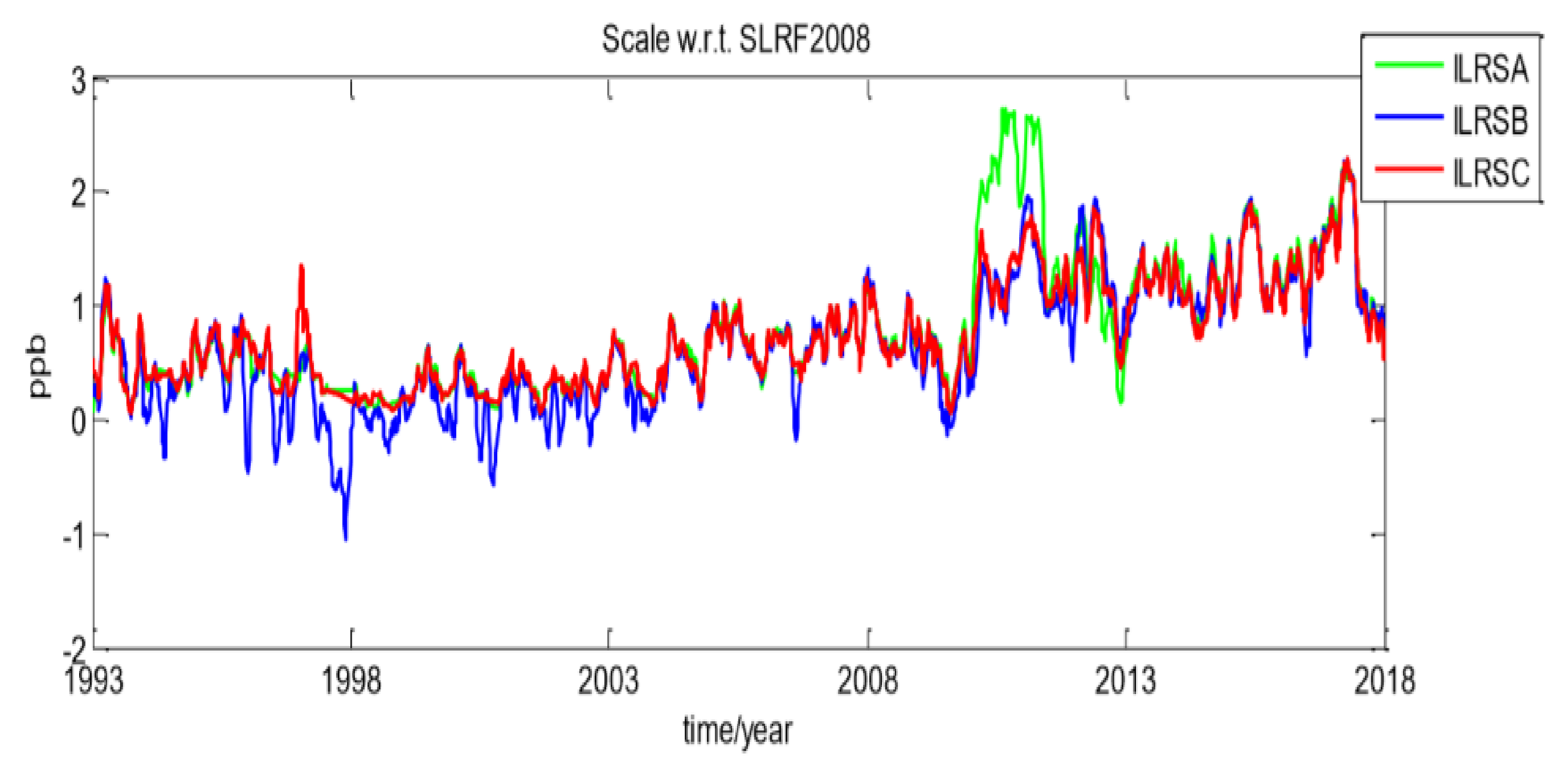


Figure. Time series comparison of the scale parameter of our combined solution ,ILRSA and ILRSB with respect to SLRF2008

4.Results and analysis

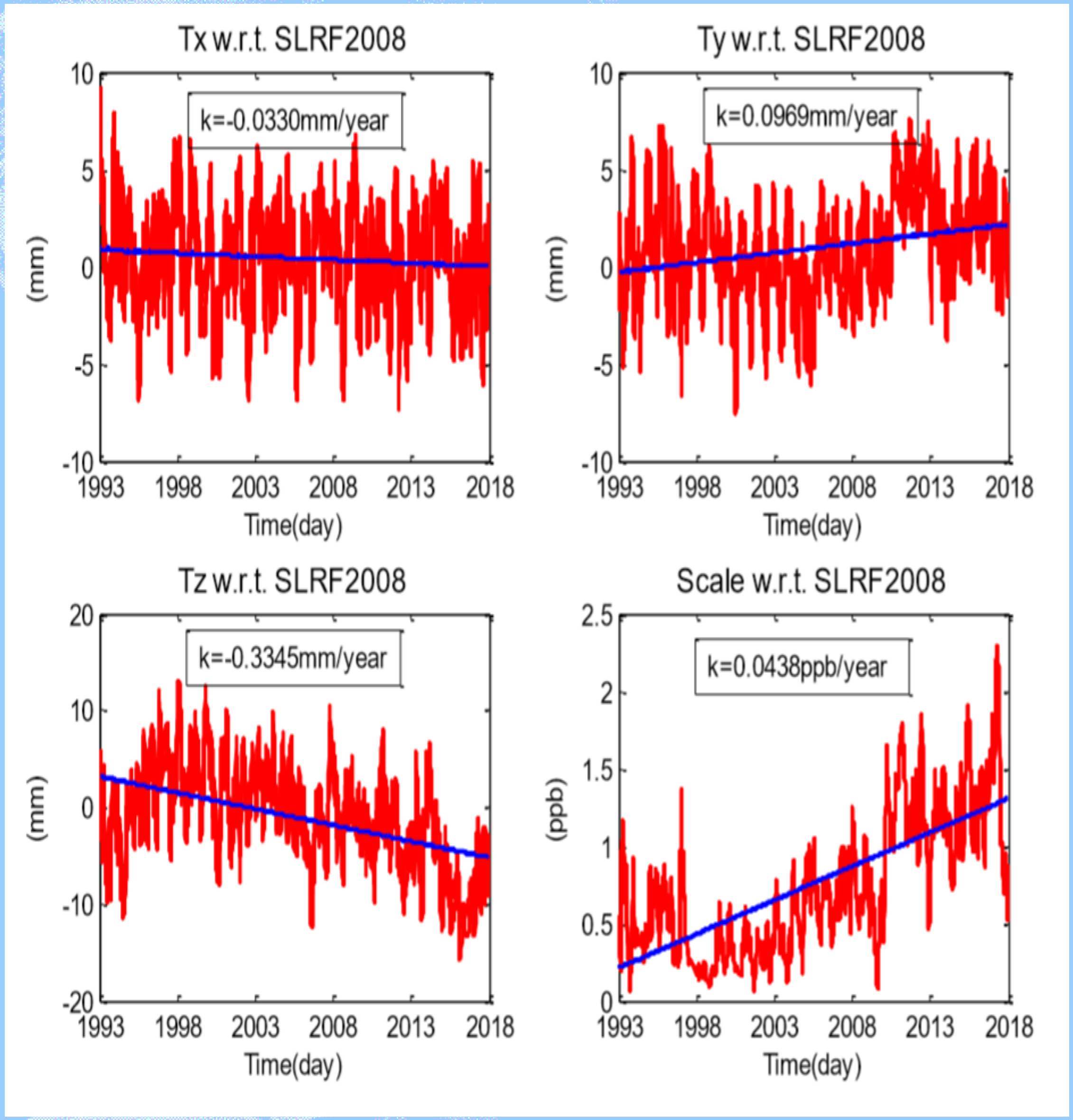
4.3 Analysis of translation parameters and scale factor

	Tx(mm)	Ty(mm)	Tz(mm)	Scale(ppb)
ILRSA	0.63(±3.80)	0.82(±3.46)	-1.08(±6.57)	0.85(±0.62)
ILRSB	0.75(±4.38)	0.78(±3.85)	-1.51(±8.63)	0.64(±0.63)
ILRSC	0.43(±3.76)	0.94(±3.55)	-0.99(±6.40)	0.79(±0.62)

From the table we can see that the mean values of ILRSC translation parameters and scale parameters are close to those of ILRSA and ILRAB. But the standard deviations are little smaller and more stable w.r.t ILRSA and ILRSB.

4.Results and analysis

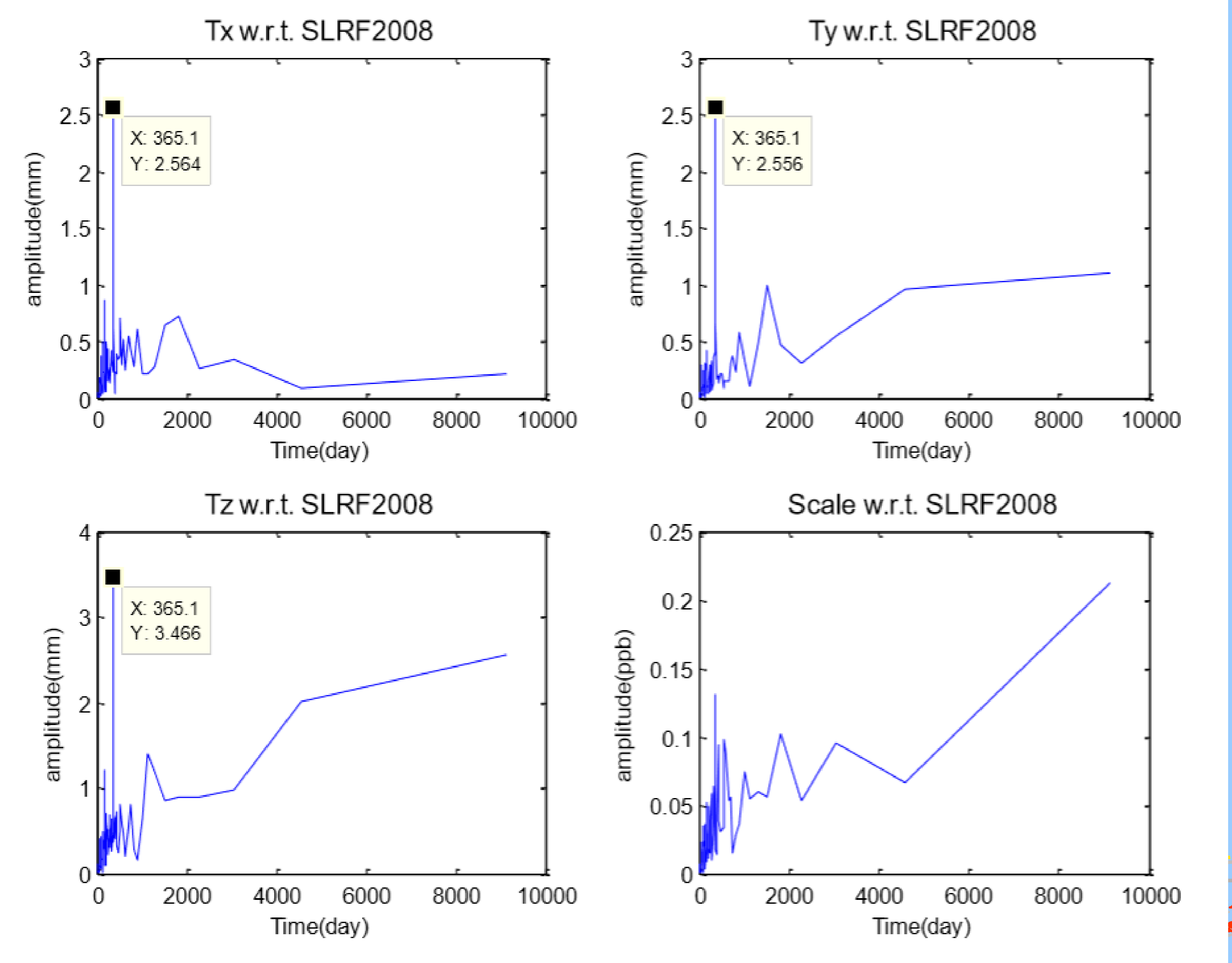
4.3 Analysis of translation parameters and scale factor



we detected the characteristic of origin and scale parameter of the ILRSC. By fitting dominant linear terms of translation and scale parameters, we get linear change rate of translation parameters and scale parameters are 0.0330mm/yr, 0.0969mm/yr, 0.3345mm/yr and 0.0438ppb/yr. As shown in the figure, the origins show a linear change mainly in Z direction.

4.Results and analysis

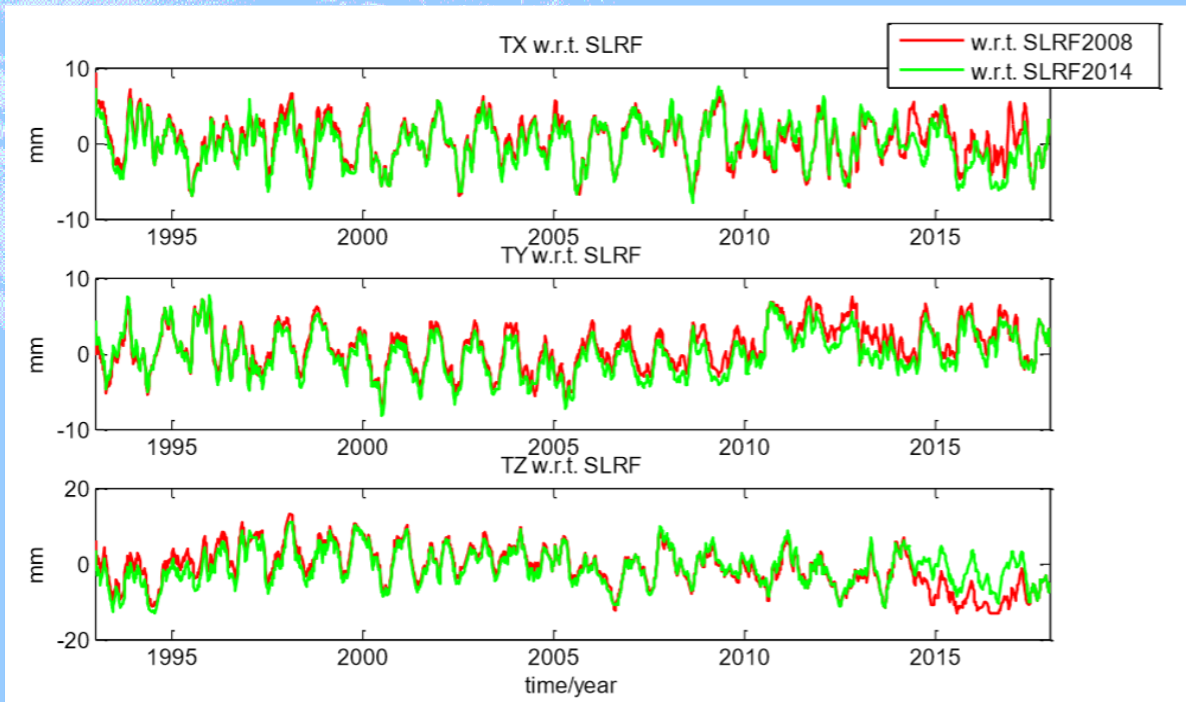
4.3 Analysis of translation parameters and scale factor

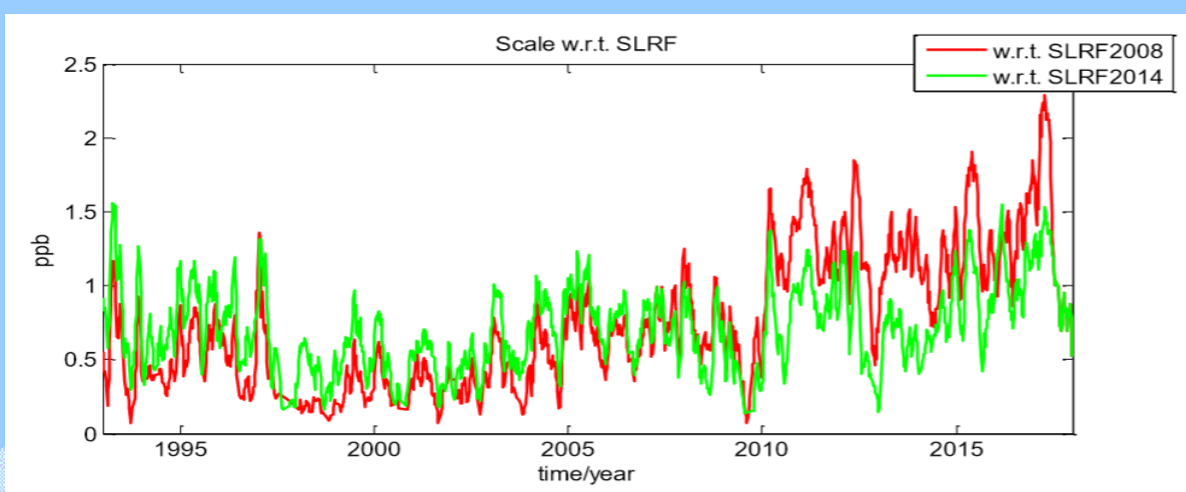


After these linear change are removed the Fourier spectrum analysis is applied to them. We find the translation parameters have an annual term of 2.564mm in X, 2.556mm in Y and 3.466mm in Z.

4.Results and analysis

4.4 Analysis of difference between SLRF2008 and SLRF2014

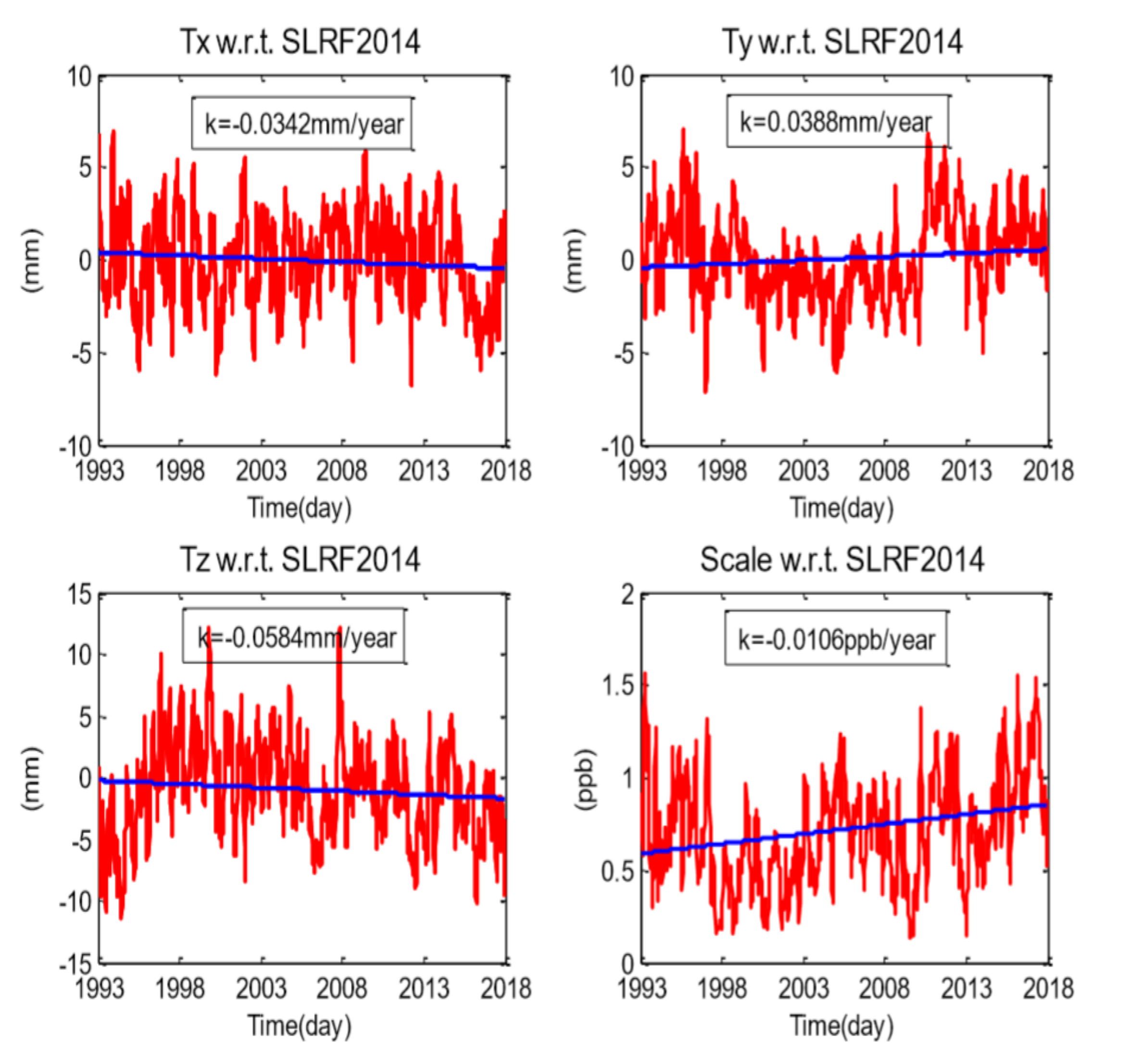




we can see that amplitude of translation parameters of SLRF2014 is obvious smaller w.r.t SLRF2008 after 2014. The change curve is more smooth both for translation parameters and the scale factor. This shows SLRF2014 is more stable than SLRF2008

4.Results and analysis

4.4 Analysis of difference between SLRF2008 and SLRF2014



Linear fitting results of translation and scale parameters of ILRSC combined weekly solution w.r.t SLRF2014 shows the linear change rate of translation and scale parameters are 0.0342mm/yr, 0.0388mm/yr, 0.0584mm/yr and 0.0106ppb/yr. The translation and scale parameters' change rate in X direction is little higher than that of SLRF2008. But in Y and Z direction they show a significant reduction than that of SLRF2008 .

4.Results and analysis

4.5 Evaluation of SHAO SINEX solutions

Table. SHAO SINEX solution relative weight factor comparison with other ACs

AC	ASI	BKG	DGFI	ESA	GFZ	GRGS	JCET	NSGF	SHAO
mean	8.85	11.43	20.21	11.67	14.97	10.5	11.75	10.45	**11.19**
Std deviation	15.37	22.15	23.04	15.66	17.78	15.57	21.77	18.86	**18.66**

4.Results and analysis

4.5 Evaluation of SHAO SINEX solutions

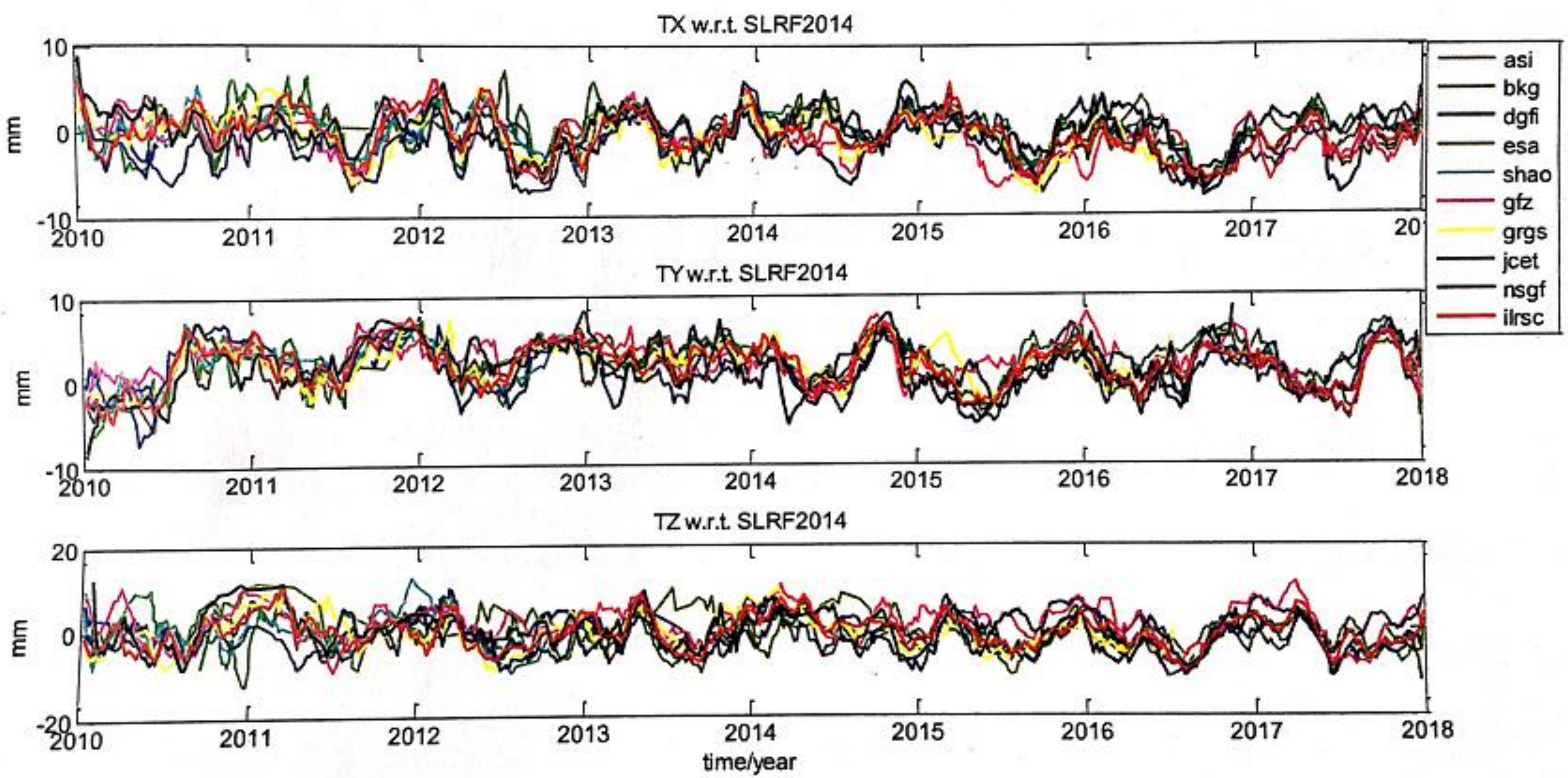


Figure. Translation parameters w.r.t SLRF2014

4.Results and analysis

4.5 Evaluation of SHAO SINEX solutions

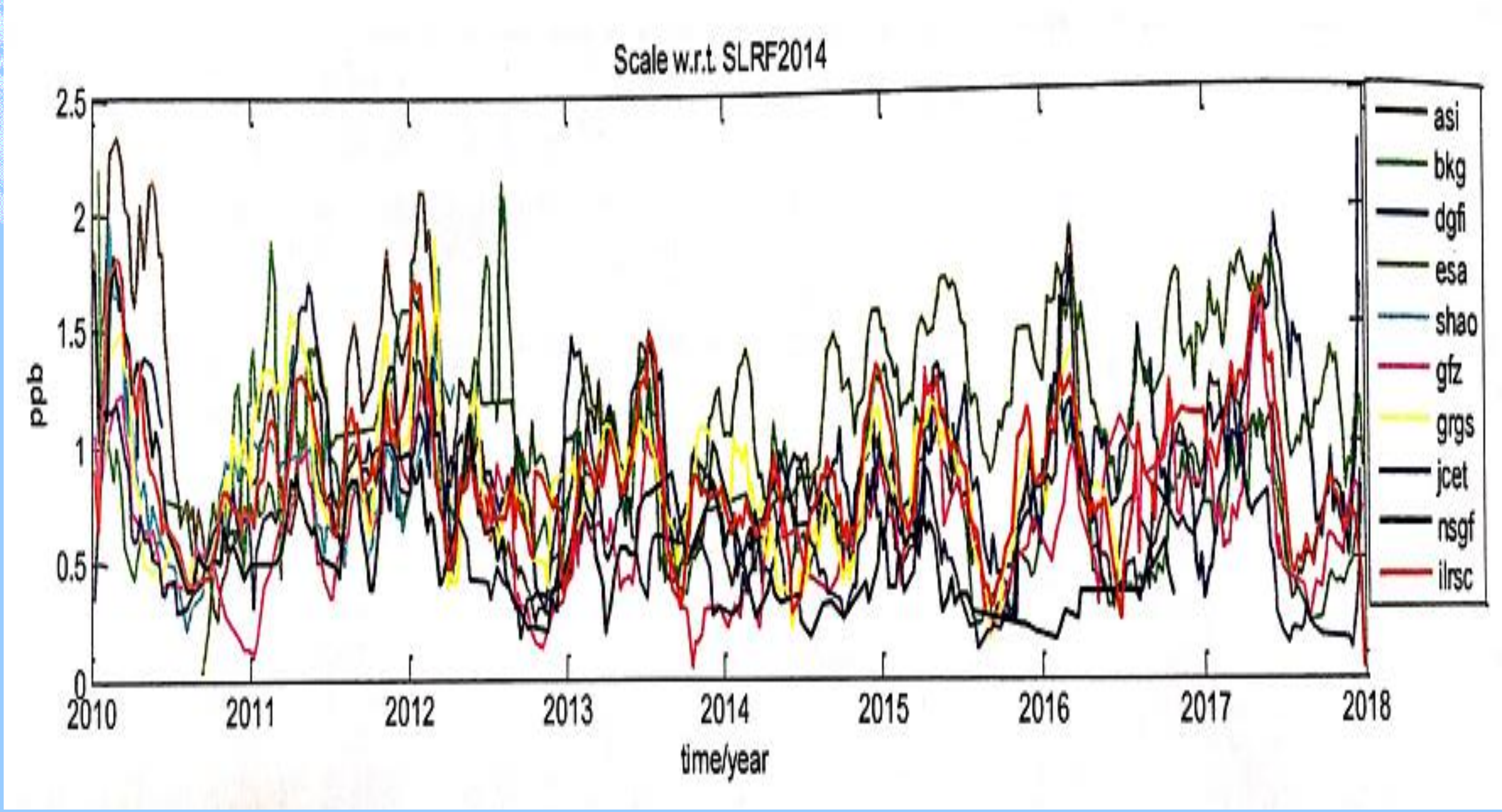


Figure. Scale parameters w.r.t SLRF2014

5. Conclusion

- SHAO could provide SLR SINEX solutions with accuracy about 1cm 3D coordinates and 0.11±0.21 mas for Xp, 0.08±0.17 mas for Yp and 0.01±0.06 ms for LOD. Its accuracy and stability are the middle of ILRS ACs
- SHAO also could provide the combination solutions with 5.67±4.33 mm for 3D coordinates and -0.035 ±0.187 mas for Xp, 0.002 ±0.176 mas for Yp and -0.001±0.048 ms for LOD.
- ILRSC relative weight factors are consistent with those of ILRSA and ILRSB. They show the same results for bad solutions at average level. ILRSC station coordinates and EOP are better than that of individual AC too.

5. Conclusion

- ILRSC translation and scale parameters show consistency with that of ILRSA and ILRSB. From 1993 to 2004, these parameters are more consistent with that of ILRSA, but from 2005 to 2017, they are more consistent with that of ILRSB. The above comparison results verify the reliability of ILRSC combination solutions and also show the existence of the third CC is specially necessary.
- The stability of TRF is discussed by ILRSC combined weekly solutions w.r.t SLRF2008 and SLRF2014 respectivily. From the characteristic comparisons of translation and scale parameters, we can verify that SLRF2014 is more stable than SLRF2008.

Thank you for your attention！

Acknowledgments

IERS/ILRS, ILRS ACs and ILRS CCs

the National Key R&D Program
(No.: 2016YFB0501405)

National Natural Science Foundation of China
(No.: 11973073)

中国科学院上海天文台
Shanghai Astronomical Observatory
Chinese Academy of Sciences