



Photometry with Gated SPAD: Theory and Approach

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ABSTRACT

- Gated single-photon avalanche diode (SPAD) is widely used in satellite laser ranging (SLR). With proper calibration, SPAD is possible to make photon flux measurement simultaneously while performing laser ranging. This presentation models the photon flux response of a gated SPAD while performing laser ranging. Algorithm to analyze range data for input photon flux is introduced.
- Experiment of photon flux measurement is done with Changchun SLR system. Fluxes are measured for dark current, night sky background, day sky background, bright stars and some ILRS targets in typical scenario.
- The method can be widely used in applications involving signal strength of echoes.



ABSTRACT

- Gated single-photon avalanche diode (SPAD) is widely used in satellite laser ranging (SLR). With proper calibration, SPAD is possible to measure photon flux. This paper introduces the response of a gated SPAD while performing laser ranging. Algorithm to analyze range data for input photon flux is introduced.
- Experiment of photon flux measurement is done with Changchun SLR system. Fluxes are measured for dark current, night sky background, day sky background, bright stars and some ILRS targets in typical scenario.
- The method can be widely used in applications involving signal strength of echoes.

Get Brightness of Echoes



Outline

1. Background and Problem
2. Theory and Algorithm
3. Experiment and Analysis
4. Discussions



1. Background and Problem

Background

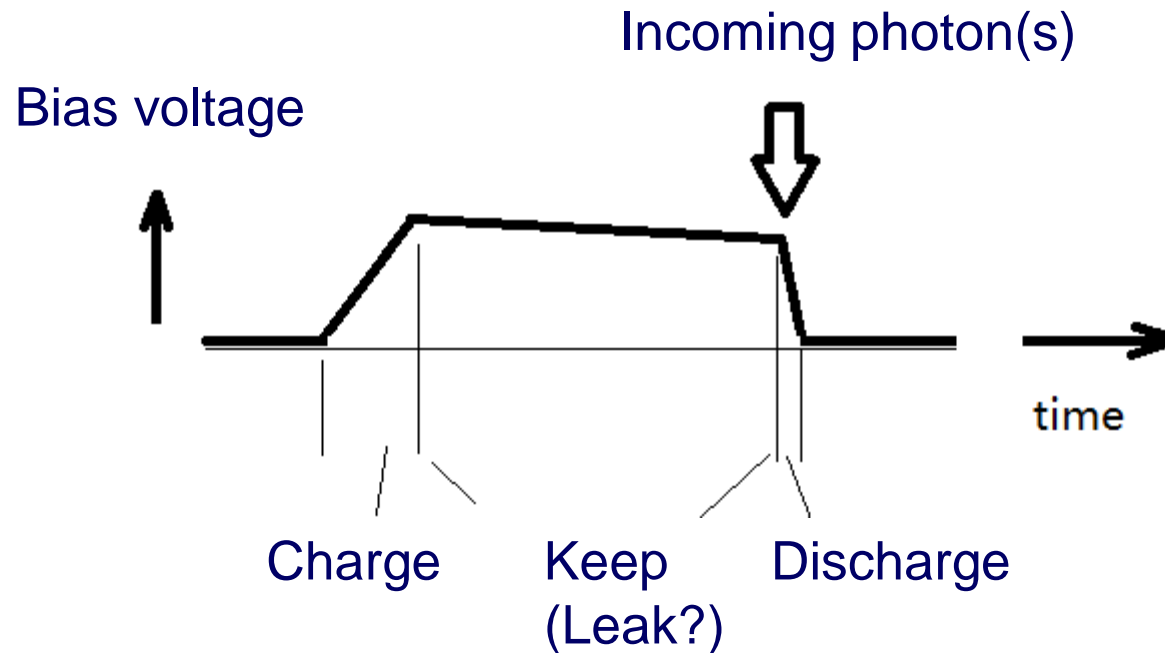
- Avalanche Photon Diode (APD) is used in laser ranging.
- Range precision/biases are hot topics, while luminance is rarely mentioned.
- The existing metric for signal luminance is return rate.



1. Background and Problem

APD work cycle:

Charge > Keep > Discharge.

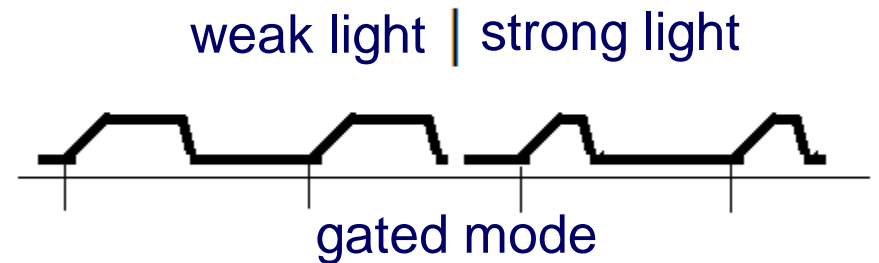
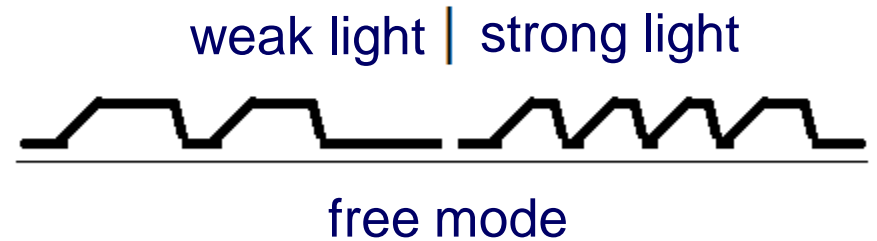




1. Background and Problem

APD Photometry

- Free Mode:
 - APD charges again immediately after avalanche.
 - Used in normal photometry.
- Gated Mode:
 - APD charges only when gate signal allows.
 - Used in SLR.





1. Background and Problem

Background

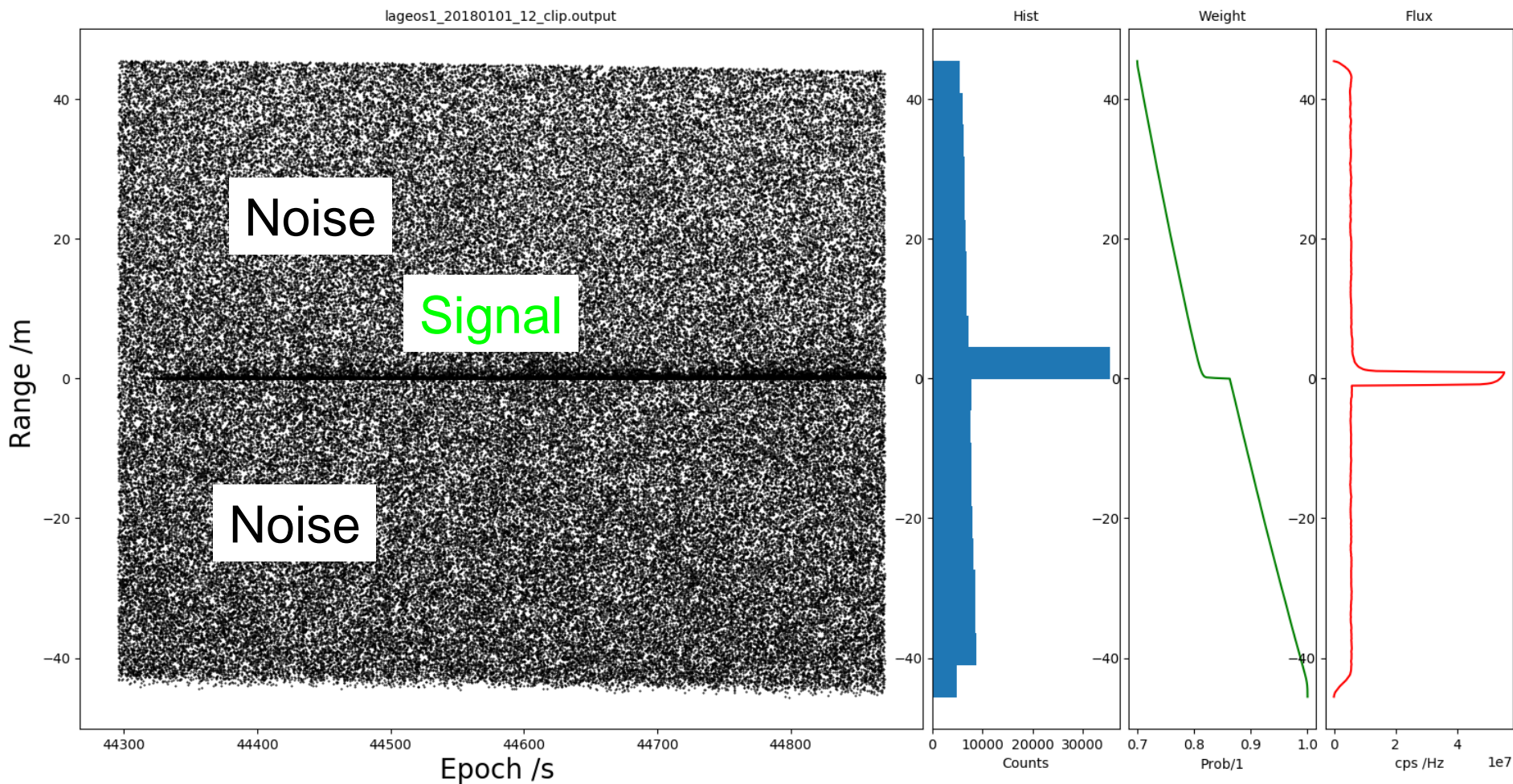
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Problem

- Return rate is inaccurate in measuring incoming photon flux.
- Heavy noise may reduce return rate.

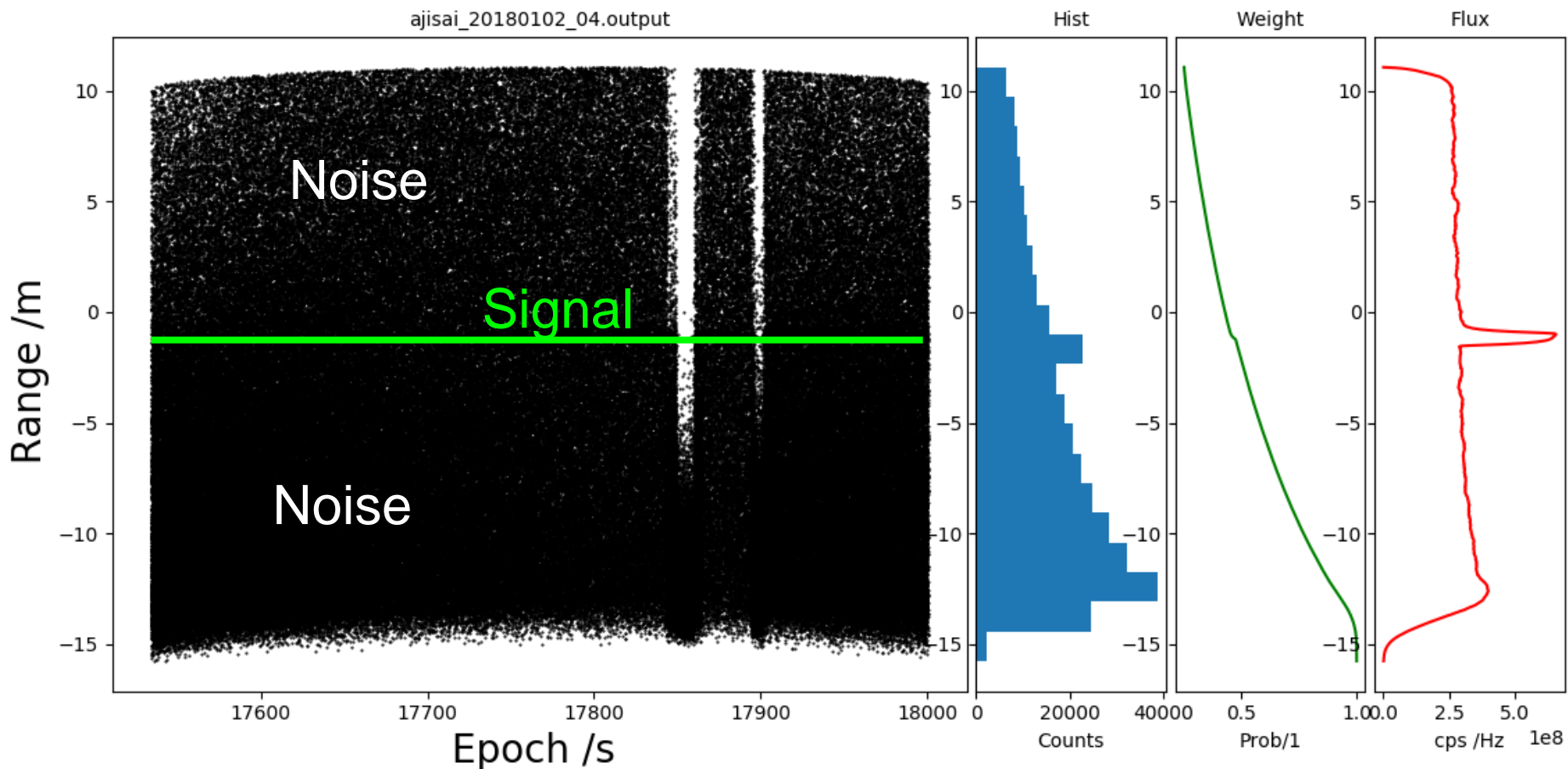


Typical O-C Plot (Night Lageos1)





Typical O-C Plot (Daylight Ajisai)

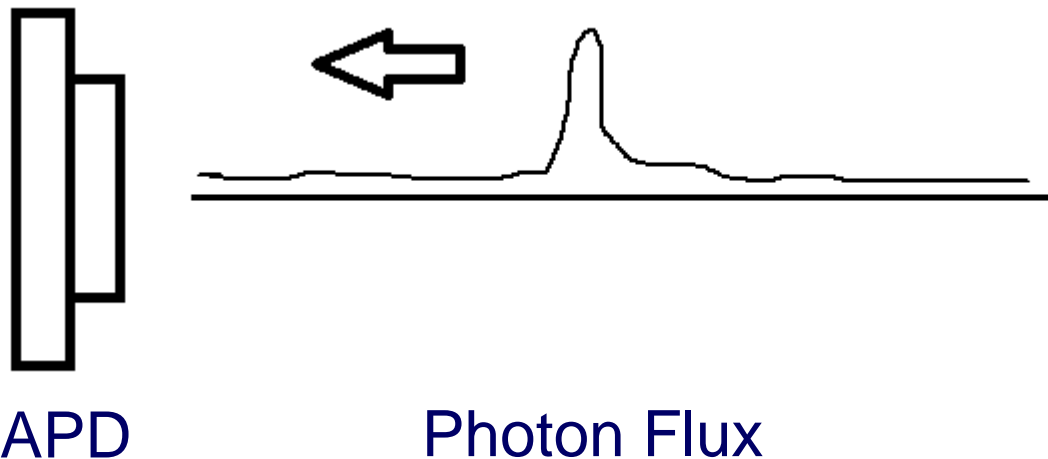




2. Theory and Algorithm

2.1 Assumption

- The photon flux $S(t)$ is repeated on every measurement.
- The APD avalanches once on every measurement.





2. Theory and Algorithm

2.2 Mathematical Modeling

- The APD gets charged up before step 1.
- Each step has Δt time-span.
- P_i – In i -th step, the probability that APD is discharged.
Therefore $P_0 = 0$
- S_i – In i -th step, the photon flux (count per second, cps)
- η – Quantum efficiency, number of photoelectrons excited by one photon.
- After a lot of mathematics (See Appendix 1)

$$S_i = \frac{1}{\eta \Delta t} \ln\left(\frac{1 - P_i}{1 - P_{i+1}}\right)$$



3. Experiment and Analysis

3.1 Photometry of Dark Noise and Sky Noise

3.2 Photometry of Stars

3.2.1 Stellar Magnitudes

3.2.2 Measurement Data Display

3.3 Photometry of Satellites

3.3.1 Avalanche Rate Measurement

3.3.2 Avalanche Rate to Magnitude



3. Experiment and Analysis

3.1 Photometry of Dark Noise and Sky Noise

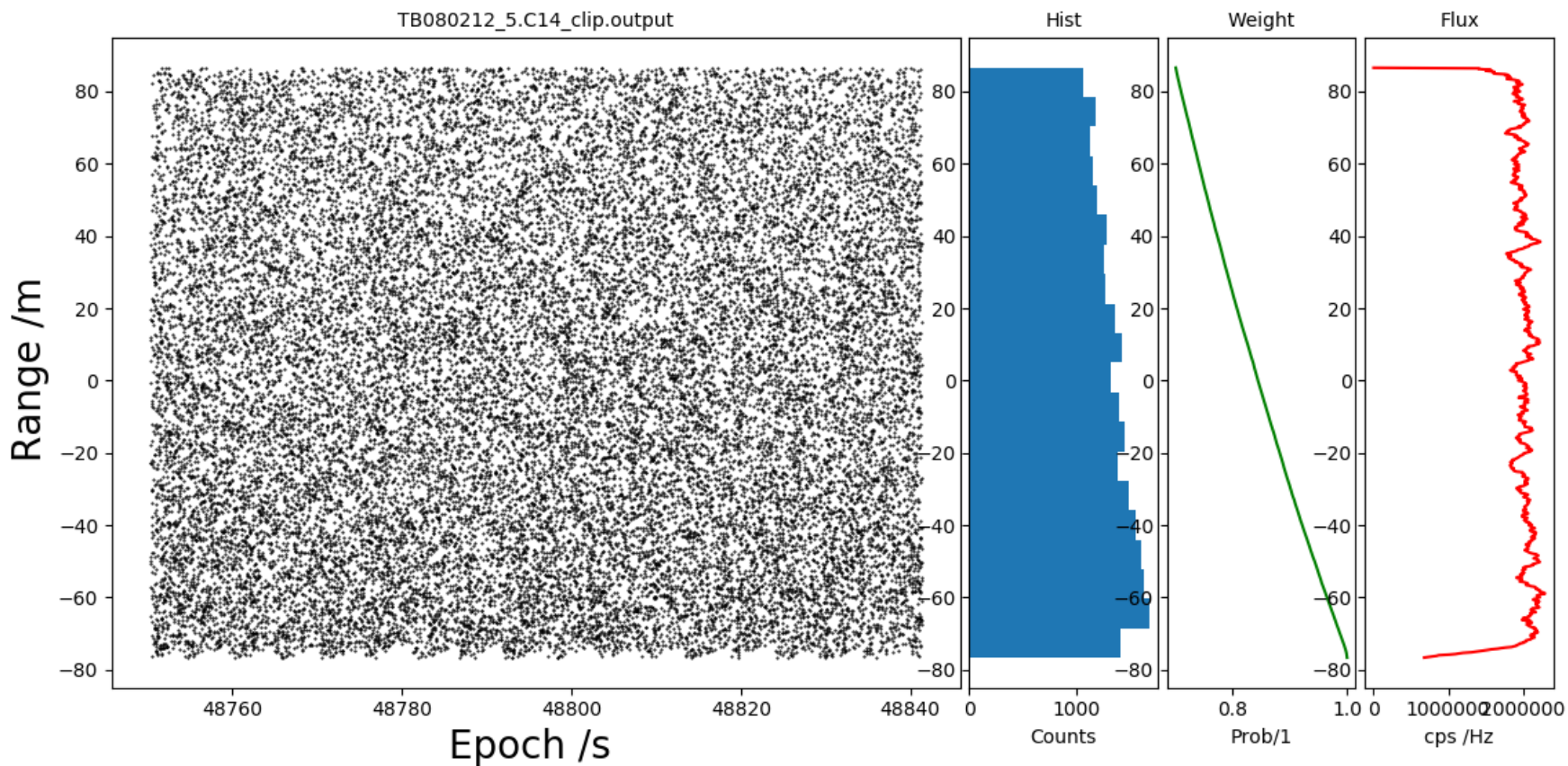
- Measure Dark Noise
 - Block transmission beam.
 - Plug receive hole. (?)
 - Do laser ranging.
- Measure Sky Noise
 - Block transmission beam.
 - Aim at sky.
 - Narrow down the FOV iris.
 - Do laser ranging.



Cork Plugs

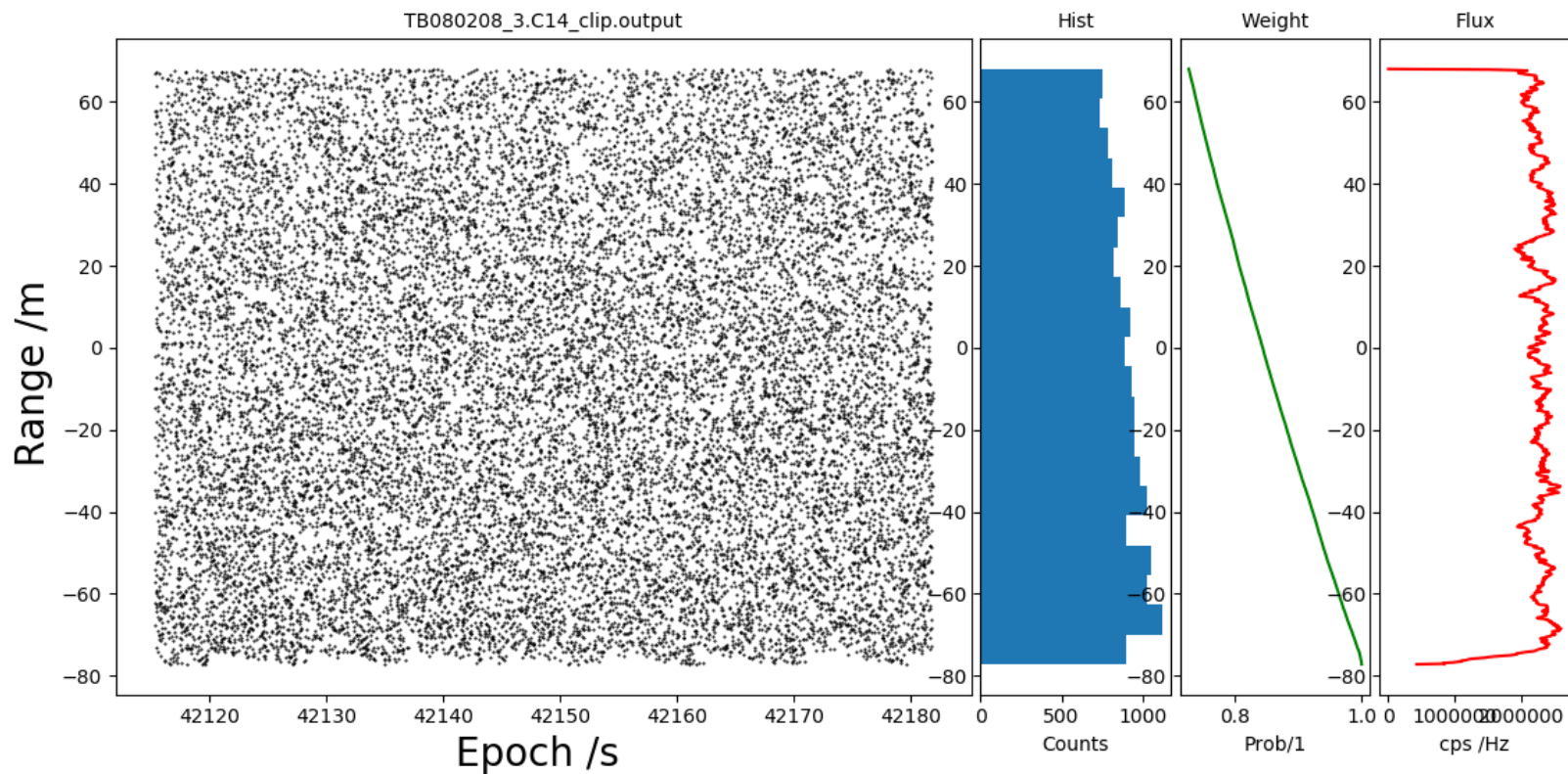


Dark Noise (Plug)





Sky Background Noise





3. Experiment and Analysis

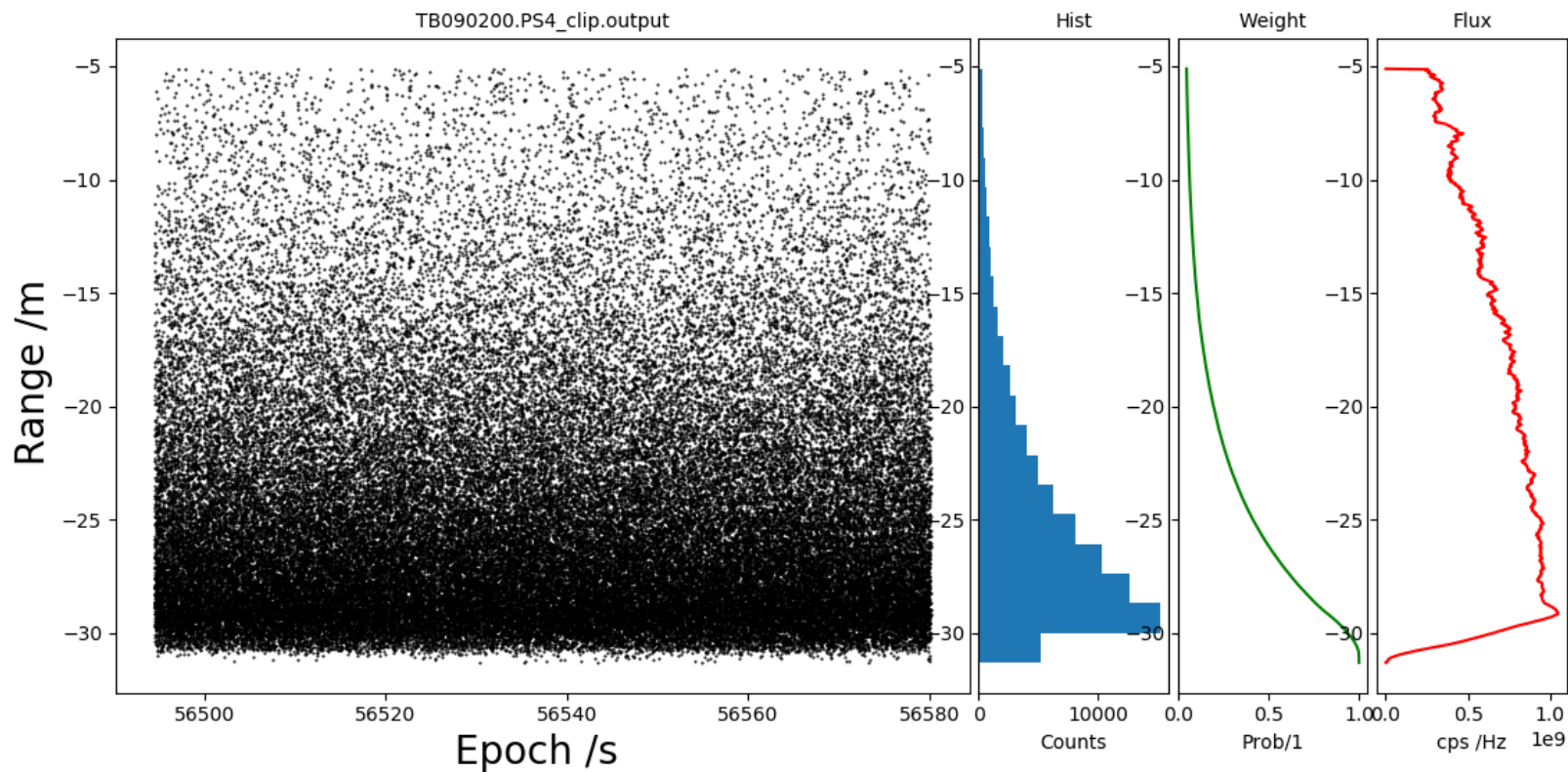
3.2 Photometry of Stars

- Arbitrarily choose bright stars around current zenith.
- Get star information from SIMBAD database.
- For 532nm wavelength, choose V magnitude among UBVRIJHK fluxes. (For 1064nm choose I.)

Catalog No.	Star Name	Vmag
HD8890	Polaris	V 2.02
HD432	bet Cas	V 2.27
HD3712	alf Cas	V 2.23
HD8538	del Cas	V 2.680
HD11415	eps Cas	V 3.37
HD20902	alf Per	V 1.79
HD137422	gam UMi	V 3.002
HD142105	zet UMi	V 4.274



Photometry for Polaris (Vmag 2.02)

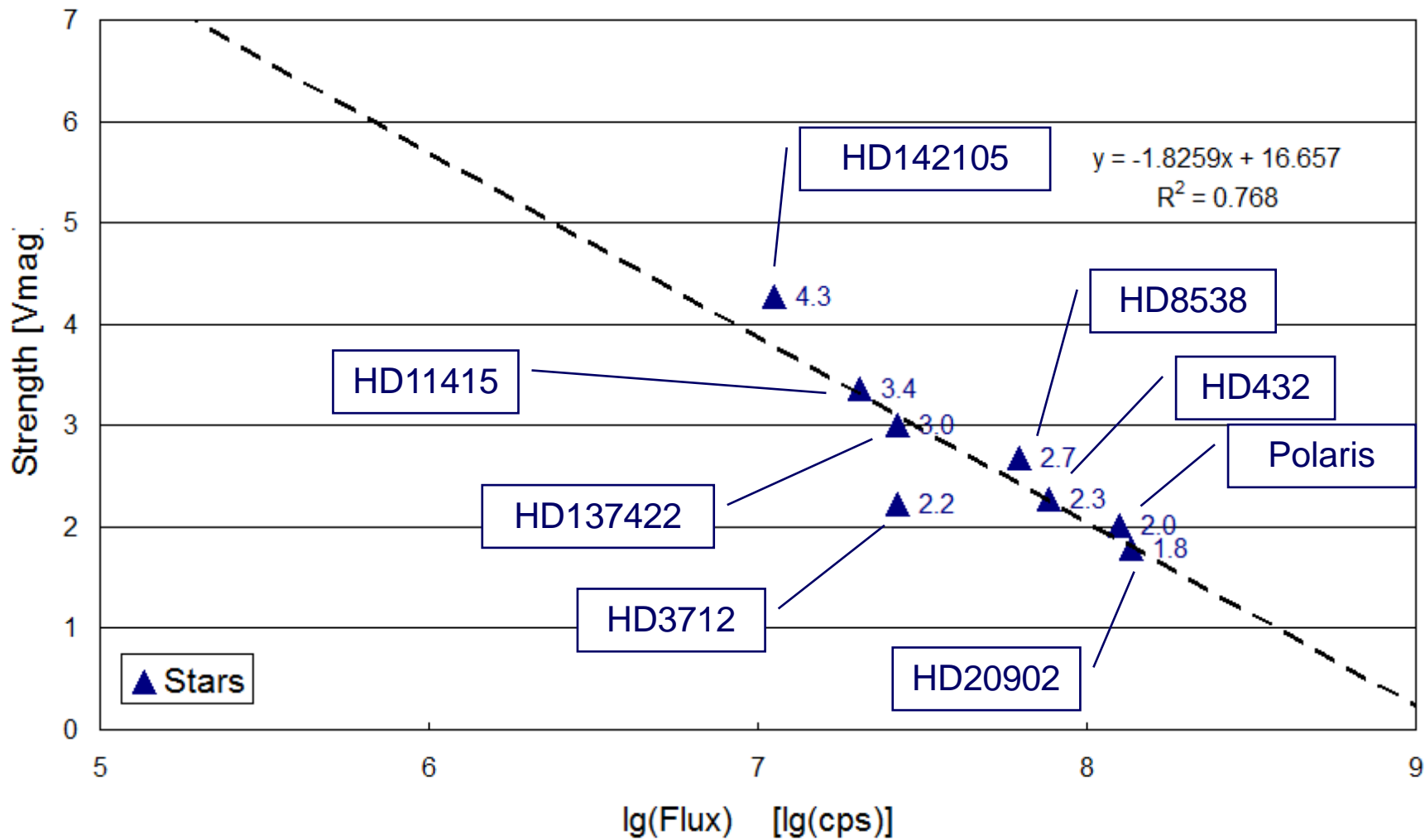




3. Experiment and Analysis

3.2 Photometry of Stars

- Steps:
 - Track a star.
 - Block transmission beam.
 - Do laser ranging.
- Remove dark noise flux from star flux measurements.
- Fit nominal V_{mag} to $\lg(\text{flux})$ with linear regression.





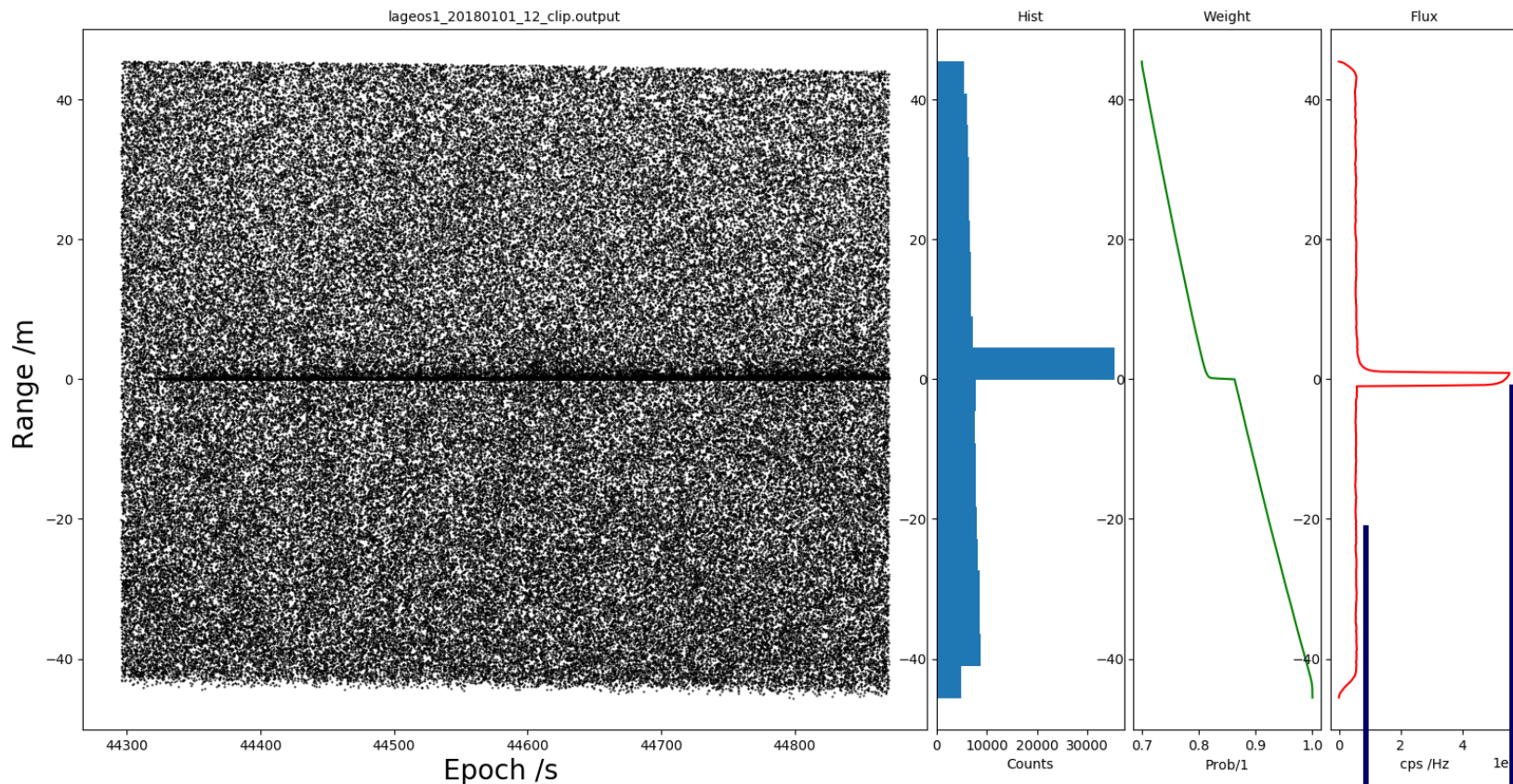
3. Experiment and Analysis

3.3 Photometry of Satellites

- Avalanche rate measurement:
 - Lageos-1 at night.
 - Starlette at night.
 - Ajisai in daylight with narrowband filter(0.15nm@532.08nm).
- The echo pulse contains dark noise + retroreflection.
- The background noise contains dark noise + sky background noise + satellite surface reflection.



Lageos-1 (Night)

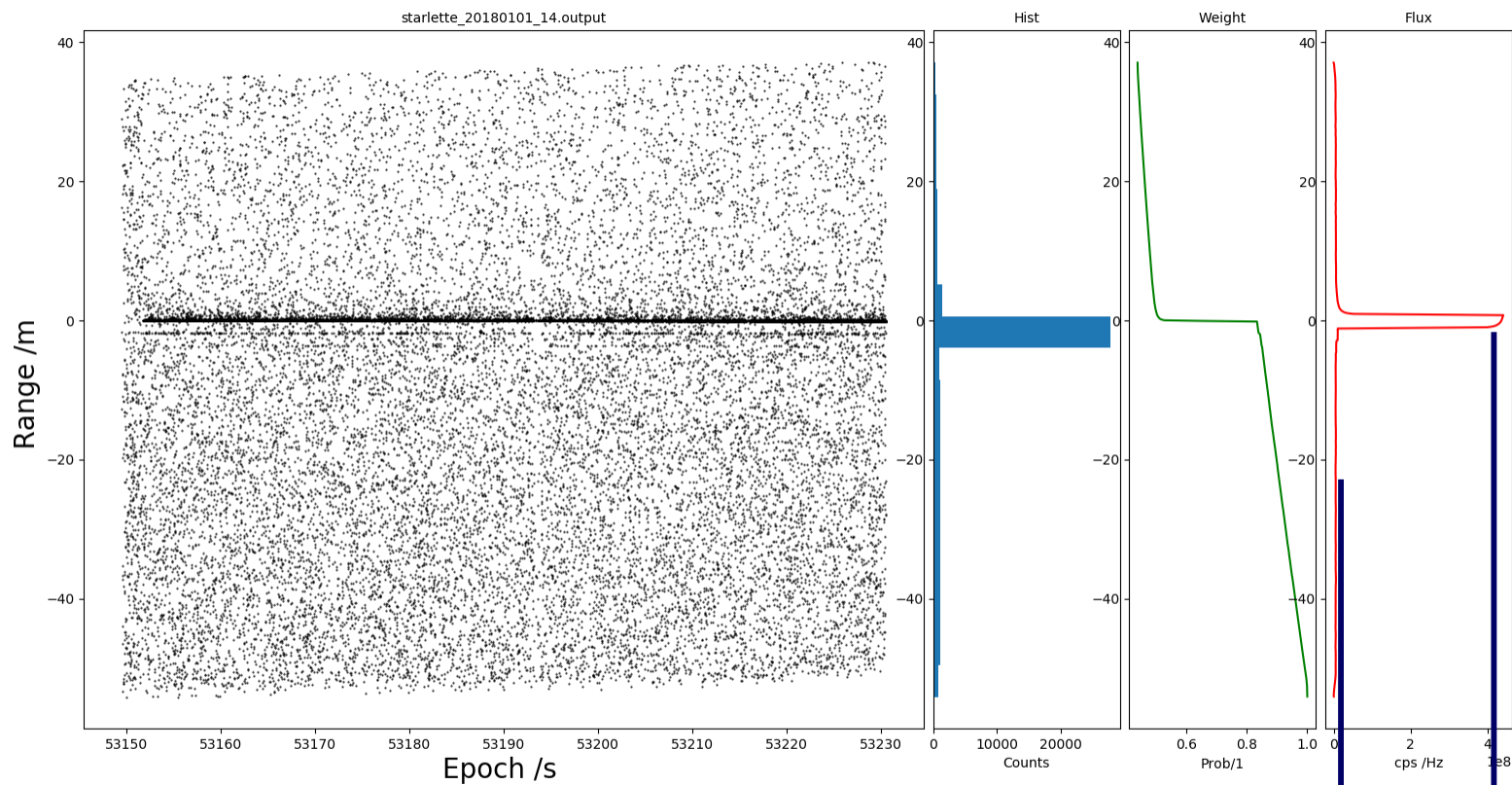


Background Noise: 4.6 mag

Lageos1 Echo: 2.6 mag



Starlette (Night)

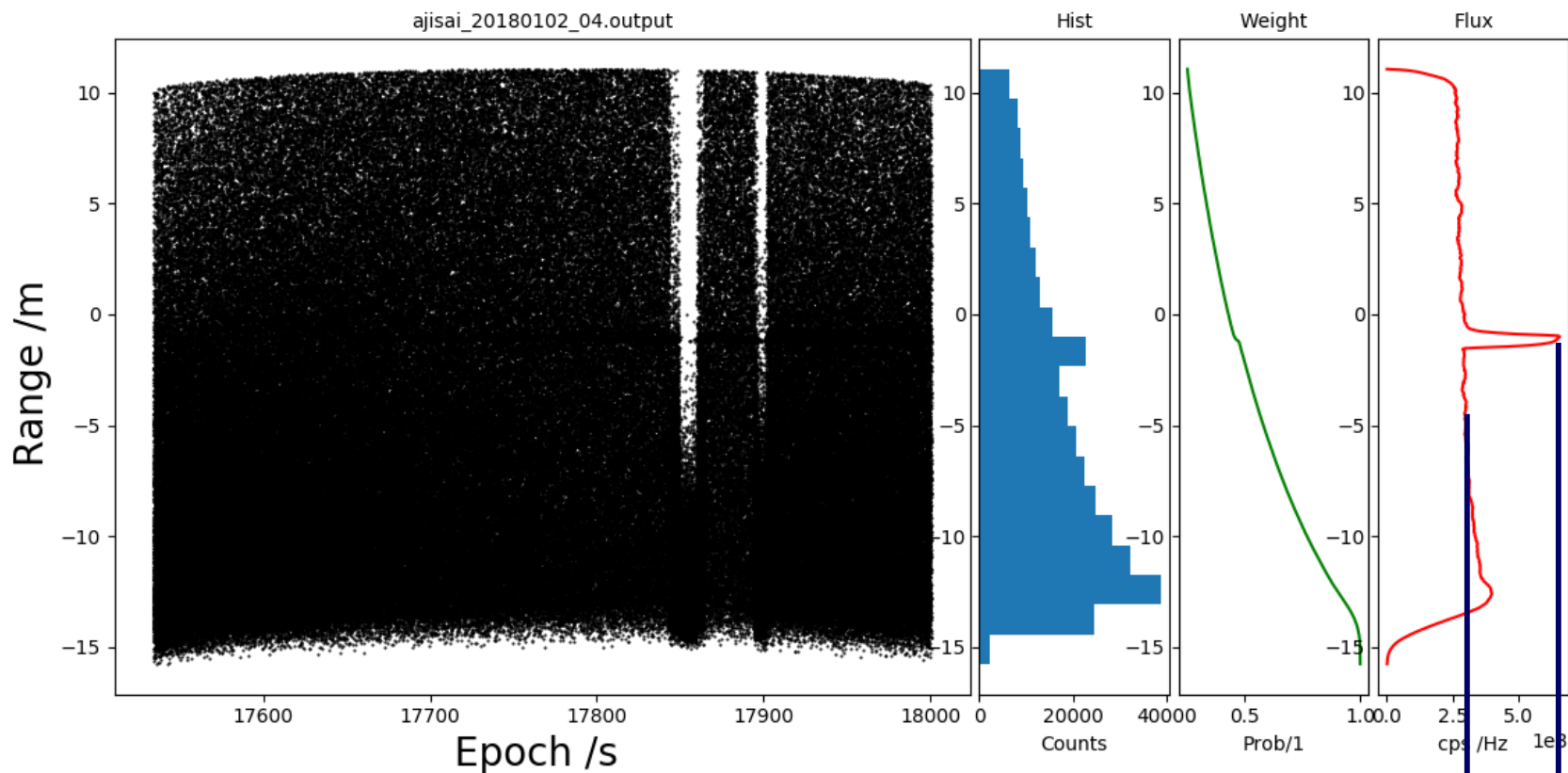


Night noise: 3.7 mag

Starlette Echo: 0.9 mag



Ajisai (Day & NBF)



Daylight noise (filtered): 1.2 mag

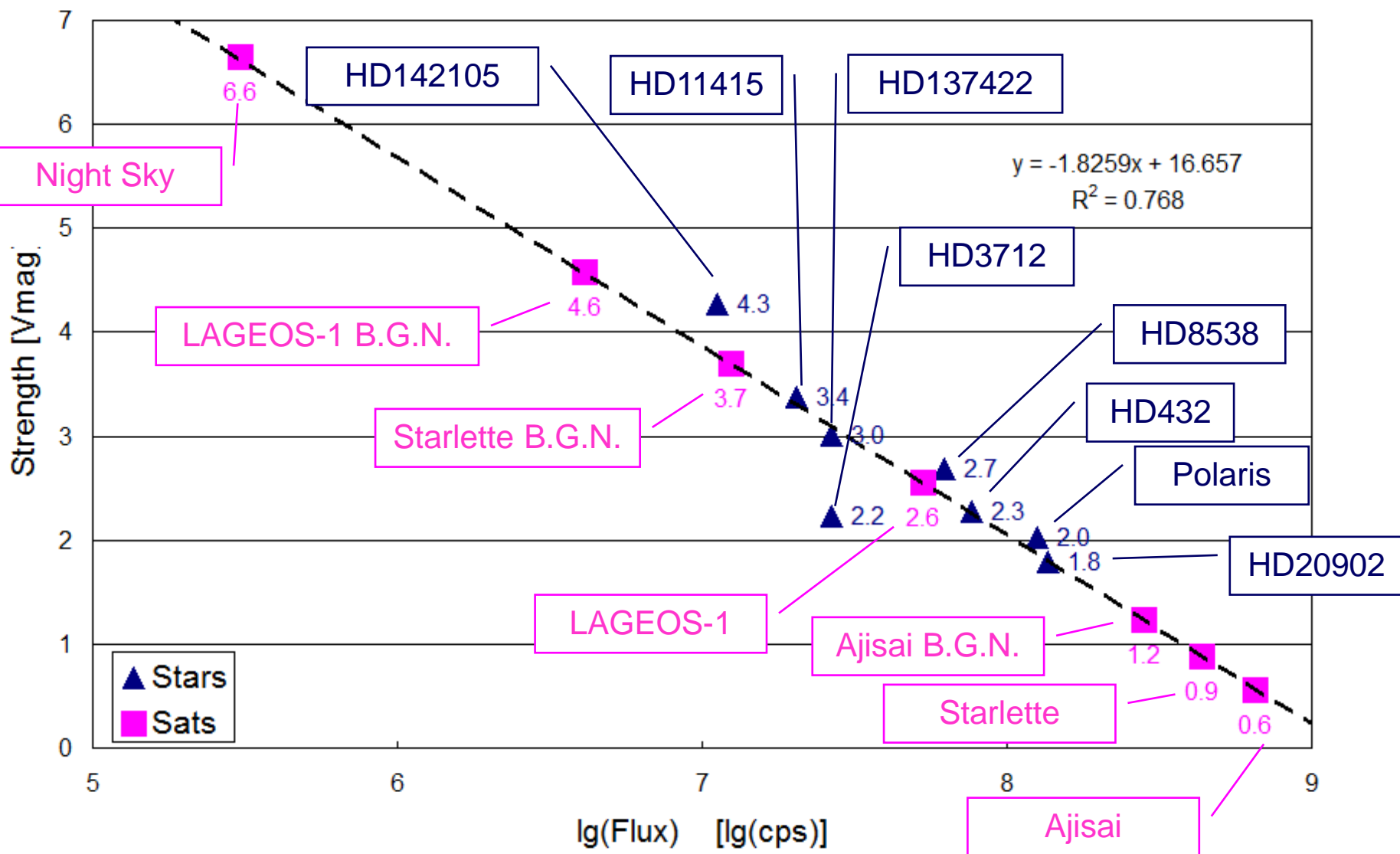
Ajisai Echo: 0.6 mag

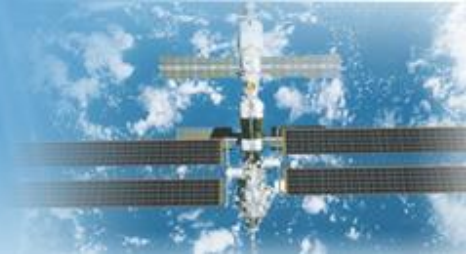


3. Experiment and Analysis

3.3 Photometry of Satellites

- Avalanche Rate to Magnitude
 - Remove dark noise flux from measurements.
 - Find corresponding $\lg(\text{flux})$
 - Find equivalent V_{mag} from the star-fit correlation.



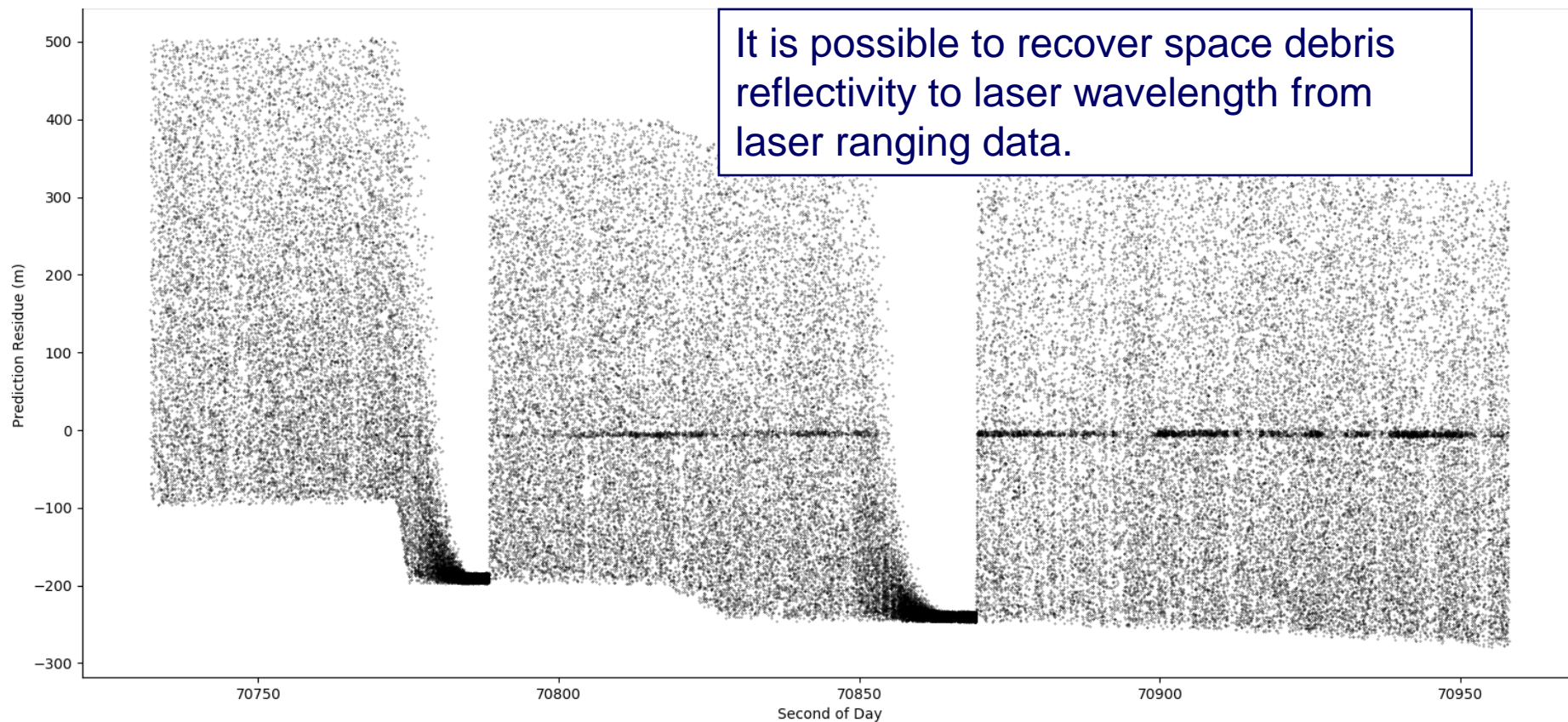


3. Discussions

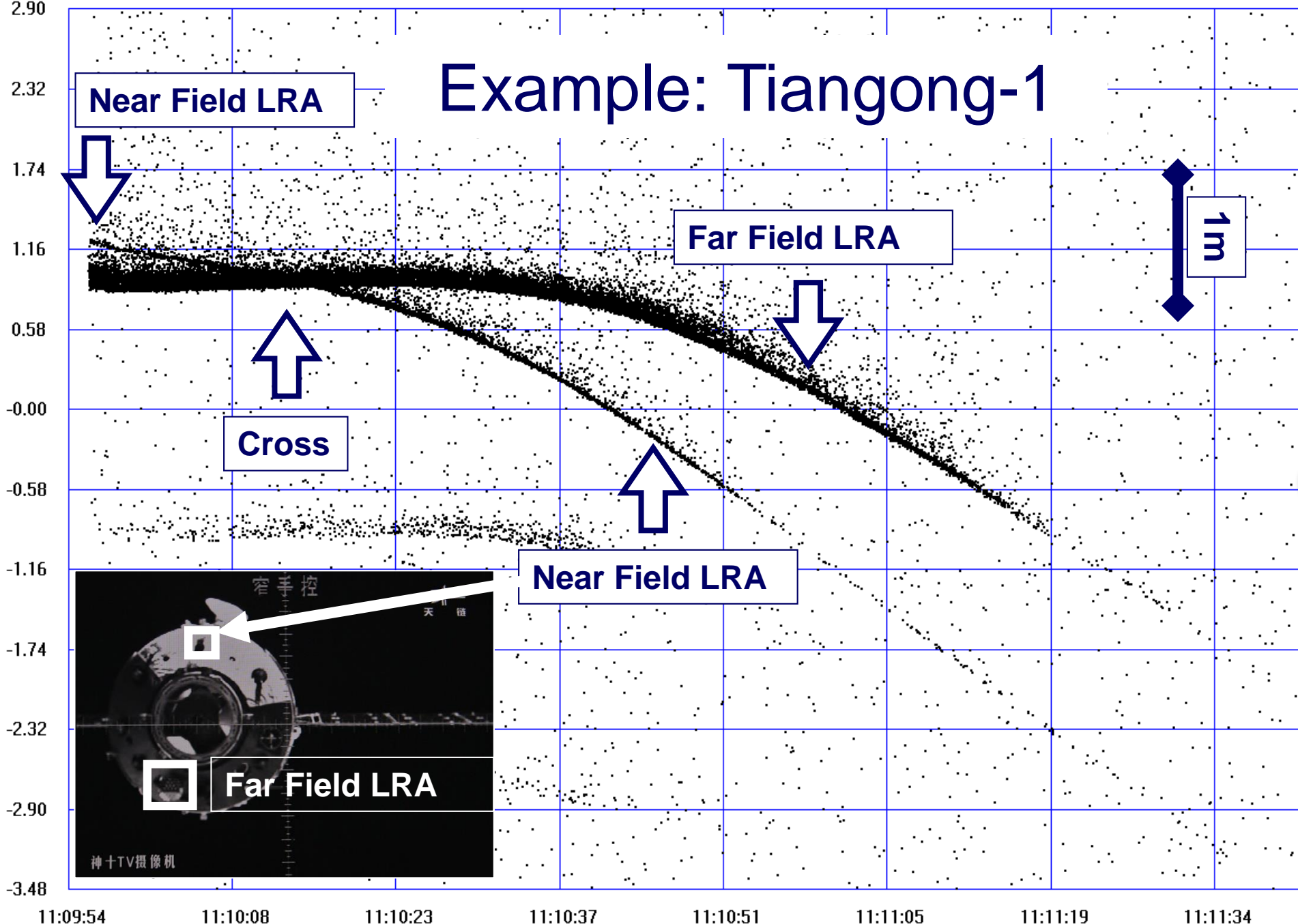
- Possible Applications:
 1. Applicable to **ALL** gated single photon detectors.
 2. SD monochrome reflectivity from LR data.
 3. Multiple reflector strengths: SNETs, Technosat, Tiangongs
 4. Rapid diagnosis of LR system.
- Possible Improvements:
 1. Photometry on more & stable stars. (Now variable stars)
 2. Adapt for inclined echoes. (Now horizontal)
 3. Uncertainty assessment. (Now none)
 4. Incorporate APD hardware tests: $QE(t)$ (Now constant QE)
 5. Investigate sunlit vs. shadowed targets. (Now none)
 6. LAMOST 532nm spectral flux of stars. (Now none)



Example: Space Debris

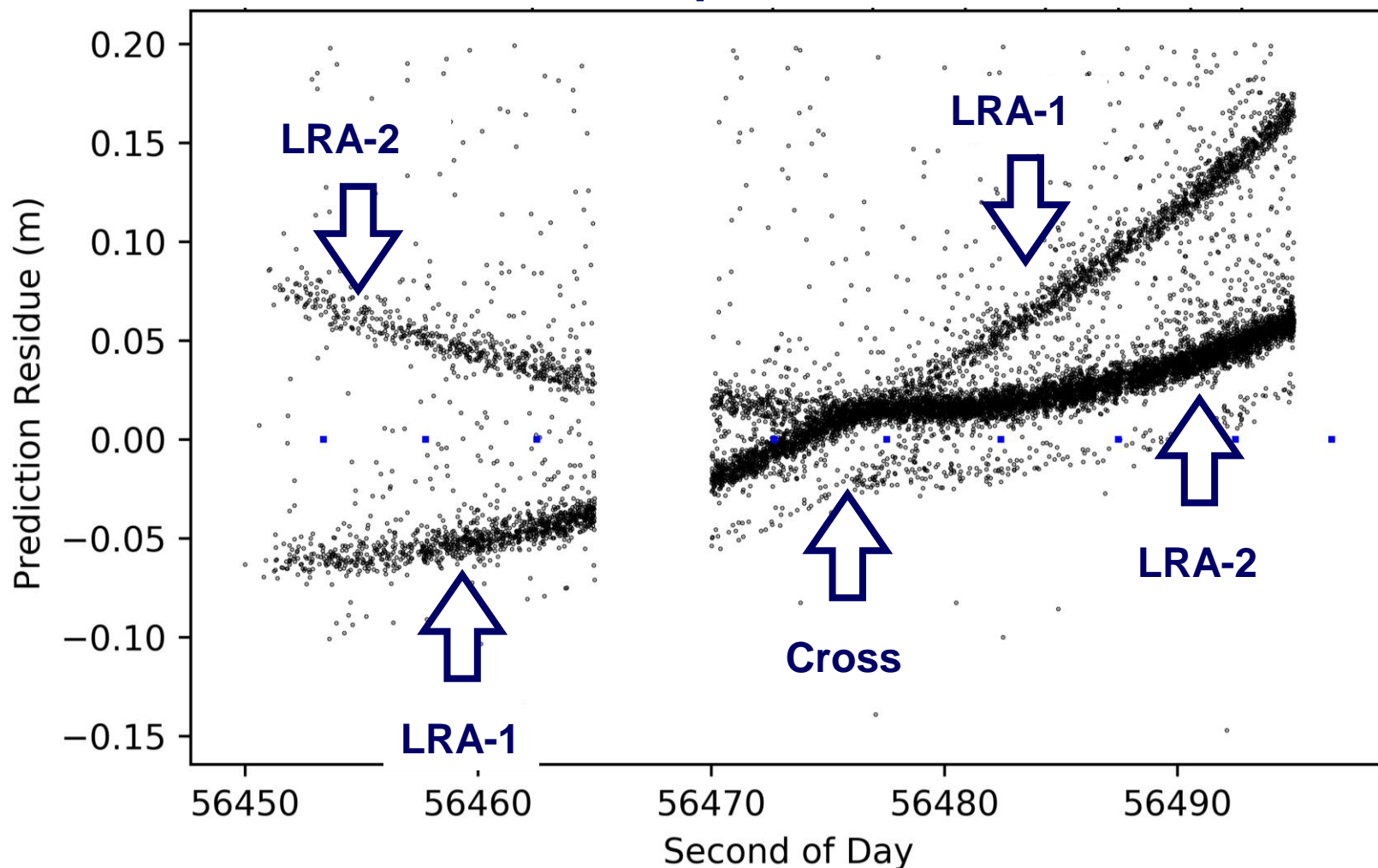


Example: Tiangong-1





Example: SNET





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Changchun Observatory, NAO, Chinese Academy of Sciences



Thank you!



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Changchun SLR



Appendix A

- APD Temporal Response Mathematical Model
- A1: Continuum Problem
- A2: Discrete Problem
 - Discretization of the continuum problem
 - Algorithm to solve flux rate



A1. Continuum Problem

Definitions

- The APD gets charged up at epoch $t_0=0$
- $P(t)$ – At epoch t , the probability that APD is discharged (avalanched).
- $F(t)$ – At epoch t , the probability that APD remains charged.
- $S(t)$ – At epoch t , the photon flux rate (count per second, cps)
- η – Time invariant quantum efficiency, i.e. number of photoelectrons excited by each photon.



A2. Discrete Problem

- Choose Δt sufficiently small, so that:
 - All photons within Δt participate in the avalanche as a whole.
 - i.e., later photons are not wasted.
 - Note that Δt is hardware related.

• Let: $i = 0, 1, 2, \dots, N$ $t_0 = 0$ $t_i = i \cdot \Delta t$

$$P_i = P(t_i)$$

$$F_i = F(t_i)$$

Discretization of the continuum problem

$$S_i = \frac{1}{\Delta t} \int_{t_{i-1}}^{t_i} S(\tau) d\tau$$



A2. Discrete Problem

- Given a charged APD, the probability to avalanche:

$$A_i = 1 - e^{-\eta S_i \Delta t}$$

- Therefore, P increases only when:

- APD remains charged
- Avalanche happens

$$P_{i+1} - P_i = F_i A_i$$

- Note that

$$F_i = 1 - P_i$$

- Solve flux as

$$S_i = \frac{1}{\eta \Delta t} \ln\left(\frac{1 - P_i}{1 - P_{i+1}}\right)$$



A2. Discrete Problem

- Explanation of the discrete variables:
- P_i – In i -th step, the probability that APD is discharged.
Therefore $P_0 = 0$
- S_i – In i -th step, the photon flux (count per second, cps)
- η – Quantum efficiency, number of photoelectrons excited by one photon.

- P_i can be estimated by partial sum of range histogram.

